Figure di Lissajou $w_x = w_y \Rightarrow$ ellisse

$\frac{w_x}{w_y} = \frac{m}{n}$

$\frac{1}{2}$

Oscillazioni smorzate

smorzamento viscoso $\vec{f_d} = -\beta \vec{v}$

$m\ddot{x} = -Kx - \beta\dot{x}$ $\frac{K}{m} = \omega^2$

$\ddot{x} = -\omega^2 x - \Gamma\dot{x}$ $\frac{\beta}{m} = \Gamma$

$\ddot{x} + \Gamma\dot{x} + \omega^2 x = 0$

$\lambda^2 + \Gamma\lambda + \omega^2 = 0$ $\lambda_\pm = -\frac{\Gamma}{2} \pm \sqrt{\frac{\Gamma^2}{4} - \omega^2}$

$x(t) = A e^{\lambda_+ t} + B e^{\lambda_- t}$ $\lambda_+ - \lambda_- = \sqrt{\Gamma^2 - 4\omega^2}$

$\Gamma^2 > 4\omega^2$ sovrasmorzato

$x(t=0) = x_0 = A + B \leftarrow$

$\dot{x}(t=0) = \dot{x}_0 = A\lambda_+ + B\lambda_-$ —

$\lambda_+ x_0 = A\lambda_+ + B\lambda_+$ —

$(\lambda_+ - \lambda_-)B = \lambda_+ x_0 - \dot{x}_0$ $B = \frac{\lambda_+ x_0 - \dot{x}_0}{\lambda_+ - \lambda_-}$

$\lambda_- x_0 = A\lambda_- + B\lambda_-$

$A(\lambda_+ - \lambda_-) = \dot{x}_0 - \lambda_- x_0$ $A = \frac{\dot{x}_0 - \lambda_- x_0}{\lambda_+ - \lambda_-}$

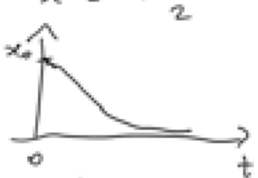
$$x(t) = e^{-\frac{\Gamma}{2}t}\left[\frac{\dot{x}_0 - \lambda_- x_0}{\lambda_+ - \lambda_-} e^{\frac{\lambda_+-\lambda_-}{2}t} + \frac{\lambda_+ x_0 - \dot{x}_0}{\lambda_+ - \lambda_-} e^{-\frac{\lambda_+-\lambda_-}{2}t}\right]$$

$$= e^{-\frac{\Gamma}{2}t}\left\{\frac{\dot{x}_0}{\lambda_+-\lambda_-}\underbrace{\left(e^{\frac{\lambda_+-\lambda_-}{2}t} - e^{-\frac{\lambda_+-\lambda_-}{2}t}\right)} + \frac{x_0}{\lambda_+-\lambda_-}\left(\lambda_+ e^{-\frac{\lambda_+-\lambda_-}{2}t} - \lambda_- e^{\frac{\lambda_+-\lambda_-}{2}t}\right)\right\}$$

$$= e^{-\frac{\Gamma}{2}t}\left\{\frac{2\dot{x}_0}{\lambda_+-\lambda_-}\operatorname{senh}\left(\frac{\lambda_+-\lambda_-}{2}t\right) + \frac{x_0}{\lambda_+-\lambda_-}\left[+\frac{\Gamma}{2}\, 2\operatorname{senh}\left(\frac{\lambda_+-\lambda_-}{2}t\right) + \frac{\lambda_+-\lambda_-}{2} e^{-\frac{(\lambda_+-\lambda_-)t}{2}} + \frac{\lambda_+-\lambda_-}{2} e^{\frac{\lambda_+-\lambda_-}{2}t}\right]\right\}$$

$$= e^{-\frac{\Gamma}{2}t}\left\{\frac{2\dot{x}_0}{\lambda_+-\lambda_-}\operatorname{senh} x + \frac{x_0\Gamma}{\lambda_+-\lambda_-}\operatorname{senh} x + x_0\cosh x\right\}$$

