UNIVERSITY OF NAPLES "FEDERICO II"

DEPARTMENT OF STRUCTURAL ENGINEERING

MSc Design of Steel Structures

DESIGN OF CAR PARK IN COMPOSITE STEEL AND CONCRETE STRUCTURE

PART III
PRIMARY BEAMS

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6 PRIMARY BEAM

Composed beam is constituted by steel profile (HEA), concrete slab and Nelson connectors. Static scheme is a continuous beam on four support. Three spans measure 10.50, 14.00 e 10.50 m respectively. The primary beams spacing is 7.00 m.

Figure 9bis – Static scheme

Continuous composed beam presents forces lower than spin-spin one, in absolute value, and lower deformability too. Nevertheless it presents unfavourable effects due to hogging moment: in fact concrete slab is subjected to tension, so steel reinforcement is necessary in the slab; moreover steel profile is subjected to compression, so may have buckling problems. Furthermore the sagging bending moment resistance is greater than hogging bending moment resistance, while the design force are similar; then an excessive sagging bending moment resistance is obtained if an elastic method of beam design is used. In order to avoid this waste an elastic analysis with bending moment redistribution can be utilized. Such analysis have to take in account of forces redistribution too, due to concrete cracking, creep and shrinkage.

It is meaningful observe that the effective width is variable along the beam. When elastic global analysis is used, a constant effective width may be assumed over the whole of each span. This value may be taken as the value $b_{eff,1}$ at mid-span for a span supported at both ends, or the value $b_{eff,2}$ at the support for a cantilever (see figure 10). Instead, for the ULS (Ultimate Limit State) and SLS (Serviceability Limit State) checks, the effective width in each cross-section should be considered.

At mid-span or an internal support, the total effective width b_{eff}, see figure 10, may be determined as:

$$b_{eff} = b_0 + \sum b_{ei}$$

where:

b_0 is the distance between the centres of the outstand shear connectors;

b_{ei} is the value of the effective width of the concrete flange on each side of the web and taken as $L_e/8$ but not greater than the geometric width b_i. The value b_i should be taken as the distance from the outstand shear connector to a point mid-way between adjacent webs, measured at mid-depth of the concrete flange, except that at a free edge b_i is the distance to the free edge. The length L_e should be taken as the approximate distance between points of zero bending moment. For typical continuous composite beams, where a moment envelope from various load arrangements governs the design, and for cantilevers, L_e may be assumed to be as shown in figure 10.

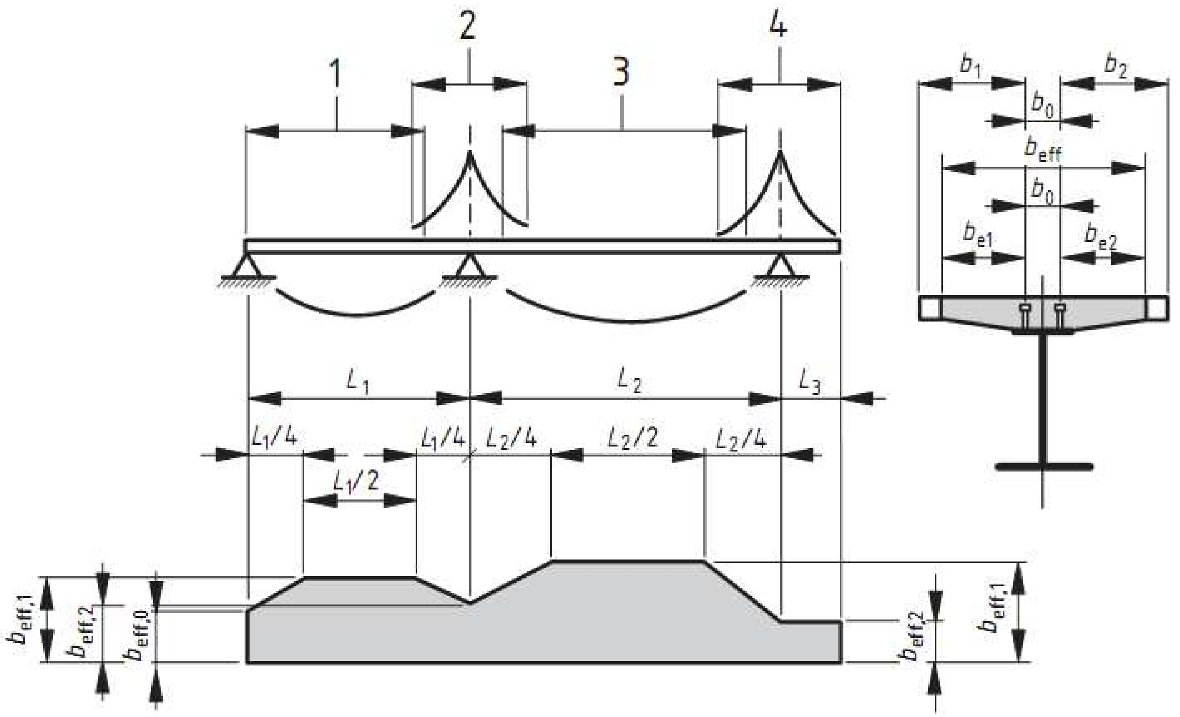


Figure 10 – Equivalent spans, for effective width of concrete flange

For the AB and CD spans, neglecting the b_0 distance between the centres of the outstand shear connectors:

$$l_{0,1} = 0.8 \cdot L_1 = 840.00cm \quad (6.1)$$

$$b_{eff,1}^{AB} = b_{e1} + b_{e2} = \frac{l_{0,1}}{8} + \frac{l_{0,1}}{8} = 210.00cm \quad (6.2)$$

