

1) Scrivere in forma algebrica i seguenti numeri complessi:

$$(3 + 2i)^2, \quad \frac{(2 + i)(2 - i)}{(3 - 2i)^2}, \quad \frac{1}{i(1 + i)^3},$$

Risposta:

$$5 + 12i, \quad \frac{25}{169} + \frac{60}{169}i, \quad -\frac{1}{4} + \frac{1}{4}i$$

2) Scrivere in forma trigonometrica i seguenti numeri complessi:

$$-3i, \quad -5, \quad \frac{1 + i}{1 - i}, \quad \sqrt{3} + i, \quad 1 - 3i, \quad -1 + 3i$$

Risposta:

$$\begin{aligned} -3i &= 3 \left(\cos \frac{3}{2}\pi + i \operatorname{sen} \frac{3}{2}\pi \right) \\ -5 &= 5 (\cos \pi + i \operatorname{sen} \pi) \\ \frac{1 + i}{1 - i} &= \left(\cos \frac{\pi}{2} + i \operatorname{sen} \frac{\pi}{2} \right) \\ \sqrt{3} + i &= 2 \left(\cos \frac{\pi}{6} + i \operatorname{sen} \frac{\pi}{6} \right) \\ 1 - 3i &= \sqrt{10} (\cos(-\arctg 3) + i \operatorname{sen}(-\arctg 3)) \\ -1 + 3i &= \sqrt{10} (\cos(\pi - \arctg 3) + i \operatorname{sen}(\pi - \arctg 3)) \end{aligned}$$

3) Calcolare:

$$(1 - i)^{11}, \quad (1 + i)^{20}, \quad \left(\frac{\sqrt{3} + i}{\sqrt{3} - i} \right)^3,$$

Risposta:

$$-32 - 32i, \quad -2^{10}, \quad -1$$

4) Calcolare:

- le radici quinte di $w = -2 + 2i$

Risposta:

$$z_k = \sqrt[5]{8} \left(\cos \frac{\frac{3}{4}\pi + 2k\pi}{5} + i \operatorname{sen} \frac{\frac{3}{4}\pi + 2k\pi}{5} \right) \quad k = 0, 1, 2, 3, 4$$

- le radici quarte di $w = -1 - \sqrt{3}i$

Risposta:

$$z_k = \sqrt[4]{2} \left(\cos \frac{\frac{4}{3}\pi + 2k\pi}{4} + i \operatorname{sen} \frac{\frac{4}{3}\pi + 2k\pi}{4} \right) \quad k = 0, 1, 2, 3$$

5) Risolvere in campo complesso le seguenti equazioni:

- $z^2 + 2z + 3 = 0$

Risposta: $z_1 = -1 + \sqrt{2}i$, $z_2 = -1 - \sqrt{2}i$

- $z^2 + 2iz - 3 = 0$

Risposta: $z_1 = \sqrt{2} - i$, $z_2 = -\sqrt{2} - i$

- $z^2 + 4i\bar{z} + 5 = 0$

Risposta: $z_1 = -i$, $z_2 = 5i$ $z_3 = \sqrt{7} - 2i$ $z_4 = -\sqrt{7} - 2i$

- $z + 3i + \operatorname{Re}(z)(i + \operatorname{Im}(z)^2) = 0$

Risposta: $z = -3i$

- $2z + 4i = \bar{z}(1 + \operatorname{Re}(z)^2 - \operatorname{Im}(z))$

Risposta: $z_1 = -i$, $z_2 = 4i$

- $4z = i|z|^2\bar{z}$

Risposta: $z_1 = 0$, $z_2 = \sqrt{2} + \sqrt{2}i$ $z_3 = -\sqrt{2} - \sqrt{2}i$