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Visualizing dipole radiation

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Abstract

The Hertzian dipole is fundamental to the understanding of dipole radiation. It provides basic insights into the genesis of electromagnetic waves and lays the groundwork for an understanding of half-wave antennae and other types. Equations for the electric and magnetic fields of such a dipole can be derived mathematically. However these are very abstract descriptions. Interpreting these equations and understanding travelling electromagnetic waves are highly limited in that sense. Visualizations can be a valuable supplement that vividly present properties of electromagnetic fields and their propagation. The computer simulation presented below provides additional instructive illustrations for university lectures on electrodynamics, broadening the experience well beyond what is possible with abstract equations. This paper refers to a multimedia program for PCs, tablets and smartphones, and introduces and discusses several animated illustrations. Special features of multiple representations and combined illustrations will be used to provide insight into spatial and temporal characteristics of field distributions—which also draw attention to the flow of energy. These visualizations offer additional information, including the relationships between different representations that promote deeper understanding. Finally, some aspects are also illustrated that often remain unclear in lectures.

Keywords: dipole radiation, Hertzian dipole, half-wave dipole, multimedia visualization, animations, illustrations

(Some figures may appear in colour only in the online journal)

1. Introduction

The dipole antenna is the simplest and most widely used type of antenna in radio and telecommunications. Understanding the Hertzian dipole is the first step toward understanding half-wave antennae and other types. The so called ‘ideal dipole’ was discussed in 1888 by Heinrich Hertz as a part of his pioneering investigations into radio waves.

The oscillating electric dipole is a central wave-producing mechanism in electromagnetism. It is a standard topic for lectures in physics, since it points to the fundamental mechanism that is responsible for electromagnetic radiation and waves. However, deriving the equations for the electric and magnetic fields is difficult. Moreover, it is not easy to explain the spatial and time-dependent behavior to physics students, in that way. Nevertheless, dipole radiation is an important topic for electrodynamic courses in mathematical and theoretical physics [1–3], as well as for bachelor university students during their first years in experimental physics [4, 5].

It can be quite instructive (and impressive) to visualize the time-dependent electric and magnetic fields that are created by a radiating dipole, and the development of electromagnetic waves. Suitable illustrations and animations are useful. The computer simulations presented here can provide additional instructive illustrations for university lectures on electrodynamics that broaden the experience well beyond what is possible with abstract equations.

The intention of this article and the corresponding computer application is to help in visualizing the characteristics of dipole radiation. The following text describes five specific illustrations focusing on various aspects (as well as combinations thereof). Corresponding animations can be created by the HTML5-applet ‘DipoleRadiation’. This app was written specifically to illustrate dipole radiation, and can even be run on smartphones. Thus, it can be used in lectures as well as for individual studies.

Special topics for the illustrations are:

- radiation pattern and structure of the electromagnetic field;
- detachment of field lines and the generation of closed loops;
- field lines of $\boldsymbol{E}$ and $\boldsymbol{H}$ in three dimensional pictures and animations;
- near field and far field, phase relationship between $\boldsymbol{E}$ and $\boldsymbol{H}$;
- flow of energy, illustrations with Poynting vectors;
- half wave dipole in contrast to a Hertzian dipole;
- electric field lines seen as contour lines of a potential function.

For devices with low computing power, a selection of pre-compiled video clips is also available.

There are many resources on the Internet that deal with dipole radiation. However, high resolution illustrations with field lines and scalable sections that permit differentiation between near field and far field are rare. Furthermore, combined illustrations, e.g. combinations of field lines and Poynting vectors to show the flow of energy, are hard to find. Often simulations are written in programming languages for specific operating systems, and are not available in HTML5. Thus, in most cases only animations (without the option to change parameter values) can be run on arbitrary devices. A selection of additional online resources with animations, which are helpful for lectures, can be found in [10–13].