$$x = \frac{\lambda_+-\lambda_-}{2}t$$



$x(t) = 0$ ammette soluzione

$$\to x(t) = e^{-\frac{\Gamma}{2}t}\left\{\frac{x_0\Gamma + 2\dot{x}_0}{\lambda_+-\lambda_-}\operatorname{senh}\left(\frac{\lambda_+-\lambda_-}{2}t\right) + x_0\cosh\left(\frac{\lambda_+-\lambda_-}{2}t\right)\right\}$$

$$\operatorname{tgh}\left(\frac{\lambda_+-\lambda_-}{2}t\right) = -\frac{x_0(\lambda_+-\lambda_-)}{x_0\Gamma + 2\dot{x}_0}$$

$$\Gamma^2 = 8\omega^2 \qquad \sqrt{\Gamma^2 - 4\omega^2} = \sqrt{4\omega^2} = 2\omega = \lambda_+ - \lambda_-$$

$$x_0 > 0 \qquad \dot{x}_0 > 0 \Rightarrow \mathrm{tgh}\, x < 0 \Rightarrow t < 0$$

non ammette soluzione

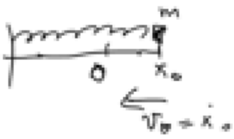
$$\dot{x}_0 < 0 \qquad 2\dot{x}_0 + x_0\Gamma < 0 \qquad \dot{x}_0 < -\frac{x_0\Gamma}{2}$$

$$\frac{\Gamma}{2} = \omega\sqrt{2} \qquad \dot{x}_0 < -\underline{\omega x_0\sqrt{2}}$$

$$\dot{x}_0 = N\omega x_0$$

$$\mathrm{tgh}\, x = -\frac{2x_0\omega}{\sqrt{2}x_0\omega + N\omega x_0} = -\frac{2}{N+\sqrt{2}}$$

$$N + \sqrt{2} < 0 \qquad N < -\sqrt{2}$$



Regime critico $\Gamma^2 = 4\omega^2 \qquad \lambda_+ = \lambda_- = -\frac{\Gamma}{2}$

$$x(t) = A e^{-\frac{\Gamma}{2}t} + B t e^{-\frac{\Gamma}{2}t}$$

$$x(0) = A = x_0 \qquad \dot{x}(0) = \dot{x}_0 = -\frac{\Gamma}{2}A + B$$

$$B = \dot{x}_0 + \frac{\Gamma}{2}x_0$$

$$x(t) = x_0 e^{-\frac{\Gamma}{2}t} + \left(\dot{x}_0 + \frac{\Gamma}{2}x_0\right) t e^{-\frac{\Gamma}{2}t}$$

Regime sottosmorzato

$$\Gamma^2 < 4\omega^2 \qquad \lambda_\pm = -\frac{\Gamma}{2} \pm \sqrt{\frac{\Gamma^2}{4} - \omega^2}$$

$$\omega^2 - \frac{\Gamma^2}{4} = \omega^{*2} \qquad \lambda_\pm = -\frac{\Gamma}{2} \pm i\omega^*$$

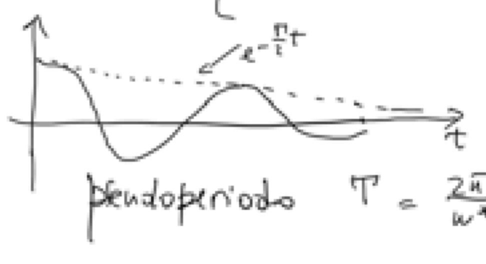
$$\frac{\lambda_+ - \lambda_-}{2} t = \frac{i\omega^* + i\omega^*}{2} t = i\omega^* t$$

$$\operatorname{sen} h\left(\frac{\lambda_+ - \lambda_-}{2} t\right) = \frac{e^{i\omega^* t} - e^{-i\omega^* t}}{2} = i \operatorname{sen} \omega^* t$$

$$\cos h\left(\frac{\lambda_+ - \lambda_-}{2} t\right) = \frac{e^{i\omega^* t} + e^{-i\omega^* t}}{2} = \cos \omega^* t$$

$$x(t) = e^{-\frac{\Gamma}{2}t}\left[\frac{x_0\Gamma + 2\dot{x}_0}{2i\omega^*} i \operatorname{sen} \omega^* t + x_0 \cos \omega^* t\right]$$