For the BC spans, neglecting the b_0 distance between the centres of the outstand shear connectors:

$$l_{0,2} = 0.7 \cdot L_2 = 980.00cm \quad (6.3)$$

$$b_{eff,2}^{BC} = b_{e1} + b_{e2} = \frac{l_{0,2}}{8} + \frac{l_{0,2}}{8} = 245.00cm \quad (6.4)$$

In order to simplify the calculation, the evaluation of the sagging bending moment resistance is conducted with the minimum effective width:

$$b_{eff}^{+} = \min\left\{b_{eff,1}^{AB}, b_{eff,2}^{BC}\right\} = 210.00cm \tag{6.5}$$

The effective width at the supports B and C is:

$$l_0 = 0.25 \cdot \left(L_1 + L_2\right) = 612.50cm \tag{6.6}$$

$$b_{eff}^{-} = b_{e1} + b_{e2} = \frac{l_0}{8} + \frac{l_0}{8} = 153.13cm \tag{6.7}$$

The primary beam is assumed uniformly propped during the concrete casting.

6.1 Loads

The construction phase is not considered because during this phase there are loads lower than during the final stage. The loads on the composite beam are own weight, slab weight, live load and snow load. The distributed loads on the composite beam are obtained by multiplying the value of the distributed loads on the slab and the primary beams spacing. The actual load on the primary beam is represented by concentrated forces in correspondence of the joints connecting primary and secondary beams. Considering an uniform distributed load on primary beam allows to simplify calculations in a conservative mane because in such way the bending effect on primary beam related to the concentrated load in correspondence of the columns is also considered.

Loads			
Self weight of the profiled steel of the primary beam	Pt	?	*[kN/m]*
Characteristic value of structural permanent loads	G_{k1}	21.01	*[kN/m]*
Characteristic value of non structural permanent loads	G_{k2}	30.45	*[kN/m]*
Live load (car park)	Q_{k1}	17.50	*[kN/m]*
Snow load	Q_{k2}	4.20	*[kN/m]*

The self weight of the profiled steel of the primary beam is not known in this phase. A supposed value equals to the self weight of the HE500B (1.55kN/m) can be assumed.

Load combinations are:

- for Ultimate Limit State

$$F_{d1,\max}^{SLU} = \gamma_{G1} \cdot G_{k1} + \gamma_{G2} \cdot G_{k2} + \gamma_Q \cdot \left(Q_{k1} + \psi_{02} \cdot Q_{k2}\right) = \\ = 1.3 \cdot 21.01 + 1.5 \cdot 30.45 + 1.5 \cdot (17.5 + 0.7 \cdot 4.20) = 103.65 kN/m \quad (6.7)$$

$$F_{d1,\min}^{SLU} = \gamma_{G1} \cdot G_{k1} = 1 \cdot 21.01 = 21.01 kN/m \quad (6.8)$$

- for Seviceability Limit State

$$F_{d2,\max}^{SLE,rara} = G_{k1} + G_{k2} + Q_{k1} + \psi_{02} \cdot Q_{k2} = 21.01 + 30.45 + 17.5 + 0.7 \cdot 4.20 = 71.90 kN/m \quad (6.9)$$

$$F_{d2,\min}^{SLE,rara} = G_{k1} + G_{k2} = 51.46 kN/m \quad (6.10)$$

$$F_{d3,\max}^{SLE,q.p.} = G_{k1} + G_{k2} + \psi_{21} \cdot Q_{k1} + \psi_{22} \cdot Q_{k2} = \\ = 21.01 + 30.45 + 0.6 \cdot 17.50 + 0 \cdot 4.25 = 61.96 kN/m \quad (6.11)$$

$$F_{d2,\min}^{SLE,q.p} = G_{k1} + G_{k2} = 51.46 kN/m \quad (6.12)$$

6.2 Design of the cross-section

Static scheme is a continuous beam on four supports; therefore different load combinations maximizing sagging and hogging bending moment have to be considered.

Linear elastic analysis with limited redistribution may be applied to continuous beams and frames for checks at limit states. The bending moment distribution given by a linear elastic global analysis may be redistributed in a way that satisfies equilibrium and takes account of the effects of inelastic behaviour of materials, and all types of buckling. The bending moments in composite beams determined by linear elastic global analysis may be modified by reducing maximum hogging moments by amounts not exceeding the percentages given in Table 5.1, EN1994-1-1.

The analysis in which the internal forces and moments for the characteristic combinations is given using the flexural stiffness $E_a \cdot I_1$ of the un-cracked sections, including long-term effects, is defined as "un-cracked analysis". The analysis in which the internal forces and moments for the characteristic combinations is given considering the concrete cracking is defined as "cracked analysis".

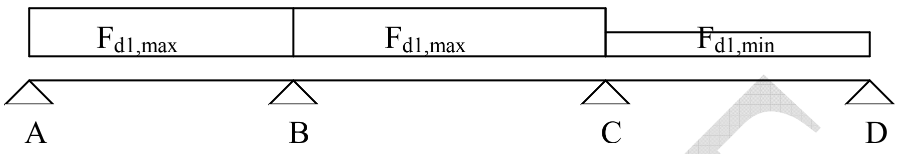
Table 5.1 : Limits to redistribution of hogging moments, per cent of the initial value of the bending moment to be reduced

Class of cross-section in hogging moment region	1	2	3	4
For un-cracked analysis	40	30	20	10
For cracked analysis	25	15	10	0

We have to determinate higher design sagging bending moment and hogging one.

