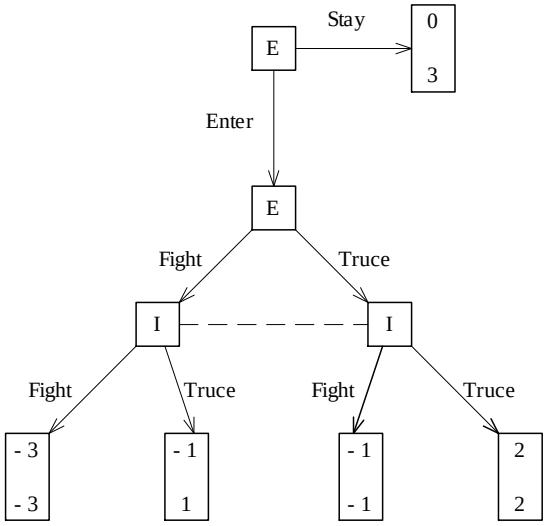
Game Theory: Problem Set II

1. [Gibbons 2.6] Three oligopolists operate in a market with inverse demand given by $P(Q) = A - Q$; where $Q = q_1 + q_2 + q_3$ and q_i is the quantity produced by firm i. Each firm has a constant marginal cost of production, c, and no fixed cost. The firms choose their quantities as follows: (1) firm 1 chooses $q_1 \geq 0$; (2) firms 2 and 3 observe q_1 and then simultaneously choose q_2 and q_3, respectively. What is the subgame-perfect outcome?

2. Consider the following extensive form game of entry deterrence (where in each final node, the first number is player E's payoff and the second number is player I's payoff):



(*a*) Describe the normal form of the game and find the pure-strategy Nash equilibria.

(*b*) What are the subgames?

(*c*) What is the subgame-perfect Nash equilibrium?

3. Five ferocious pirates are dividing their plunder (100 gold coins). They are ordered from 1 to 5, and suggests ways of sharing the coins as follows. Pirate 1 suggests a way of sharing the coins (where no single coin can be subdivided). For example, he might suggest $\{80, 5, 5, 5, 5\}$ meaning that he gets 80 coins and the others get 5 each. All five pirates then vote on the proposal. If a majority accept, the division is carried out. If the majority decline, then Pirate 1 is thrown overboard. Ties are broken in favour of the proposer. If there is agreement, the game ends. If Pirate 1 has been thrown overboard, we return to the first step, where Pirate 2 suggests a division, and so on. The process continues until an agreement is reached.

Explain carefully what happens on the path of the (unique) subgame-perfect equilibrium (assuming any voter who is indifferent (at equilibrium) between accepting or rejecting an offer votes to reject).