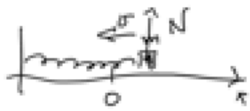
$$x(t) = e^{-\frac{\Gamma}{2}t}\left[x_0 \cos \omega^* t + \frac{\dot{x}_0 + x_0\Gamma/2}{\omega^*} \operatorname{sen} \omega^* t\right]$$



pseudoperiodo $T = \frac{2\pi}{\omega^*}$

Smorzamento radente

$$f_d = -\mu N \, \text{sign}(\dot{x})$$



$$m\ddot{x} = -Kx + f_d$$

$\dot{x} < 0 \qquad \text{sign}(\dot{x}) = -1 \qquad f_d = \mu m g$

$\ddot{x} = -\omega^2 x + \mu g \qquad \tilde{x} = \dfrac{\mu g}{\omega^2}$

$$x(t) = A\cos(\omega t + \varphi) + \frac{\mu g}{\omega^2}$$

$x(0) = A\cos\varphi + \dfrac{\mu g}{\omega^2} = x_0 \qquad \dot{x}(0) = 0$

$$\dot{x}(0) = -\omega A \,\text{sin}\,\varphi = 0 \Leftrightarrow \varphi = 0$$

$A + \dfrac{\mu g}{\omega^2} = x_0 \qquad A = x_0 - \dfrac{\mu g}{\omega^2}$

$$\rightarrow x(t) = \left(x_0 - \frac{\mu g}{\omega^2}\right)\cos\omega t + \frac{\mu g}{\omega^2} \qquad [t \leq t_1]$$

$\dot{x}(t_1) = 0 \qquad -\omega\left(x_0 - \dfrac{\mu g}{\omega^2}\right)\text{sen}\,\omega t_1 = 0$

$\omega t_1 = \pi \qquad t_1 = \dfrac{\pi}{\omega}$

$t > t_1 \Rightarrow \dot{x} > 0 \qquad \text{sign}(\dot{x}) = 1$

$$\ddot{x} = -\omega^2 x - \mu g$$

$$x(t) = B\cos(\omega(t-t_1)+\varphi_1) - \frac{\mu g}{\omega^2}$$

$$x(t_1) = \left(x_0 - \frac{\mu g}{\omega^2}\right)\cos\pi + \frac{\mu g}{\omega^2} = \frac{2\mu g}{\omega^2} - x_0$$

$$x(t_1) = B\cos\varphi_1 - \frac{\mu g}{\omega^2} = B - \frac{\mu g}{\omega^2}$$

$$\dot{x}(t_1) = -\omega B \,\text{sen}\,\varphi_1 = 0 \Rightarrow \varphi_1 = 0$$

$$B - \frac{\mu g}{\omega^2} = \frac{2\mu g}{\omega^2} - x_0 \Rightarrow B = \frac{3\mu g}{\omega^2} - x_0$$

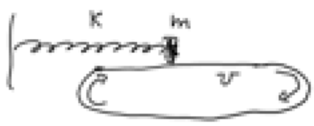
→ $t > t_1$ $\quad x(t) = \left(\frac{3\mu g}{\omega^2} - x_0\right)\cos\omega(t-t_1) - \frac{\mu g}{\omega^2}$

$$\text{sen}\left[\omega(t-t_1)\right] = 0 \qquad t - t_1 = \frac{\pi}{\omega}$$

$$t = t_1 + \frac{\pi}{\omega} = \frac{\pi}{\omega} + \frac{\pi}{\omega} = \frac{2\pi}{\omega}$$