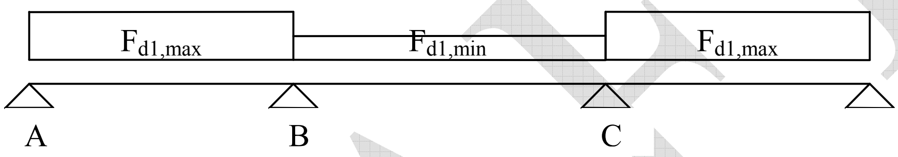
Load combination for higher hogging bending moment on both pin B and C, due to the symmetry of the scheme, is:

Condition 1



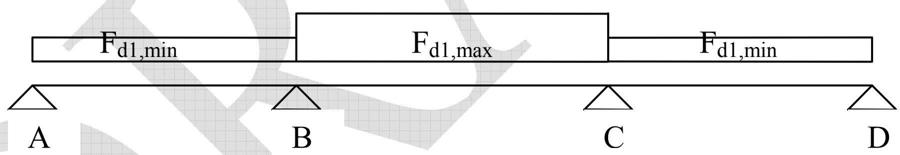
Load combination for higher sagging bending moment in lateral spans AB and CD is:

Condition 2



Load combination for higher sagging bending moment in the central span BC is:

Condition 3



So, adopting a constant inertia along the continuous beam, the hogging bending moments on the spins for the three combination are:

Elastic analysis – Design bending moment		
Condition 1		
Bending Moment in B	[kN m]	-1756.68
Bending Moment in C	[kN m]	-1073.35
Condition 2		
Bending Moment in B	[kN m]	-704.98
Bending Moment in C	[kN m]	-704.98
Condition 3		
Bending Moment in B	[kN m]	-1225.20
Bending Moment in C	[kN m]	-1225.20

Known highest sagging moment on the spins by means of an elastic analyses, in order to design the beam, a coefficient of redistribution δ=0.7, higher or equal to 0.6 present in table 5.1 of EC4-1-1, is assumed. Multiplying higher negative moment for this redistribution coefficient:

Bending moment ridistrubution – δ=0.70		
Condition 1		
Bending Moment in B	[kN m]	-1229.68
Bending Moment in C	[kN m]	-1229.68
Condition 2		
Bending Moment in B	[kN m]	-704.98
Bending Moment in C	[kN m]	-704.98
Condition 3		
Bending Moment in B	[kN m]	-1225.20
Bending Moment in C	[kN m]	-1225.20

Is opportune note that for conditions load 2 and 3 the values of hogging bending moment is lower than the value of the hogging bending moment for combination load 1 reduced by the redistribution coefficient δ. So for these load combinations a redistribution coefficient δ=1 is considered. If for a load combination, elastic moments on spin are higher than the hogging bending moment for combination load 2 reduced for the redistribution coefficient, it mans that we assumed a redistribution coefficient δ' between 1 and δ.

Based on such values of the hogging moment on the spins, corresponding sagging moment in the spans can be obtained as follows.

Condition 1:

shear in A (kN)	427
null point of shear diagram in span AB (m)	4.12
moment in span AB (kN*m)	**879.79**
shear in B (right) (kN)	726
null point of shear diagram in span BC (m)	7.00
moment in span BC (kN*m)	**1309.83**
shear in C (right) (kN)	227
null point of shear diagram in span CD (m)	10.80
moment in span CD (kN*m)	**1.1**

Condition 2:

shear in A (kN)	477
null point of shear diagram in span AB (m)	4,60
moment in span AB (kN*m)	**1097.73**
shear in B (right) (kN)	147
null point of shear diagram in span BC (m)	7,00
moment in span BC (kN*m)	**-190.14**
shear in C (right) (kN)	611
null point of shear diagram in span CD (m)	5.90
moment in span CD (kN*m)	**1097.73**

Condition 3:

shear in A (kN)	-6
null point of shear diagram in span AB (m)	0.3
moment in span AB (kN*m)	**0.96**
shear in B (right) (kN)	726
null point of shear diagram in span BC (m)	7,00

moment in span BC (kN*m)	**1314.30**
shear in C (right) (kN)	227
null point of shear diagram in span CD (m)	10.8
moment in span CD (kN*m)	**0.96**

Maximum sagging bending moment is: ***1314.3 kNm*** in order to design section in span.

Maximum hogging bending moment is: ***-1229.68 kNm*** in order to design section on spin.

6.2.1 Design of the cross-section to sagging bending moment

Effective width section that must be considered is: b_{eff}=2,10m.

Section height is defined as $h_{tot}=h_a+h_t$ (steel metallic height + composed slab height):

$$h_{tot} = \left[\frac{L}{20};\frac{L}{16}\right] \quad \Rightarrow \quad h_{tot} = [700;875]\ mm$$

where L is highest span length.

$h_t=120mm$, so steel profile height $(h_a=h_{tot}-h_t)$ is included between 580 mm and 755 mm.

Known highest sagging bending moment, area of steel profile can be obtained assuming that neutral axes is in correspondence of superior edge of steel profile.

$$M_{Sd}^{+} = A_a \cdot f_{yd} \cdot \left(\frac{h_a}{2} + h_p + \frac{h_c}{2}\right) = 1314.30\ kN \cdot m$$

$$A_a = \frac{M_{Sd}^{+}}{f_{yd} \cdot \left(\frac{h_a}{2} + h_p + \frac{h_c}{2}\right)} = \frac{131430 kN \cdot cm}{26.19 \,{}^{kN}\!/_{cm^2} \cdot \left(\frac{58cm}{2} + 5.5cm + \frac{6.5}{2}cm\right)} = 132.94 cm^2$$

But for translation equilibrium steel profile area must be lower than:

$$A_a \le \frac{b_{eff} \cdot h_c \cdot f_{cd}}{f_{yd}} = \frac{2100 \cdot 75 \cdot 11.75}{261.9} = 70.66\ cm^2$$

Area obtained by rotation equilibrium is higher than this one, then neutral axes is in the profile. In safety may be assumed that:

$$M_{pl,rd} = M_{pl,a} + b_{eff} \cdot h_c \cdot f_{cd} \cdot \left(\frac{h_c}{2} + h_p\right)$$ where h_p is the height of rib.

