Electricity

ELECTRIC CHARGE evidences

In an atom Protons and Neutrons costitute the nucleus. They are called nucleons and interact between each other mainly due to the strong nuclear force.

ATOMIC MODELS

Thomson (plum pudding model with point-like negative charges all around an extended region of positive charge) → Rutherford (planetary model with a positive charge in the centre, having very small spatial size, and negative point-like charges orbiting around)→ Bohr (quantized planetary model with allowed energy levels and electron stable orbits around the nucleus for selected values of the orbit radius)

An attractive force was measured between charge of the same «sign» and repulsive between charges of opposite «sign»: **ELECTRIC CHARGE of 2 «signs»**

CHARGE: QUANTIZATION – INVARIANCE (from a frame to another) – CONSERVATION

⇓

Electrostatic Interaction

The Coulomb Law

$$\boldsymbol{F}_e = K \frac{q_1 q_2}{r^2} \boldsymbol{e}_r$$

Analogies and differences with respect to the gravitational force between masses

$$\boldsymbol{F}_g = G \frac{m_1 m_2}{r^2} \boldsymbol{e}_r$$

ANALOGIES	DIFFERENCES
- Depend on the source product - Inversely proportional to the square of the distance - Directed along the joining straight line - Spherical symmetry (radial forces)	- intensity G $\sim$ 6,67 · 10^{-11} N· m^2/kg^2 K $\sim$ 8,99 · 10^9 N· m^2/C^2 The electrostatic one is much more intense! - Repulsive (the Coulomb force is both attractive and repulsive depending on cases , the Newton force only attractive)

Examples

Let us proove that:

$$F_g \ll F_e$$

In fact

Electron mass (e^-) = 9,11 · 10^{-31} kg
Proton mass (e^+) = 1,67 · 10^{-27} kg
Charge having equal absolute value | e+| = |e-| ≅ 1,6 · 10^{-19} C

$$|F_g| = G\frac{m_{e^+}.\, m_{e^-}}{r^2} = \frac{6{,}67 \cdot 10^{-11} \cdot (\,9{,}11 \cdot 10^{-31} \cdot 1{,}67 \cdot\ 10^{-27})}{r^2} = \frac{1{,}01 \cdot\ 10^{-67}}{r^2}$$

$$|F_e| = K\,\frac{e^2}{r^2} = \frac{9 \cdot\ 10^9 \cdot (\,1{,}6 \cdot\ 10^{-19})^2}{r^2} = \frac{2{,}3 \cdot\ 10^{-28}}{r^2}$$

$$|F_e| \sim 10^{39}\,|F_g|$$

The reason relies on the different values of the coupling universal constants, *K* and *G*, and on the values of the sources *q* (in C) and *m* (in kg). *q* is much larger than *m*!

The dielectric constant

***K* IN VACUUM:**

$$K = \frac{1}{4\,\pi\,\epsilon_0}$$

$\boldsymbol{\epsilon_0}$= dielectric constant of vacuum ~ 8,85 · 10^{-12} C^2/ N · m^2

***K* IN THE MEDIUM:**

$$K = \frac{1}{4\,\pi\,\epsilon_m} \rightarrow \epsilon_m = \epsilon_0 \epsilon_r$$

$\boldsymbol{\epsilon_r}$ ≥**1** => relative dielectric constant

RELATIVE DIELECTRIC CONSTANTS

Air = 1,00054
Water = 78,5
Paper = 3,5
Glass = 4,7

Electric and Gravitational fields

In analogy with the gravitational field we define the electric field:

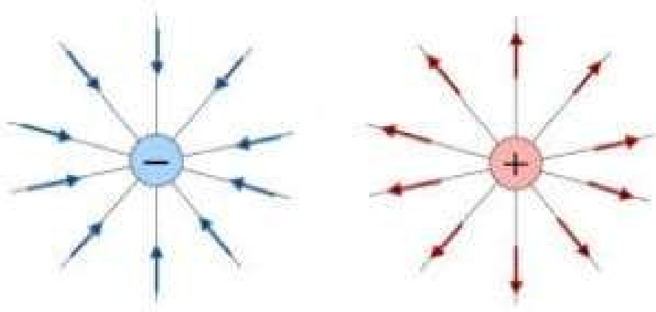
GRAVITAZIONAL FIELD: $\bar{g} = \frac{\bar{F}_g}{m}$