2. Methods

2.1. Illustrations and multiple representations

Images can highlight different features of electromagnetism from what is captured by mathematical equations and can illustrate some of the unique characteristics of fields more clearly. Additionally, animations provide specific valuable insights into time-dependent processes. Moreover, beside a spatial and temporal overview, the combination of graphical

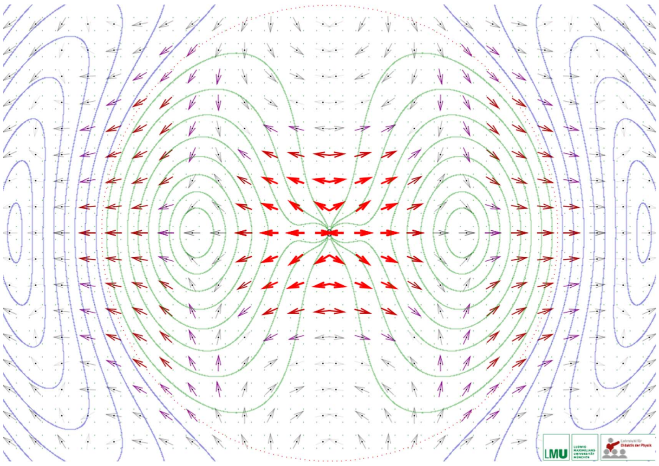

Figure 1. Snapshot showing electric field lines of a Hertzian dipole in combination with the flow of energy.

tools can focus on relations between different aspects, e.g. between a field distribution and its corresponding flow of energy (see figure 1 for an example).

From the perspective of multimedia theory, Ainsworth in 1999 identified various functions of computer illustrations for learning and understanding [6]. Multiple representations can '*provide complementary information*', '*constrain possible (mis)interpretations*' that arise from one illustration in the use of another, and, finally, help learners to '*construct a deeper understanding*' of a phenomenon [6]. Understanding can be aided by promoting abstraction from specific representations (which encourages generalization), and by teaching the relationships between different representations. Thus, various representations and also combinations of several images should be offered. Multiple visualizations help to highlight and to interrelate different aspects of the electromagnetic field formed by an oscillating electric dipole.

2.2. Dipole radiation (Hertzian dipole)

The *Hertzian dipole* is more of a theoretical construction than a physical antenna. It has an infinitesimal length, which means that the wavelength of the emitted electromagnetic radiation is much larger than the length of the antenna. The electromagnetic radiation from a Hertzian dipole can be calculated precisely by mathematical means. From this point, the superposition principle permits analytical or numerical calculation of the radiation from more complex antennae, based on the solution for the fields from a Hertzian dipole. Hence, becoming familiar with fundamental characteristics of the Hertzian dipole radiation is a first step.

Electric dipoles are characterized by their dipole moment, a vector quantity. Here, we examine an oscillating electric dipole, with a maximum dipole moment p_0 along the y-direction and a harmonic angular frequency ω. It can be described by the following time-dependent electric dipole moment

$$\vec{p}(t) = p_0 \cdot \hat{y} \cdot \sin \omega t. \tag{1}$$

As the dipole moment varies sinusoidally in magnitude, the electric field produced by the dipole also varies in magnitude. Furthermore, because the electric current changes in time, the magnetic field produced by the current varies in magnitude and direction. However, changes in the electric and magnetic fields do not come about everywhere instantaneously. Oscillations in these fields move away from the dipole at the speed of light (at least in the far field, see below) and with diminishing amplitude. Thus, the electric and magnetic fields depend on position and time.

In a vacuum, the field produced by this oscillating dipole can be derived exactly by using the retarded potential formulation (see textbooks on electrodynamics). The results for the electric field vector $\vec{E}$ and the magnetic field vector $\vec{H}$ can be described by the following equations (2) and (3)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(-\frac{\ddot{\vec{p}}}{c^2 \cdot r} - \frac{\dot{\vec{p}}}{c \cdot r^2} + \frac{3(\dot{\vec{p}} \cdot \vec{r}) \cdot \vec{r}}{c \cdot r^4} + \frac{(\ddot{\vec{p}} \cdot \vec{r}) \cdot \vec{r}}{c^2 \cdot r^3} + \frac{3(\vec{r} \cdot \vec{p}) \cdot \vec{r}}{r^5} - \frac{\vec{p}}{r^3}\right) \quad (2)$$

$$\vec{H} = \frac{1}{4\pi}\left(\frac{\ddot{\vec{p}} \times \vec{r}}{c \cdot r^2} + \frac{\dot{\vec{p}} \times \vec{r}}{r^3}\right); \quad \text{with } \vec{p} = \vec{p}\left(t - \frac{r}{c}\right). \quad (3)$$

Equations (2) and (3) are taken from [3, p 432].