Imposing that $M_{sd,r}^{+} = M_{pl,rd}$, $M_{pl,a}$ (plastic bending moment resistance of the profiled steel) can be obtained:

$$M_{pl,a} = M_{sd,r}^{+} - b_{eff} \cdot h_c \cdot f_{cd} \cdot \left(\frac{h_c}{2} + h_p\right) = 131430 kN \cdot cm - 210 cm \cdot 117.5 \,{}^{kN}\!/_{cm^2} \cdot 6.5 cm \cdot \left(\frac{6.5cm}{2} + 5.5cm\right) = 1271.96 kN \cdot m$$

Then moment of area of cross-section, or the resistance plastic modulus, can be obtained and steel profile section can be chosen.

$$M_{pl,a} = 2 \cdot S_x \cdot f_{yd} \Rightarrow S_x = \frac{M_{pl,a}}{2 \cdot f_{yd}} = \frac{1271960000 N \cdot mm}{2 \cdot 261.9 N/mm^2} = 2428.33 cm^3$$

$$W_{pl,x} = 2 \cdot S_x = 2 \cdot 2428.33 cm^3 = 4856.66 cm^3$$

In the following table there are reported the geometric and static characteristics of the chosen profiled steel.

Geometric and static characteristics of HE500B			
Height	h_a	500	*[mm]*
Area	A_a	23860.0	*[mm²]*
Flange width	b_f	300.00	*[mm]*
Flange thickness	t_f	28.00	*[mm]*
Web thickness	t_w	14.50	*[mm]*
Web height	h_w	444.00	*[mm]*
Web height less root radius lenght	$h_{w,rid}$	390.00	*[mm]*
Root radius	r	27.00	*[mm]*
Root radius area	A_r	156.44	*[mm²]*
Second moment of area about x-x axis	J_x	1072000000	*[mm⁴]*
Second moment of area about y-y axis	J_y	126200000	*[mm⁴]*
Radius of gyration about x-x axis	ρ_x	201.20	*[mm]*
Radius of gyration about y-y axis	ρ_y	72.70	*[mm]*
Resistance modulus about x-x axis	$W_{el,x}$	4287000	*[mm³]*
Resistance modulus about y-y axis	$W_{el,y}$	841600	*[mm³]*
Resistance plastic modulus about x-x axis	$W_{pl,x}$	4815000	*[mm³]*
Resistance plastic modulus about y-y axis	$W_{pl,y}$	1292000	*[mm³]*
Moment of area about x-x axis	S_x	2407500	*[mm³]*

6.2.2 Design of the reinforcement to hogging bending moment

The plastic bending moment resistance of the profiled steel is:

$$M_{pl,a} = W_{pl,x} \cdot f_{yd} = 1261.05 kNm$$

This value is higher than the hogging bending moment reduced for the redistribution coefficient. For this, the reinforcement in the concrete slab is used only for limiting the concrete cracking.

For the limitation of crack width, the general considerations of EN 1992-1-1, 7.3.1(1) - (9) apply to composite structures. The limitation of crack width depends on the exposure classes according to EN 1992-1-l. As a simplified and conservative alternative, crack width limitation to acceptable width can be achieved by ensuring a minimum reinforcement defined in 7.4.2, EN1994-1-1, and bar spacing or diameters not exceeding the limits defined in 7.4.3, EN1994-1-1. According to 7.4.2, EN1994-1-1, unless a more accurate method is used in accordance with EN 1992-1-1, 7.3.2(1), in all sections without pre-stressing by tendons and subjected to significant tension due to restraint of imposed deformations (e.g. primary and secondary effects of shrinkage), in combination or not with effects of direct loading the required minimum reinforcement area As for the slabs of composite beams is given by:

$$A_s = k_s \cdot k_c \cdot k \cdot f_{ct,eff} \cdot \frac{A_{ct}}{\sigma_s} = 11.74 cm^2 \qquad (6.54)$$

where:

k_s is a coefficient which allows for the effect of the reduction of the normal force of the concrete slab due to **initial cracking and local slip of the shear connection**, which may be taken as 0,9;

k_c is a coefficient which takes account of the **stress distribution** within the section immediately prior to cracking, it can be assumed equals to 1;

k is a coefficient which allows for the effect of **non-uniform self-equilibrating stresses** which may be taken as 0,8;

$f_{ct,eff}$ is the mean value of the tensile strength of the concrete effective at the time when cracks may first be expected to occur. Values of $f_{ct,eff}$ may be taken as those for f_{ctm}:
f_{ctm}= 0.3*fck^(2/3);

A_{ct} is the **area of the tensile zone** (caused by direct loading and primary effects of shrinkage) immediately prior to cracking of the cross section. For simplicity the area of the concrete section within the effective width may be used.

σ_{st} is the **maximum stress** permitted in the reinforcement immediately after cracking. This may be taken as its characteristic yield strength f_{sk}. A lower value, **depending on the bar size**, may however be needed to satisfy the required crack width limits. This value is given in Table 7.1 EN1994-1-1.