ELETTRICO FIELD: $\bar{E} = \frac{\bar{F}_e}{q}$

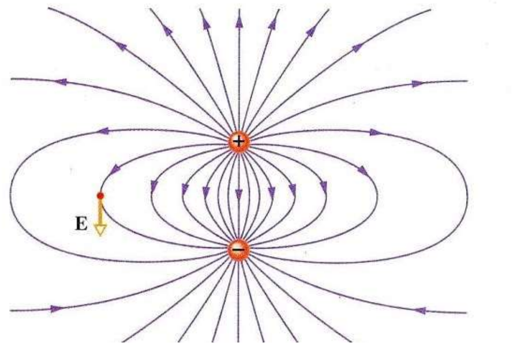
For POINT-LIKE CHARGES we have (q_0 being a test charge):

$$\bar{E} = \lim_{q_0 \to 0} \frac{\bar{F}_e}{q_0} = K \frac{q_1}{r^2} \bar{e}_r$$

The field of a point-like charge has spherical symmetry. We define the field lines as those oriented for which the field is tangent at any point.

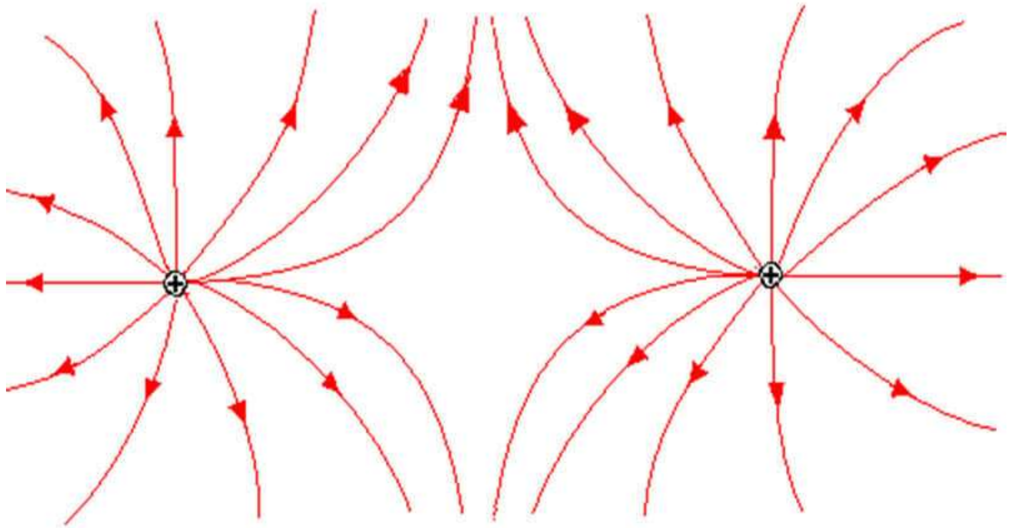


For 2 opposite charges (electric dipole) in vacuum we have:



Various electric field configurations

For two equal charges in vacuum we have:



The field is additive since is originates from the force. For *N* point-like charges we have: $\bar{E} = \sum_{i=1}^{N} \frac{q_i}{4\,\pi\,\epsilon_0} \frac{1}{{r_i}^2}\ \bar{e}_{r_i}$

In general charges are considered **«not necessarely» point-like**. The sum becomes then an integral.

Distributions of electric charges

CHARGES DISTRIBUTED IN A VOLUME

$$\bar{E} = \int \frac{dq}{r^2}\,\bar{e}_r = \int_V d(\boldsymbol{r})\frac{dV}{r^2}\,\bar{e}_r$$

$d(\boldsymbol{r})$ = density of volume charges

CHARGES DISTRIBUTED ON A SURFACE

$$\bar{E} = \int \frac{dq}{r^2}\bar{e}_r = \int_S \sigma(\boldsymbol{r})\frac{dS}{r^2}\bar{e}_r$$

$\sigma(\boldsymbol{r})$ = surface density of charges

CHARGES DISTRIBUTED ON A LINE

$$\bar{E} = \int \frac{dq}{r^2}\bar{e}_r = \int \lambda\,(\boldsymbol{r})\frac{dl}{r^2}\bar{e}_r$$

$\lambda\,(\boldsymbol{r})$ = linear density of charges

Electrostatic potential energy

$\bar{E}$ is a conservative field:

$$\oint \bar{F}_e \cdot d\bar{l} = 0$$

In fact, the Coulomb is conservative. Let's exhibit examples of forces and fields that are conservative and non-conservative.