Here, $\overrightarrow{p}$ represents the electric dipole moment, $\dot{\vec{p}}$ and $\ddot{\vec{p}}$ are the first and the second derivative of this quantity with respect to time, ε_0 is the electric permittivity of free space, c is the speed of light and $\vec{r}$ is the distance from the dipole.

2.3. Methods for calculating field lines

Electric field lines can be calculated using Euler's method, which can be understood even by freshmen: from an arbitrary starting point, a nearly infinitesimally short line is drawn in the direction of the field vector (according to equation (1)). From the position at the end of the line, the local field vector is computed anew, and again a very short line segment is drawn. This procedure is repeated until the starting point is reached again or the line ends up at the position of the dipole.

However, even when using very short line segments, this procedure will lead to systematic deviations. Therefore, a sort of Runge–Kutta-procedure is used here. Instead of one calculation for the direction of the field line at a certain point, four estimates of the slope (i.e. the calculation of the E_x/E and E_y/E components) are combined to gain more accuracy.

The first direction is given by E_{x1}/E and E_{y1}/E at the initial position (x_0, y_0, in Cartesian coordinates).

The next slope (E_{x2}/E and E_{y2}/E) results from a calculation of E at the position ($x_0 + E_{x1}/E \cdot d/2$, $y_0 + E_{y1}/E \cdot d/2$), where d is a very short distance (if converted into display coordinates it is even smaller than one pixel).

The third slope (E_{x3}/E and E_{y3}/E) comes from the field at the position ($x_0 + E_{x2}/E \cdot d/2$, $y_0 + E_{y2}/E \cdot d/2$), and the fourth slope (E_{x4}/E and E_{y4}/E) from ($x_0 + E_{x3}/E \cdot d$, $y_0 + E_{y3}/E \cdot d$).

Then a 'weighted sum' of these slopes is used to get the final estimate for the next point the field line runs through

$$x = x_0 + (E_{x1}/6 + E_{x2}/3 + E_{x3}/3 + E_{x4}/6)/E \cdot d; \text{ and} \quad (4)$$

$$y = y_0 + (E_{y1}/6 + E_{y2}/3 + E_{y3}/3 + E_{y4}/6)/E \cdot d. \quad (5)$$

The distance d is so small that the accuracy of the field lines is better than one pixel[1].

[1] One graphical test for this is to create field lines by starting from different positions and stepping into the opposite direction.

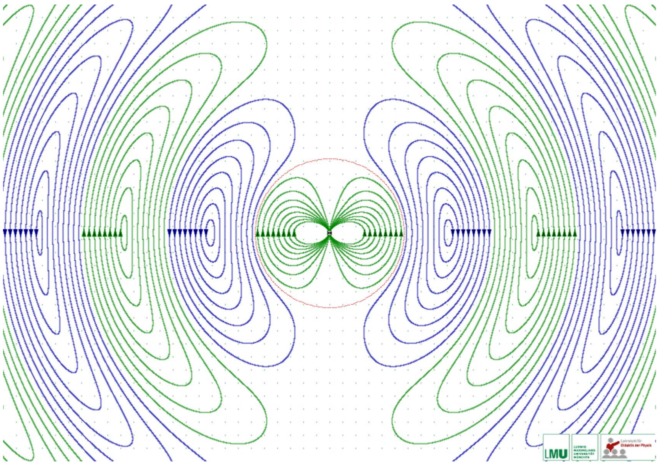

Figure 2. Snapshot showing electric field lines of a Hertzian dipole.

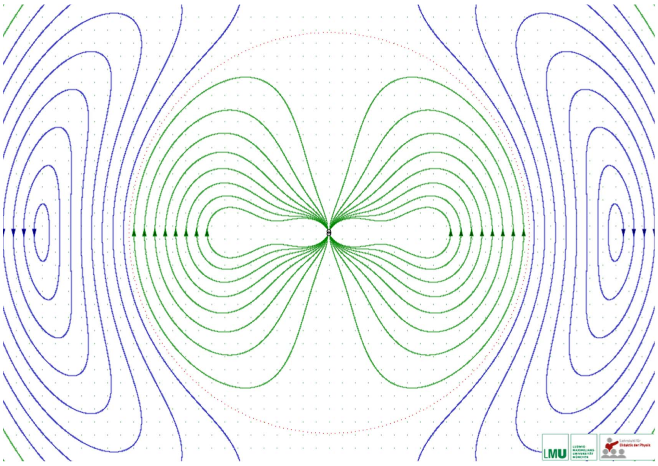

Figure 3. Snapshot showing electric field lines of a Hertzian dipole, focusing at the transition between near field and far field.