Table 7.1 : Maximum bar diameters for high bond bars

Steel stress σ_s (N/mm^2)	Maximum bar diameter ϕ^* (mm) for design crack width w_k		
	w_k=0,4mm	w_k=0,3mm	w_k=0,2mm
160	40	32	25
200	32	25	16
240	20	16	12
280	16	12	8
320	12	10	6
360	10	8	5
400	8	6	4
450	6	5	-

$f_{ct,eff} = f_{ctm}$ 0.26kN/cm^2
$A_{ct} = b^{-}_{eff} * h_c$ 995.3125cm^2
σ_{st} 32kN/cm^2 (table 7.1 - w_k=0.3 mm - Φ=10)

$A_{s(min)}=k*k_c*f_{ct,eff}*A_{ct}/\sigma_{st}$ 5.74cm^2

Where at least the minimum reinforcement given by 7.4.2, EN1994-1-1 is provided, the limitation of crack widths to acceptable values may generally be achieved by limiting bar spacing or bar diameters. Maximum bar diameter and maximum bar spacing depend on the stress in the reinforcement and the design crack width. Maximum bar diameters are given in Table 7.1 and maximum bar spacing in Table 7.2, EN1994-1-1.

In this case the maximum bar spacing is 100mm. Therefore, we use 15ϕ10 (As = 11.7 cm^2 > $A_{s(min)}$) with spacing of 100mm into support effective width. The concrete gross cover c is equals to 2.5 cm (the net concrete cover is equal to 2.0 cm).

Table 7.2 Maximum bar spacing for high bond bars

Steel stress σ_s (N/mm^2)	Maximum bar spacing (mm) for design crack width w_k		
	w_k=0,4mm	w_k=0,3mm	w_k=0,2mm
160	300	300	200
200	300	250	150
240	250	200	100
280	200	150	50
320	150	100	-
360	100	50	-

6.3 Ultimate Limit State

6.3.1 Bending moment resistance

The design sagging bending resistance is determined by rigid-plastic theory. In case in which the plastic neutral axis is in the profiled steel web:

$$y_c = \frac{A_a \cdot f_{ad} - b_{eff} \cdot h_c \cdot f_{cd} - 2 \cdot t_f \cdot b_f \cdot f_{ad} - 4 \cdot A_r \cdot f_{ad}}{2 \cdot t_w \cdot f_{ad}} + (h_t + t_f) = 15.86 cm \qquad (6.13)$$

The hypothesis is correct. The sagging plastic bending moment resistance of the cross-section is:

$$M^+_{pl,Rd} = b_{eff} \cdot h_c \cdot f_{cd} \cdot \left(y_c - \frac{h_c}{2} \right) + 2 \cdot b_f \cdot t_f \cdot f_{ad} \cdot \left(y_c - h_t - \frac{t_f}{2} \right) +$$

$$+2 \cdot f_{ad} \cdot t_w \cdot \left(\frac{y_c - h_t - t_f}{2} \right)^2 + A_a \cdot f_{ad} \cdot \left(y_s + h_t - y_c \right) + 4 \cdot A_r \cdot f_{ad} \cdot (y_c - h_t - t_f - d_r) \qquad (6.14)$$

$$M^+_{pl,Rd} = 1632.86 kNm$$

Also the design hogging bending resistance is determined by rigid-plastic theory. In case in which the plastic neutral axis is in the profiled steel web:

$$y_c = \frac{A_a \cdot f_{ad} + 2 \cdot t_w \cdot \left(h_t + t_f \right) \cdot f_{ad} - 2 \cdot t_f \cdot b_f \cdot f_{ad} - 4 \cdot A_r \cdot f_{ad} - A_s \cdot f_{sd}}{2 \cdot t_w \cdot f_{ad}} = 32.94 cm \qquad (6.15)$$

The hypothesis is correct. The hogging plastic bending moment resistance of the cross-section is:

$$\left| M^-_{pl,Rd} \right| = +A_a \cdot f_{ad} \cdot \left(y_s + h_t - y_c \right) + 2 \cdot b_f \cdot t_f \cdot f_{ad} \cdot \left(y_c - h_t - \frac{t_f}{2} \right) + t_w \cdot f_{ad} \cdot \left(y_c - h_t - t_f \right)^2 +$$

$$+4 \cdot A_r \cdot f_{ad} \cdot \left(y_c - h_t - t_f - d_r \right) + A_s \cdot f_{sd} \cdot \left(y_c - c \right) = 1359.19 kNm \qquad (6.16)$$

6.3.2 Class section check

In the section subjected to hogging bending moment, the neutral plastic axis cuts the web of the profiled steel; lower flange and part of the web of the profiled steel are in compression and buckling problems are possible. So the class of steel profile must be verified.

The classification system defined in EN 1993-1-1, 5.5.2 applies to cross-sections of composite beams. The role of cross section classification is to identify the extent to which the resistance and rotation capacity of cross sections is limited by its local buckling resistance. According to EN1993-1-1, 5.5.2, four classes of cross-sections are defined, as follows:

- Class 1 cross-sections are those which can form a plastic hinge with the rotation capacity required from plastic analysis **without reduction of the resistance**;
- Class 2 cross-sections are those which can develop their plastic moment resistance, but have limited rotation capacity because of local buckling;

- Class 3 cross-sections are those in which the stress in the extreme compression fibre of the steel member assuming an elastic distribution of stresses can reach the yield strength, but local buckling is liable to prevent development of the plastic moment resistance;
- Class 4 cross-sections are those in which local buckling will occur before the attainment of yield stress in one or more parts of the cross-section.

The classification of a cross-section depends on the width to thickness ratio of the parts subject to compression (Compression parts include every part of a cross-section which is either totally or partially in compression under the load combination considered).

A composite section should be classified according to the least favourable class of its steel elements in compression. The class of a composite section **normally depends on the direction of the bending moment at that section**. A steel compression element restrained by attaching it to a reinforced concrete element may be placed in a more favourable class, provided that the resulting improvement in performance has been established.