OTHER CONSERVATIVE FIELD: **gravitational**
NON-CONSERVATIVE FIELD: **magnetic**
NON-CONSERVATIVE FORCE: **friction**

A conservative force can be seen as the «gradient» (derivative) of a quantity, *U*, which is a function of the spatial point and is called potential energy.

$$-\frac{dU}{dr} \equiv |\,F_e| \rightarrow -\int |F_e|\,dr = U$$

Thus:

$$-\oint |F|\,dr = \Delta U = 0$$

Potential energy of point-like charges

For point-like charges at a distance *r* from one another:

$$U = -\int |F|\, dr = \int K\,\frac{q_1 q_2}{r^2}\,dr$$

When possible, for the sake of simplicity the constant is set 0.

$$= \int_r^{\infty} K\,\frac{q_1 q_2}{r^2}\,dr = -K\,\frac{q_1 q_2}{r}\Big|_r^{\infty}$$

$$= K\,\frac{q_1 q_2}{r} + \text{constant}$$

The potential energy of two electric charges is negative if the two charges are oppostite in sign (as they attract each other) while it is positive if the two chaarges have the same sign (they repel). What is the physical meaning of this energy? It is the needed energy to break the system and infintely separate the two charges or, equivalently, the energy required to constitute the system in that confguration carrying the charges to the set spatial points from infinitely far away.

Electrostatic potential

$$V = \frac{U}{q_0}\bigg|_{q_0 \to 0} \qquad \Delta V = \frac{\Delta U}{q_0}\bigg|_{q_0 \to 0}$$

For **POINT-LIKE CHARGES:** $V = K\,\frac{q}{r}$ $\quad \Delta V_{12} = K\,q\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

$V(r)$ (is a scalar) = work to carry a unit charge from infinitely far away to the assigned point. It is measured in Volt (V). 1V=1J/1C

ΔV_{12} = is the work per unit charge, needed to bring a unit charge from 1-point (r_1 from the origin) to 2-point (r_2 from the origin) in the presence of q, which creates the electric field.

RELATIONS between $\bar{E}$ and V

$$\Delta V_{12} = \frac{\Delta U}{q_o} = -\int_{r_1}^{r_2} \frac{\bar{F}}{q_0} \cdot d\bar{r} = -\int_{r_1}^{r_2} \bar{E} \cdot d\bar{r} = \int_{r_2}^{r_1} \bar{E} \cdot d\bar{r}$$

In fact, for point-like charges we have: $\Delta V_{12} = \int_{r_2}^{r_1} K\,\frac{q}{r^2}$

Field flux

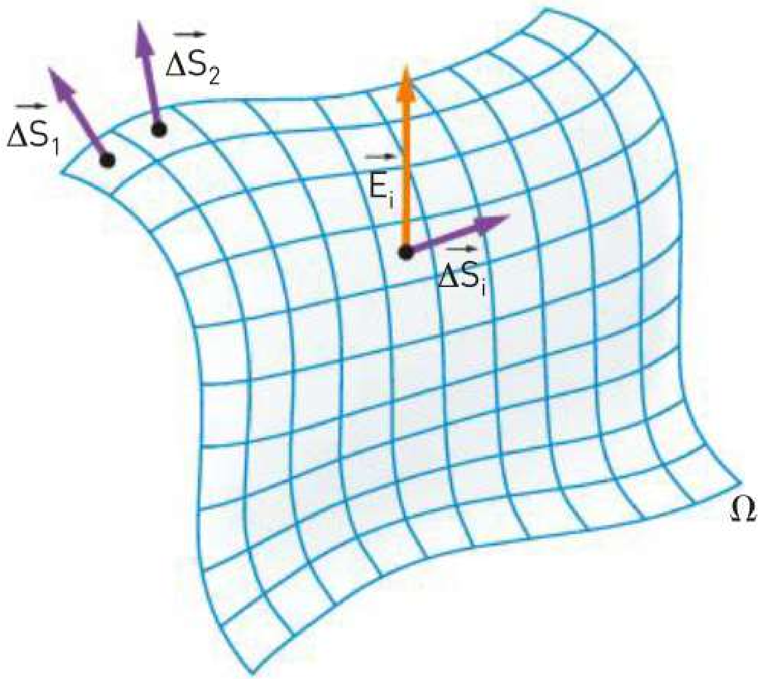
The field flux through a surface is defined as:

$$\phi_E = \int_{\Omega} \boldsymbol{E} \cdot \boldsymbol{n}\, dS$$

The flux is proportional to the number of field lines that cross a given surface Ω

$$\Delta\vec{S}_1 = \vec{n}_1 dS$$

$$\Delta\vec{S}_2 = \vec{n}_2 dS$$



In the IS the flux is measured in N·m^2/C.