A completely different technique, which was also used by Heinrich Hertz, calculates a potential function where the field lines appear as ‘contour lines’. A corresponding image is shown and explained further below.

3. Results and application

3.1. Visualizing the field structure and radiation pattern

Common images of field lines are well known from various textbooks. A local electric field vector is always tangent to these field lines. In figures 2 and 3, the directions of the field lines are indicated by arrows, and different phases of the field are color-coded, indicating different directions of circulation. The animations show electric field lines due to a vertically oscillating electric dipole. In this article, only snapshots from a movie clip can be shown, but the

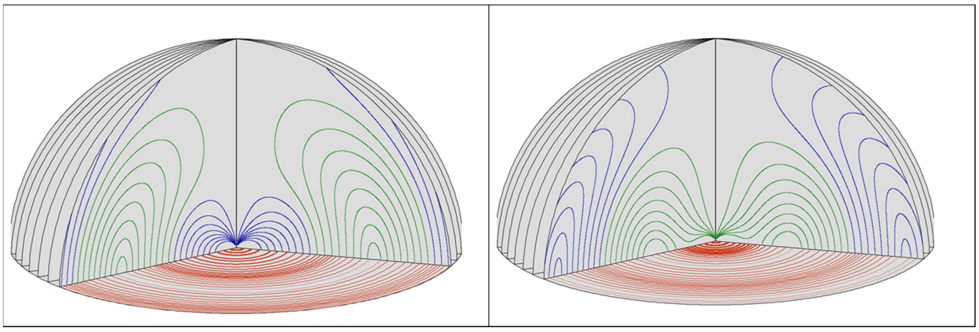

Figure 4. Electric field lines (blue and green) and magnetic field lines (red) at right angles, composing the electromagnetic wave radiated by the oscillating dipole.

animations show time-dependent spreading and progression of field properties more clearly than a series of pictures. When the oscillating dipole vector is at its maximum, the field lines next to the dipole are nearly those of a static dipole, emerging from a positive charge and ending up at a negative charge. However, at distances on the order of half a wavelength or greater, the field lines are completely detached from the dipole, characterizing a radiation field which propagates freely (without being attached to charges) in a vacuum at speed c.

The animation can be viewed either frame-by-frame with adaptable time intervals or by repeating in a loop.

3.2. 3D-view for E- and H-lines

Since we are examining a three-dimensional process, it is helpful to show three-dimensional representations as well. A spatial impression can be useful for future applications—such as the superposition of the radiation from two or more dipoles. 3D representations are also valuable in discussing the absorption of electromagnetic radiation as a function of the alignment of transmitter and receiver.

The radiation pattern of a vertical dipole is equal in all directions perpendicular to the axis of the antenna. In figure 4 two frames are shown, also indicating the magnetic field by H-lines in the equatorial plane.

The animated visuals illustrate that the magnetic field lines are concentric circles and always perpendicular to the electric field. Both of these fields are perpendicular to the direction in which the wave is travelling. The fields vary with the same frequency and in phase with each other in the far field, which can be seen when the display window is enlarged to 2λ or 4λ. The far field pattern shows a transverse electromagnetic wave, with electric and magnetic fields at right angles to each other and at right angles to the direction of propagation. The electric polarization is coplanar with the direction of the dipole vector.

3.3. Detachment of field lines, reversal of the dipole and zero-crossing of the electric field

To discuss the separation of field lines from the dipole, the strength of the electric field in the equatorial plane is plotted on top of the field lines in figure 5 as a line graph. In this illustration it is apparent that the field lines detach from the dipole when it reverses itself and thus the field strength directly beside the dipole has a zero-crossing.