For classification, the plastic stress distribution should be used except at the boundary between Classes 3 and 4, where the elastic stress distribution should be used taking into account sequence of construction and the effects of creep and shrinkage. For classification, design values of strengths of materials should be used. Concrete in tension should be neglected. The distribution of the stresses should be determined for the cross-section of the steel web and the effective flanges.

For the cross-section with class 1 (Table 5.2, EN1993-1-1):

$$\frac{h_{w,rid}}{t_w} \leq \frac{396 \cdot \varepsilon}{13 \cdot \alpha - 1} \Rightarrow \frac{39.00}{1.45} = 26.9 \leq 40.44 \qquad (6.18)$$

$$\frac{b_f/2 - r}{t_f} \leq 10 \cdot \varepsilon \Rightarrow \frac{30.00/2 - 2.7}{2.80} = 4.39 \leq 9.20 \qquad (6.19)$$

where:

h_{wc} is the **height of web in compression less root radius lenght** equals to $h_{wc}=h_a+h_t-y_{c(-)}-t_f-r = 23.56\ cm$;

$h_{w,rid}$ is **web height less root radius length**

α is a coefficient given by: $\alpha = \dfrac{h_{wc}}{h_{w,rid}} = 0.77$

ε $\varepsilon = \sqrt{\dfrac{235}{f_{ak}}} = 0.92$;

The cross-section is class 1.

6.3.3 Vertical shear resistance

The resistance to vertical shear $V_{pl,Rd}$ should be taken as the resistance of the structural steel section $V_{pl,a,Rd}$ unless the value for a contribution from the reinforced concrete part of the beam has been established. The design plastic shear resistance $V_{pl,a,Rd}$ of the structural steel section should be determined in accordance with EN 1993-1-1, 6.2.6:

$$V_{pl,Rd} = A_v \cdot \frac{f_{ad}}{\sqrt{3}} = 1357.57kN \quad (6.20)$$

The shear area is:

$$A_v = A_a - 2 \cdot b_f \cdot t_f + \left(t_w + 2 \cdot r\right) \cdot t_f = 89.78cm^2 \quad (6.21)$$

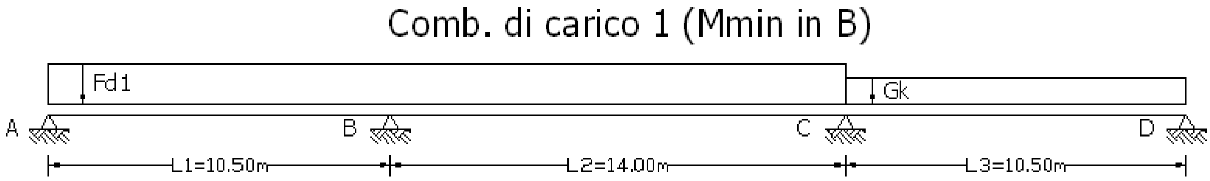


Figura 12 – Combinazione di carico massimizzante il taglio a destra dell'appoggio B.

The maximum design shear force, obtained in the case in which on the intenal span there is a plastic jogging moment resistance, is given by:

$$V_{sd} = \frac{F_{d1,\max}^{SLU} \cdot L_2}{2} = 725.0kN > 0.5 \cdot V_{pl,Rd} \quad (6.22)$$

Where the vertical shear force V_{Ed} exceeds half the shear resistance V_{Rd} given by $V_{pl,Rd}$ in whichever is the smaller, allowance should be made for its effect on the resistance moment (EN 1994-1-1,6.2.2.4). For cross-sections in Class 1 or 2, the influence of the vertical shear on the resistance to bending may be taken into account by a reduced design steel strength $(1-\rho)\cdot f_{yd}$ in the shear area where:

$$\rho = (2 \cdot \frac{V_{sd}}{V_{pl,Rd}} - 1)^2 = 0.004 \cong 0$$

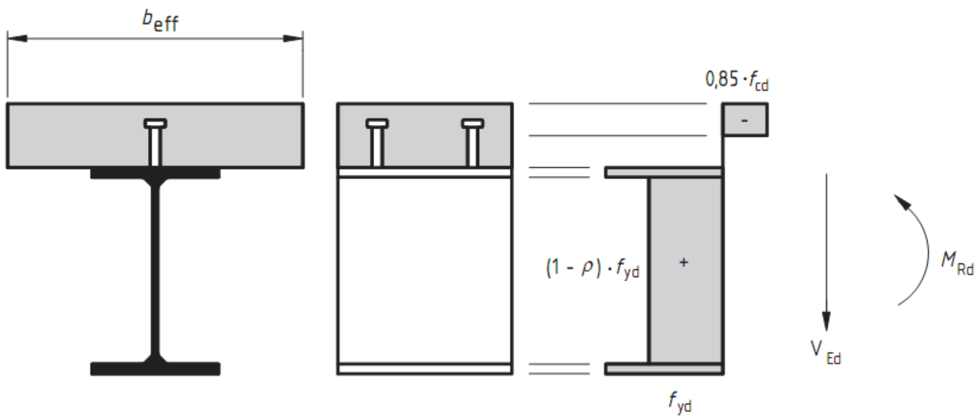


Figure 6.7 : Plastic stress distribution modified by the effect of vertical shear

Therefore the interaction bending and vertical shear can be neglecting.

6.3.4 Elastic analysis

After the design of cross-section the self weight of the profiled steel is known and also its second moment of inertia is known. Therefore it is necessary evaluating the design elastic bending moment at ULS by considering the effective width for each span. For simplicity, we supposed that the bending moment obtained from the previous analysis is correct.