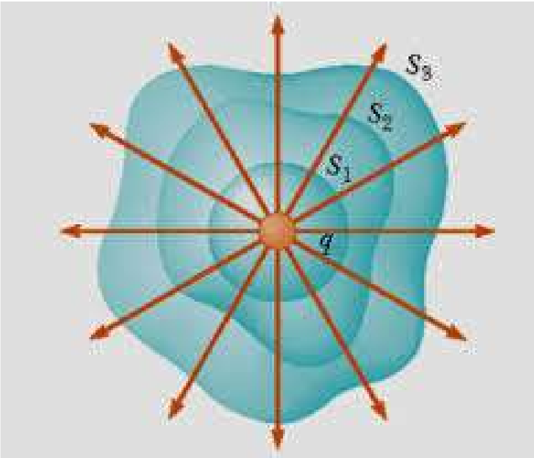
Gauss theorem

The field flux through any **closed** surface, Ω, is equale to the algebraic sum of the internal charges to that surface divided by the dielectric constant of the medium we are in. The external charges do NOT contribute to the flux through Ω.

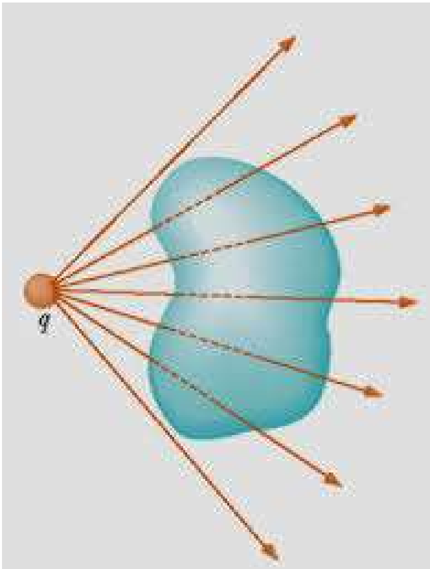
$$\Phi_E = \oint_{\Omega} \overline{E} \cdot \overline{n} dS = \frac{Q_{tot,i}}{\varepsilon_0}$$

PROOF (graphical)

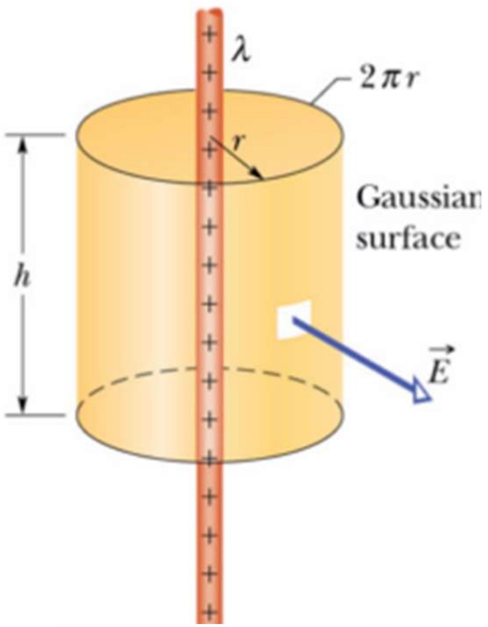
Internal charge: all the field lines that exit or enter the internal charge cross the closed surface in a single point and, therefore, they all contribute to the flux

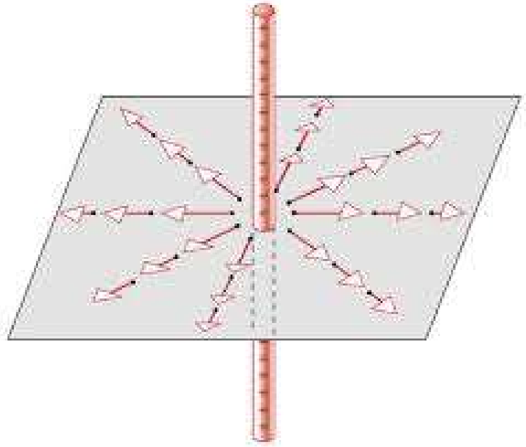


External charge: all the field lines that exit or enter the external charge do not cross the closed surface at all or they cross it twice, in two points, once entering and other time exiting, and therefore they all do NOT contribute to the flux