This becomes more familiar by considering a simple dipole built up by two equal charges of opposite sign. These charges define the start and end points for electric field lines, and they

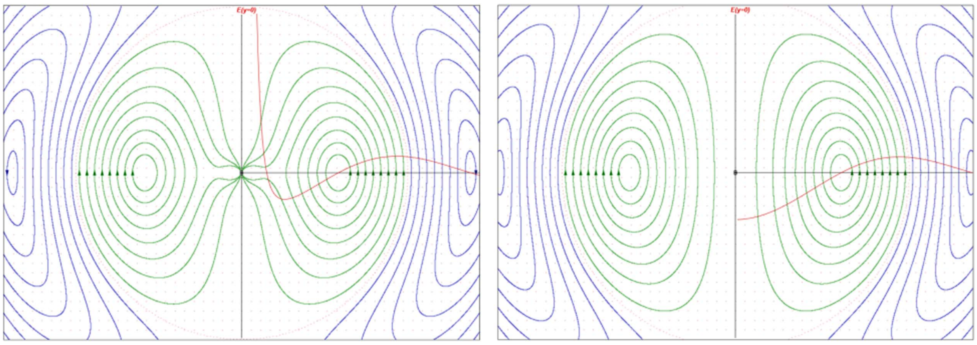

Figure 5. Drawing of field lines combined with a diagram to indicate the value of E in the equatorial plane.

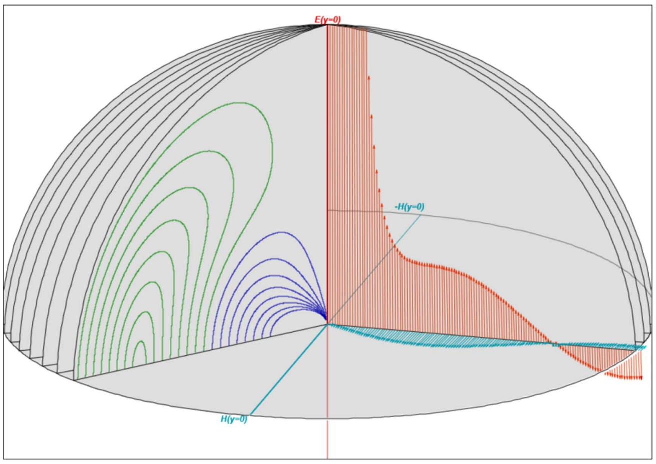

Figure 6. 3D plot of field lines, combined with a diagram to indicate the behavior of E and H in the equatorial plane.

oscillate up and down in alternating directions harmonically. As they approach one another at the midpoint of the system, the start and end points of the field lines coalesce. As the charges cross the midpoint, they neutralize one another, leaving no net charge to terminate electric field lines. Thus they form closed loops. Thereafter, the direction of the dipole vector as well as the direction of circulation of the field lines is reversed.

3.4. Electric and magnetic field—phase shift in the near field

The electric and magnetic fields near the oscillating dipole/antenna are quite different from the fields at greater distances. Close to the dipole, there is a phase shift between the alternating electric and magnetic field. When the electric field reaches its maximum the magnetic field becomes zero, and vice versa. Conversely, far away from the antenna, the electric and magnetic fields oscillate in phase with one another, perpendicular to each other and to the propagation of the electromagnetic wave. The strength of the electric and magnetic fields on the horizontal x-axis are indicated by line graphs (see figure 6). Thus, it can be seen that close

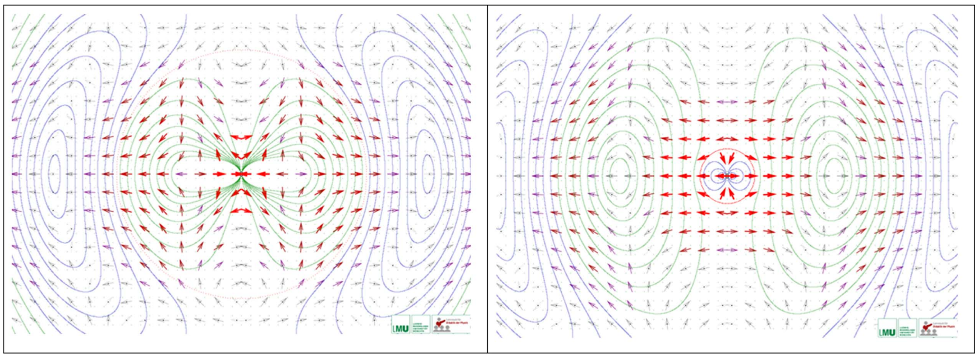

Figure 7. Poynting vectors at two different times.

to the dipole there is a phase shift of almost $\pi/2$ between the electric and the magnetic field. With increasing distance, the phase shift decreases more and more. In the far field the oscillations are in phase.

Why is the phase shift changing? To understand this, the following considerations and illustrations on the flow of energy will be useful.