6.3.4.1 Redistribution coefficient check

Total redistribution coefficient is the ratio of design hogging bending moment and bending moment resistance.

$$\delta = \delta_0 \cdot \delta_U = \frac{M^-_{Rd}}{\left|M^-_{Sd,el,\max}\right|} = \frac{1359.19}{1756.68} = 0.77 \qquad (6.23)$$

The redistribution of 23% is compatible with the value in table 5.1 EN1994-1-1 for the un-cracked analysis.

Table 5.1 : Limits to redistribution of hogging moments, per cent of the initial value of the bending moment to be reduced

Class of cross-section in hogging moment region	1	2	3	4
For un-cracked analysis	40	30	20	10
For cracked analysis	25	15	10	0

Furthermore total redistribution coefficient is also the product of the serviceability redistribution coefficient δ_E and the ultimate redistribution coefficient δ_U. Serviceability redistribution coefficient consider the redistribution due to the concrete cracking, viscosity and shrinking. In order to define δ_E, it is possible to utilize the procedure proposed by Prof. Nigro.

In order to consider concrete viscosity EM method (effective elastic modulus), without distinguishing between short and long term loads:

$$E_c^{'} = \frac{E_{cm}}{2} = 15100 N/mm^2 \quad (6.24)$$

$$n_{eff} = \frac{E_a}{E_c^{'}} = 13.91 \quad (6.25)$$

The position of the centroid and the inertia moment of the un-cracked section are:

$$y_{n1} = \frac{\frac{b_{eff}^{+} \cdot h_c}{n_{eff}} \cdot \frac{h_c}{2} + A_a \cdot \left(h_t + y_s\right)}{\frac{b_{eff}^{+} \cdot h_c}{n_{eff}} + A_a} = 27.16 cm \quad (6.26)$$

$$I_1 = \frac{1}{n_{eff}} \cdot \frac{b_{eff}^{+} \cdot h_c^3}{12} + \frac{b_{eff}^{+} \cdot h_c}{n_{eff}} \cdot \left(y_{n1} - \frac{h_c}{2}\right) + J_x + A_a \cdot \left(h_t + y_s - y_{n1}\right)^2 = 186760 cm^4 \quad (6.27)$$

The position of the centre of gravity and the inertia moment of the cracked section are:

$$y_{n2} = \frac{A_s \cdot c + A_a \cdot \left(h_t + y_s\right)}{A_s + A_a} = 35.92 cm \quad (6.28)$$

$$I_2 = A_s \cdot \left(y_{n2} - c\right)^2 + J_x + A_a \cdot \left(h_t + y_s - y_{n2}\right)^2 = 115989 cm^4 \quad (6.29)$$

Cracking moment value is:

$$M_{cr} = \frac{neff \cdot f_{cfd} \cdot I_1}{y_{n1}} = 137.34 kNm \quad (6.30)$$

Author relationship proposed in order to define δ_E in absence of shrinkage is:

$$\delta_E = \delta_0 = \left(\frac{I_2}{I_1}\right)^{\alpha} = \left(\frac{45580}{82169}\right)^{0.25} = 0.89 \quad (6.31)$$

where α depends from the length of the cracked part of the beam:

$$\alpha = 0.25 \cdot e^{\beta} = 0.25 \cdot e^{-0.02} = 0.25 \quad (6.32)$$

$$\beta = -2.85 \cdot \mu_{cr}^{2.15} = -2.85 \cdot 0.10^{2.25} = -0.02 \quad (6.33)$$

$$\mu_{cr} = \frac{M_{cr}}{M_{Sd,el,\max}^{-}} = \frac{86,72 kNm}{896.81 kNm} = 0.08 \quad (6.34)$$

Author relationship proposed in order to define $\delta_{E,\theta}$ in presence of shrinkage is:

$$\delta_{E,\vartheta} = \delta_0 \cdot (1 + k_{\vartheta} \cdot \overline{\vartheta}) = 0.89 \cdot \left(1 + 0.63 \cdot 0.09\right) = 0.94 \quad (6.35)$$

where:

$$\overline{\vartheta} = \frac{E_c^{'} \cdot S_c \cdot \varepsilon_{sh}}{M^{-}_{Sd,el,\max}} = \frac{15100 \cdot 32641 \cdot 3.30 \cdot 10^{-4}}{175668} = 0.09 cm^{-1} \qquad (6.36)$$

$$S_c = b_{eff}^{+} \cdot h_c \cdot \left(y_{n1} - \frac{h_c}{2} \right) = 210 \cdot 12 \cdot \left(27.16 - \frac{12}{2} \right) = 32641 cm^3 \qquad (6.37)$$

$$k_{\vartheta} = \left(0.60 + 0.532 \cdot \mu_{cr}\right) \cdot \left(\frac{I_2}{I_1} \right)^{0.496 \cdot \mu_{cr}} = \left(0.6 + 0.532 \cdot 0.1\right) \cdot \left(\frac{115989}{186760} \right)^{0.496 \cdot 0.08} = 0.63 \qquad (6.38)$$

Known δ_E, ultimate redistribution coefficient δ_U can be easily obtained by:

$$\delta_{U,0} = \frac{M^{-}_{Rd}}{\delta_E \cdot \left| M^{-}_{Sd,el,\max} \right|} = 0.87 \qquad (6.39)$$

$$\delta_{U,\vartheta} = \frac{M^{-}_{Rd}}{\delta_{E,\vartheta} \cdot \left| M^{-}_{Sd,el,\max} \right|} = 0.82 \qquad (6.40)$$

The ultimate redistribution coefficient δ_U for sections of class 1 is higher than 0.75 like EN1994-1-1 prescribes for the cracked analysis.