Applications of the Gauss theorem: charged wire and charged plate

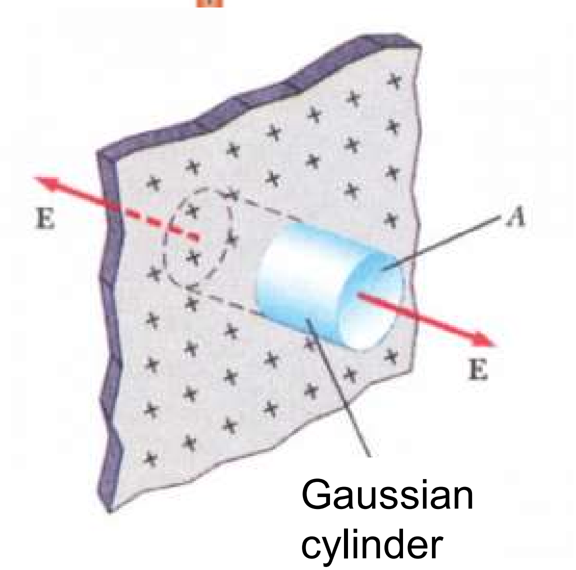




$$E \cdot S = \frac{q}{\epsilon_0}$$

$$q = \int \lambda \, dl = \lambda \, l$$

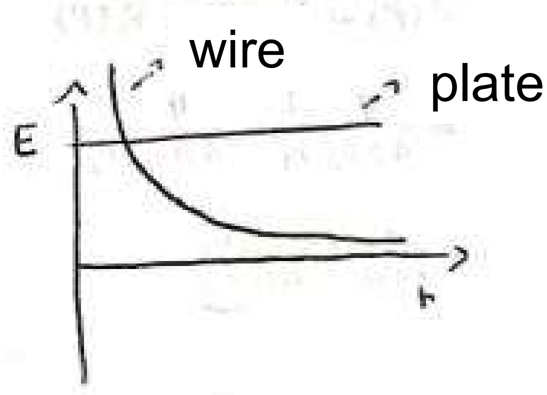
$$E \cdot 2\pi l r = \frac{\lambda \, l}{\epsilon_0} \rightarrow E(r) = \frac{\lambda}{2 \, \pi r \, \epsilon_0}$$



$$2S \cdot E = \frac{q}{\epsilon_0} \rightarrow SE = \frac{\sigma S}{2\epsilon_0}$$