3.5. Poynting vector and flow of energy

According to Poynting the directed energy flux density can be calculated by the cross product of E and H, using equation (6):

$$\vec{S} = \vec{E} \times \vec{H} \tag{6}$$

Figure 7 presents snapshots from an animation, displaying Poynting vectors in various positions. The thickness and colors indicate the magnitude of the Poynting vectors.

It should be clear that the directions of the vectors vary strongly with time especially in the near-field. These vectors are even transiently directed towards the dipole; this is due to the phase difference between E and H. For the horizontal plane the situation is quite simple: During periods when E and H have different signs, the Poynting vector has a negative sign, and energy flows back toward the dipole. This happens, for example, when the dipole vector is just building up again after a zero crossing. When the dipole vector is zero, the electric current is at its maximum, as is, therefore, the surrounding magnetic field. However, according to Lenz's law, the magnetic field and the electric current are linked. The electric current cannot collapse instantly when the oscillating electric charges pass their midpoint and the electric dipole disappears. The electric current is sustained and the energy for that is partially recovered from the surrounding field. Hence, the Poynting vectors briefly indicate a flow of energy back to the dipole. Due to the finite velocity of signal propagation, this is restricted to the near-field. There is no energy flow back towards the dipole in the far field and thus also the phase shift between E and H dissolves with increasing distance.

To visualize this, a representation is available which focuses on the situation in the equatorial plane (perpendicular to the dipole). The energy current densities are shown by arrows, and the values of E, H and S are plotted in line graphs as functions of time and distance from the dipole (see figure 8). Very near to the origin, the E and H fields are rather strong but almost 90° out of phase. The cross product of the electric and the magnetic field vectors indicates the direction in which the wave travels. Thus, the phase shift has a striking impact on the flow of energy. At certain times, when the sign of the Poynting vector is

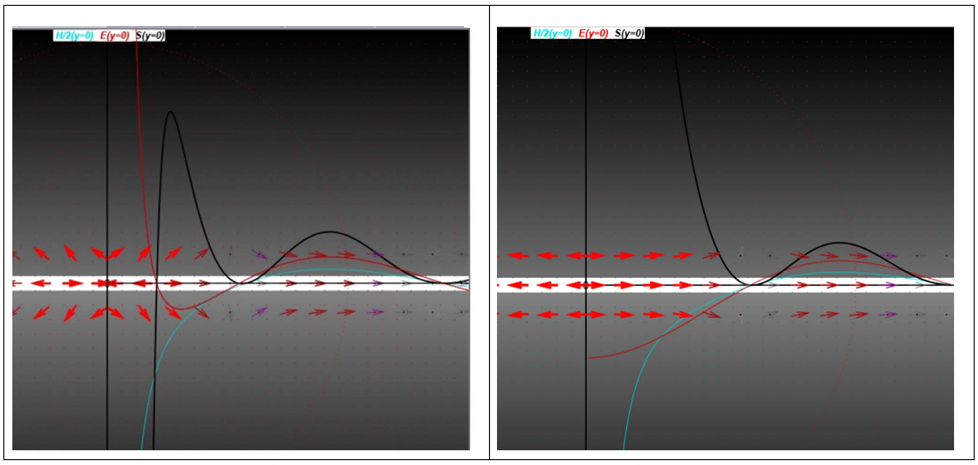

Figure 8. Magnitude of the Poynting vector S shown in a line graph over distance (black) together with the magnitudes of E (red) and H (light blue) in the equatorial plane. Furthermore, the flow of energy is indicated by arrows.

negative, energy is returned to the oscillating dipole. The animation is meant to make the interplay of several physical quantities clear.

It should be observed that more than one wavelength away from the dipole E and H oscillate almost in phase. Hence, a significant reverse flow of energy can be observed only in the near field. This can be seen even more clearly by displaying the Poynting vectors for a larger area in the program.

3.6. Half-wave dipole and electric-dipole radio antenna

The simplest and most common antenna is composed of two straight rods or wires aligned on the same axis. In this example, a sinusoidal alternating voltage is applied to the wires, causing electric charges in the antenna to oscillate. The antenna serves as a resonator, with standing waves of electrical current flowing back and forth between its ends. Thus, electric dipoles emerge and disappear periodically. The length of the wire or rod determines the wavelength of the electromagnetic radiation that is emitted. The whole antenna is one-half the wavelength of the radiation.