6.3.5 Bending moment resistance

Design forces redistribuited – U.L.S.					
	Sagging bending moment [kNm]			*Hogging bending moment [kNm]*	
	AB	BC	CD	B	C
Cond. 1	829.70	1180.31	8.7	-1359.19	-1359.19
Cond. 2	1097.73	190.14	1094.73	-704.98	-704.98
Cond. 3	0.96	1314.30	0.96	-1225.20	-1225.20

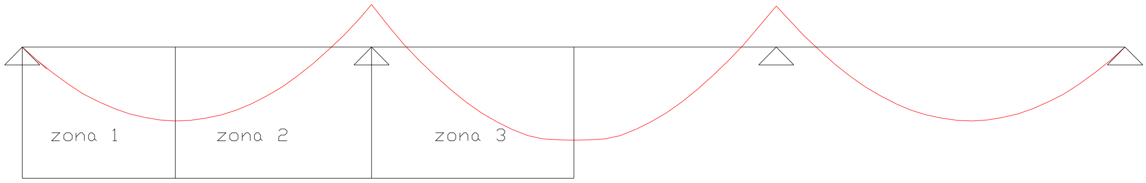
The maximum design sagging bending moment must be lower than the segging bendong moment resistance.

$$M^{+}_{sd} = 1314.30 kNm < M^{+}_{pl,Rd} = 1632.86 \Rightarrow \eta = 0.80 \qquad (6.41)$$

The check is ok.

6.3.6 Longitudinal shear resistance

Shear connection and transverse reinforcement shall be provided to transmit the longitudinal shear force between the concrete and the structural steel element, ignoring the effect of natural bond between the two. The critical length are indicated in the following figure.



The critical lenght are: L_{cr1} =460 cm, L_{cr2} =590 cm, L_{cr3} =700 cm.

The longitudinal shear force are:

$$V_{l1} = \min\left\{F_{cf,AB}^{cls}, F_{cf}^{acc.}\right\} = 1605kN \quad (6.42)$$

$$V_{l2} = \min\left\{F_{cf,AB}^{cls}, F_{cf}^{acc.}\right\} + A_s \cdot f_{sd} = 1912kN \quad (6.43)$$

$$V_{l3} = \min\left\{F_{cf,BC}^{cls}, F_{cf}^{acc.}\right\} + A_s \cdot f_{sd} = 2180kN \quad (6.44)$$

where:

$$F_{cf,AB}^{cls} = h_c \cdot b_{eff}^{AB} \cdot f_{cd} = 1605kN \quad (6.45)$$

$$F_{cf,BC}^{cls} = h_c \cdot b_{eff}^{BC} \cdot f_{cd} = 1873kN \quad (6.46)$$

$$F_{cf}^{acc.} = A_a \cdot f_{ad} = 6249kN \quad (6.47)$$

$$A_s \cdot f_{sd} = 307kN \quad (6.48)$$

The design shear resistance of a headed stud automatically welded in accordance with EN 14555 should be determined, for solid slab, from:

$$P_{Rd} = \frac{0{,}8 f_u \pi d^2 / 4}{\gamma_V}$$

or:

$$P_{Rd} = \frac{0{,}29 \alpha d^2 \sqrt{f_{ck} E_{cm}}}{\gamma_V}$$

whichever is smaller, with:

$$\alpha = 0{,}2\left(\frac{h_{sc}}{d} + 1\right) \quad \text{for } 3 \leq h_{sc}/d \leq 4$$

$$\alpha = 1 \quad \text{for } h_{sc}/d > 4$$

where:

- γ_V is the partial factor;
- d is the diameter of the shank of the stud, 16 mm $\leq d \leq$ 25 mm;
- f_u is the specified ultimate tensile strength of the material of the stud but not greater than 500 N/mm^2;
- f_{ck} is the characteristic cylinder compressive strength of the concrete at the age considered, of density not less than 1750 kg/m^3;
- h_{sc} is the overall nominal height of the stud.

For sheeting with ribs parallel to the supporting beams, the design shear resistance should be taken as the resistance in a solid slab, calculated as given by 6.6.3.1 (except that f_u should not be taken as greater than 450 N/mm^2) multiplied by the reduction factor k_l given by:

$$k_l = 0.6 \cdot \frac{b_{p,0}}{h_p} \cdot \left(\frac{h}{h_p} - 1 \right) = 1 \qquad (6.49)$$

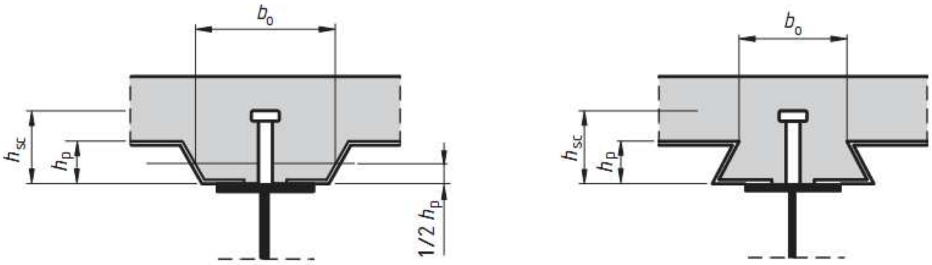


Figure 6.12 : Beam with profiled steel sheeting parallel to the beam

This provisions are satisfied for the case in exam. Therefore, the design shear resistance is:

$$P_{Rd,rid} = k_l \cdot P_{Rd} = 66kN \qquad (6.50)$$

The numbers of connectors for a complete shear connection are:

$$N_{f1} = \frac{V_{l1}}{P_{Rd,rid}} = 24 \qquad (6.51)$$

$$N_{f2} = \frac{V_{l2}}{P_{Rd,rid}} = 29 \qquad (6.52)$$

$$N_{f3} = \frac{V_{l3}}{P_{Rd,rid}} = 33 \qquad (6.53)$$