$$E(constant) = \frac{\sigma}{2\epsilon_0}$$

$$q = \int \sigma \, dS = \sigma S$$



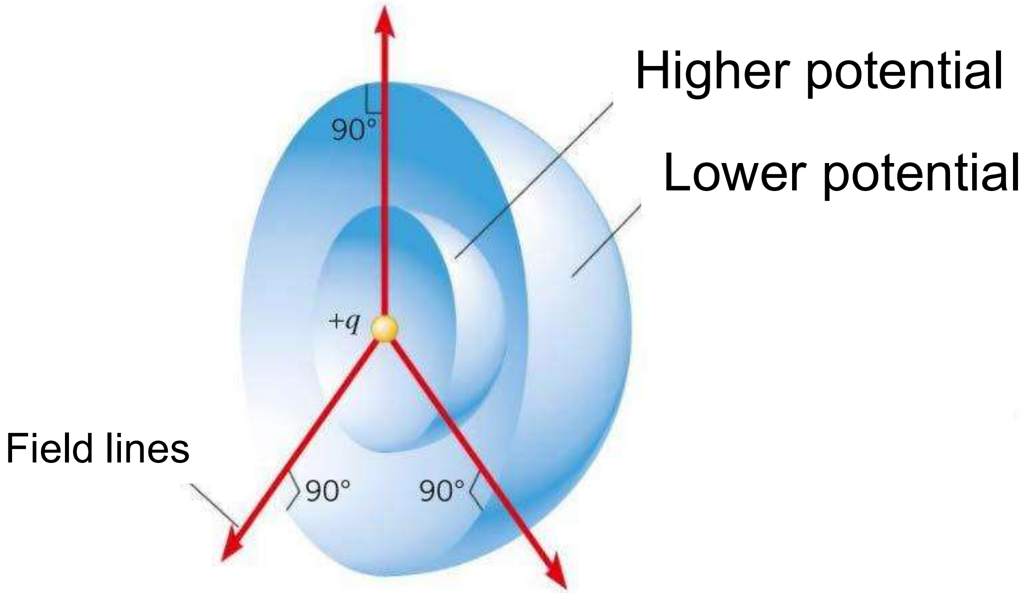
Equipotential surfaces

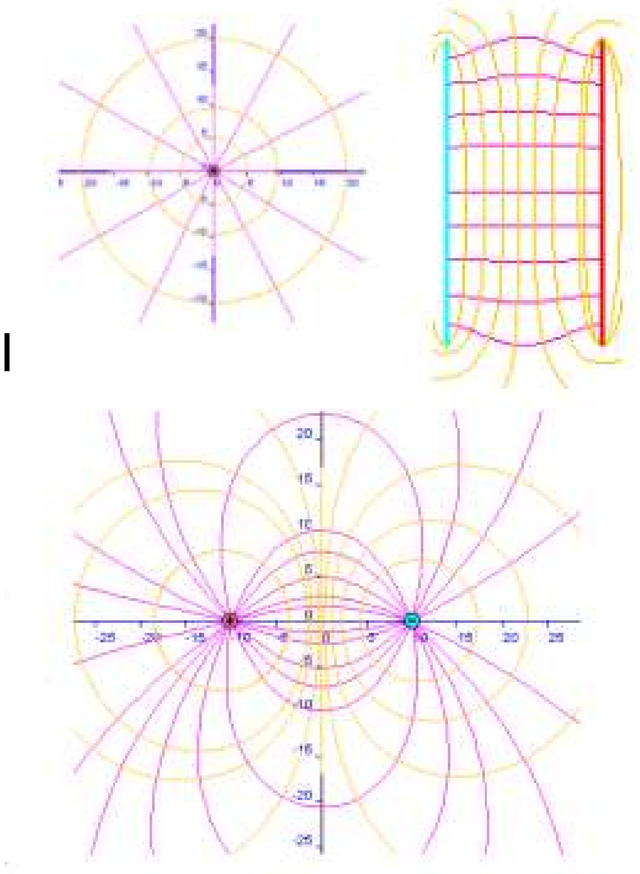
They are deined as the locus of the points having the same potential value.

(a) when q_0 moves from A to B lying on an equipotential surface the work is 0.

$$L_{AB} = -q_0 \cdot \Delta V = q_0 \left(V_A - V_B \right) = 0 \qquad \left(V_A = V_B \right)$$

(b) The field lines of $\overrightarrow{E}$ are orthogonal to an equipotential.





Field and equipotential surfaces for a point-like charge, a dipole and two parallel plates

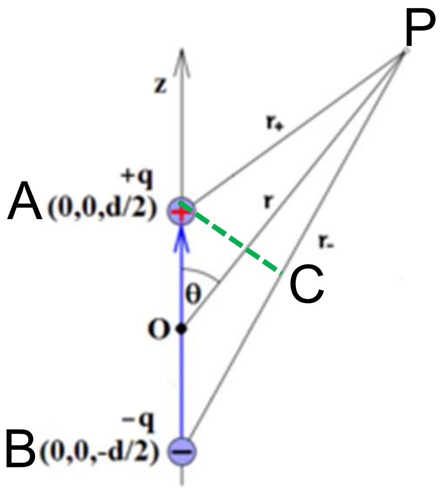
Electric dipole

It is contituted by 2 charges having equale value but opposite sign, +q and –q, separated by ,a distance *d*.

It is characterized by the **DIPOLE MOMEMT**:

$$\vec{p} = q\,\vec{d}$$

The vector ***d*** is directed from the negative to the postive charge



Let us calculate the potential of a dipole at a point, P:

$$V(P) = V_+(P) + V_-(P) = \frac{q}{4\pi\epsilon_0}\frac{1}{r_+} - \frac{q}{4\pi\epsilon_0}\frac{1}{r_-} = \frac{q}{4\pi\epsilon_0}\left(\frac{r_- - r_+}{r_+ r_-}\right)$$

Let us assume P as very far from +q e -q, i.e. $r \gg d$. Then:

$$\Delta r = r_- - r_+ \cong BC = d\cos\theta \qquad r_+ r_- \approx r^2$$

$$V(P) = \frac{q}{4\pi\epsilon_0}\cdot\frac{d\cos\theta}{r^2} = \frac{|\vec{P}|\cdot\cos\theta}{4\pi\epsilon_0 r^2}$$

REMARKS.

(a) It decreases as $\frac{1}{r^2}$, whereas for a point-like charge it goes as $\frac{1}{r}$

(b) On the equator plane $\theta = \frac{\pi}{2}$ we have $V\left(\frac{\pi}{2}\right) = 0$

The dipole moment is measured in $|\vec{p}|$ = C · m