For the current, a standing wave is assumed, approximately sinusoidal along the length of the dipole, with a node at each end and maximum amplitude in the middle. Such a linear antenna may be treated as a series of very short conductors. Based on the superposition principle, a mathematical expression for the radiation of a linear antenna can be derived from the field equations of a Hertzian dipole (e.g. see [7, 8]).

At close ranges, there are significant differences between the wire example considered above and the Hertzian dipole. In the far field, the differences between the Hertzian dipole and the half-wave dipole are smaller. This can be shown using the dipole simulation program (see figure 9).

3.7. Electric field lines as contour lines of a potential function

Differential calculus makes it possible to derive a potential function, with the electric field lines being contour lines of this potential field [9]. The values of this potential function ϕ can

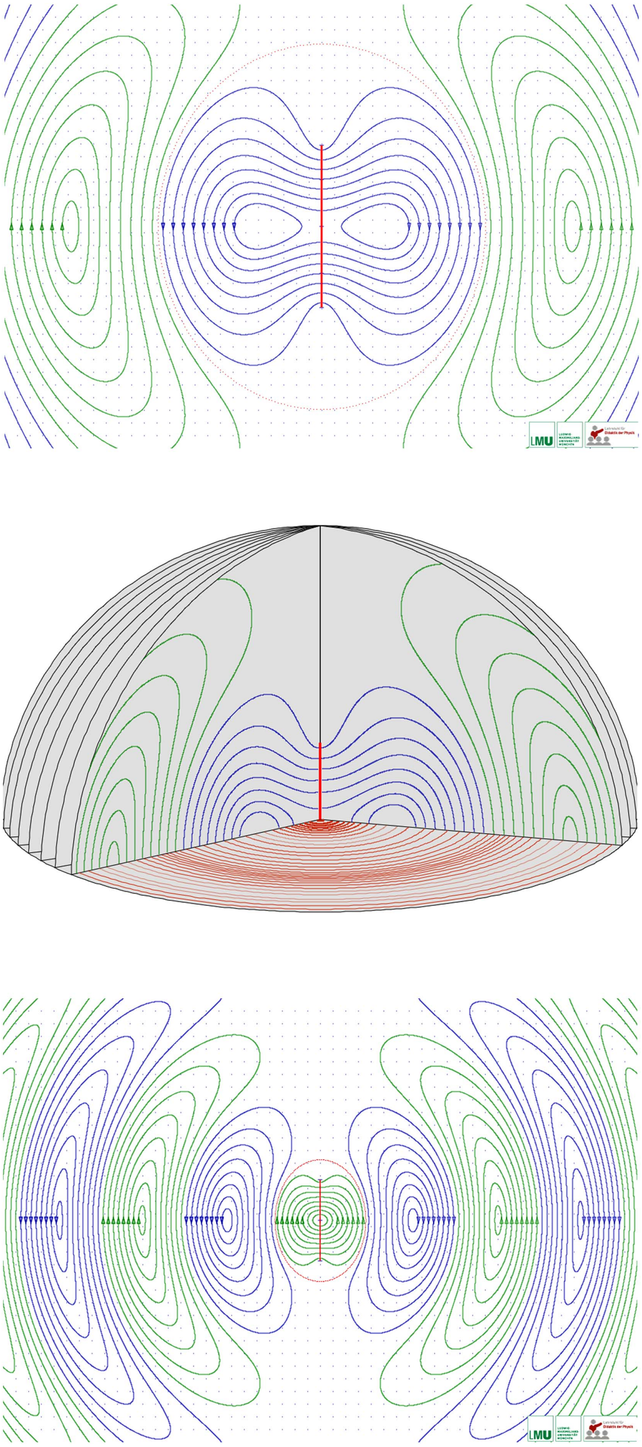

Figure 9. Snapshots from an animation showing momentarily fields of a radiating half-wave dipole.