$$K x_n \le \mu_s m g$$

Modello della corda di violino



$f_d \sim 0$

attrito statico

$$K(x - x_0) = K v t \le f_{s\,max} = f_s m g$$

$$x - x_0 = v t \qquad t < t_1 = \frac{m g f_s}{K v}$$

$$t > t_1 \Rightarrow m\ddot{x} = -K(x - x_0)$$

→ $x - x_0 = A\,\text{sen}\left[\omega(t - t_1) + \varphi\right]$

$t_1 \rightarrow \quad x - x_0 = v t_1 \Rightarrow x(t_1) = x_0 + v t_1$

$\dot{x}(t_1) = v$

$$x(t_1) = x_0 + A\,\text{sen}\,\varphi = x_0 + v t_1$$

$$A\,\text{sen}\,\varphi = v t_1 \qquad A\cos\varphi = \frac{v}{w}$$

$$\dot{x}(t_1) = A\,w\cos\varphi = v$$

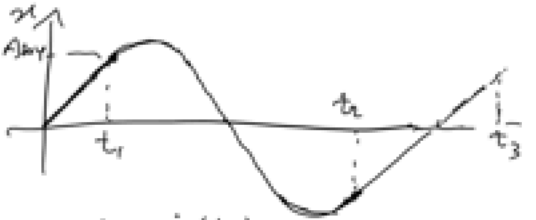
$$\underline{\text{tg}\,\varphi = w t_1} \qquad A = \sqrt{\frac{v^2}{w^2} + v^2 t_1^2}$$

$$= v\sqrt{t_1^2 + \frac{1}{w^2}}$$

$$t_2 \qquad \dot{x}(t_2) = v$$

$$\dot{x}(t_2) = A\,w\cos[w(t_2 - t_1) + \varphi] = v$$

$$v = \dot{x}(t_1) = A\,w\cos\varphi$$



$$\dot{x}(t_2) = \dot{x}(t_1)$$

$$A\,w\cos[w(t_2 - t_1) + \varphi] = A\,w\cos\varphi$$

$$\cos\beta = \cos\alpha \qquad \beta = 2\pi - \alpha$$

$$w(t_2 - t_1) + \varphi = 2\pi - \varphi$$

$$t_2 - t_1 = \frac{2\pi - 2\varphi}{w} = \frac{2}{w}(\pi - \varphi)$$

$$x(t_2) = x_0 + \frac{A}{\omega} \operatorname{sen}[\omega(t_2-t_1)+\varphi] \Rightarrow$$

$$x(t_2) - x_0 = A \operatorname{sen}[\omega(t_2-t_1)+\varphi] = -A \operatorname{sen}\varphi$$

$$\omega(t_2-t_1) = 2\pi - 2\varphi$$

$$2A \operatorname{sen}\varphi = v(t_3-t_2)$$

$$t_3 - t_2 = \frac{2A\operatorname{sen}\varphi}{v}$$

$$T = t_3 - t_1 = t_3 - t_2 + t_2 - t_1 =$$

$$T = \frac{2A\operatorname{sen}\varphi}{v} + \frac{2}{\omega}(\pi - \varphi) = \frac{2\pi}{\omega} + 2\underbrace{\left(\frac{A}{v}\operatorname{sen}\varphi - \frac{\varphi}{\omega}\right)}$$

$$\frac{A}{v}\operatorname{sen}\varphi - \frac{\varphi}{\omega} = \sqrt{t_1^2 + \frac{1}{\omega^2}}\operatorname{sen}\varphi - \frac{\varphi}{\omega}$$

$$= \frac{\sqrt{1+\omega^2 t_1^2}}{\omega}\operatorname{sen}\varphi - \frac{\varphi}{\omega} = \frac{\omega t_1}{\omega} - \frac{\varphi}{\omega} = t_1 - \frac{\varphi}{\omega}$$

$$\operatorname{tg}\varphi = \omega t_1 \qquad \varphi = \operatorname{arctg}(\omega t_1)$$

$$\operatorname{sen}\varphi = \frac{\operatorname{tg}\varphi}{\sqrt{1+\operatorname{tg}^2\varphi}} = \frac{\omega t_1}{\sqrt{1+\omega^2 t_1^2}}$$

$$t_1 - \frac{\varphi}{\omega} = t_1 - \frac{\operatorname{arctg}(\omega t_1)}{\omega} \sim 0$$

$$T \sim \frac{2\pi}{\omega}$$

nastro → archetto

corpo + molla → corda