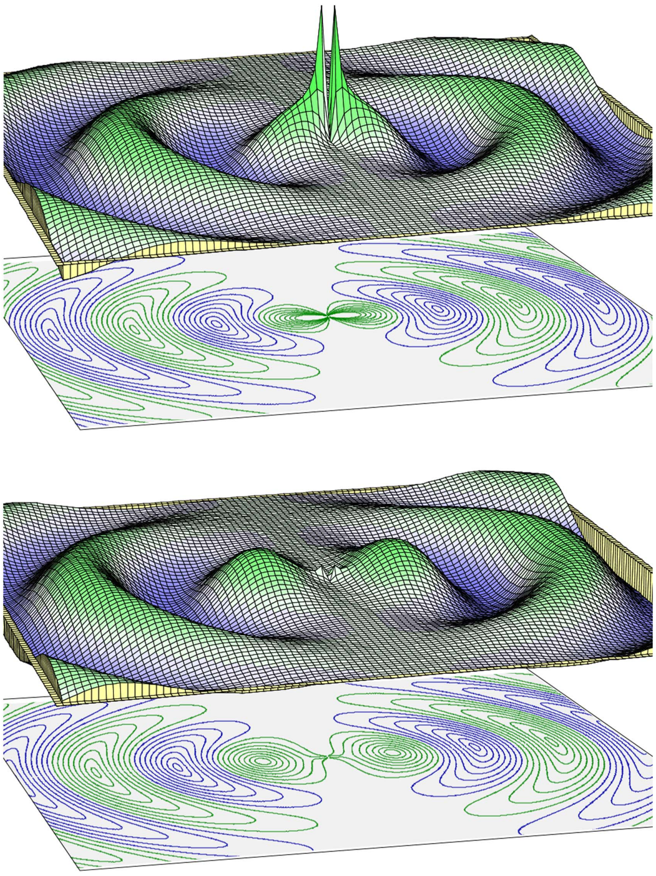

Figure 10. Snapshots showing the potential function and the corresponding lines of the electric field.

be computed using equation (7)

$$\phi = \frac{E_0}{k_0}\sin^2\vartheta\left[\frac{\cos(\omega t - k_0 r + \varphi_0)}{k_0 r} - \sin(\omega t - k_0 r + \varphi_0)\right]. \tag{7}$$

See [9, p 226], where r and ϑ describe the position in polar coordinates, ω and k_0 are the angular frequency and angular wave number, and E_0 and φ_0 are constants to represent the initial conditions.

In figure 10, snapshots from a corresponding animation are shown. This illustration can also offer a meaningful context to clarify when and why the field lines detach from the dipole and form closed loops.

Though these three-dimensional images can fascinate physicists and even laymen, caution is advised in order to avoid misinterpretation and misconceptions. The above quantities are computed only in the equatorial plane, with the values of the potential function shown as displacements perpendicular to this plane. This should not be confused with the behavior of a two-dimensional surface wave (such as, for example, a water wave). In this context, limits of spatial imagination should be considered.

4. Conclusions

The different animations above can be tools to vividly illustrate various aspects of electromagnetic dipole radiation. However, the animations should be used to highlight only the one or two specific aspects that they are designed to illustrate. Beyond this, these visualizations should be used with caution. They are theoretical constructs, used to illustrate certain properties and processes that are not visible in principle. Critical thinking is always required.

4.1. Field line drawings and their dynamic presentations

In general, it should be noted that field lines and field line images are theoretical constructs. They help to describe spatial distributions of physical quantities. However, assigning genuine mechanical properties to them is dangerous. This can be seen, for example, in the case of electromagnetic wave propagation and its representation in the form of 'travelling' field lines with specific velocities assigned to line sections.

It makes sense, in the far field, to take the zero crossings of the field strengths in the equatorial plane as points of reference that move at the speed of light. This can lead to a meaningful notion of signal propagation in the outer region (in the far field). However, in the near field, the waves are extremely inharmonic, and it would be wrong to associate a conventional speed to a field line segment. A rubber band, for example, would be a misleading analogy for field lines.

Nevertheless, there are strict limits (e.g. the dotted line in figure 2) on the areas of field lines with the same sense of circulation. For the Hertzian dipole, these are half spherical shells that are relatively easy to calculate. Their radius grows with the speed of light.

To summarize: modern media can help to illustrate many characteristics of dipole radiation. However, when using a tool such as the one described above, one should always take into account its limitations.

4.2. Computing dipole radiation

The program described above is a HTML5 application and runs on computers, tablets and smartphones. Ready-made animations (animated gifs) are also available to overcome a limited computing power (which is especially useful for the case of smartphones).

For more details and downloads see: http://www.en.didaktik.physik.uni-muenchen.de/multimedia/dipolstrahlung/index.html.

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