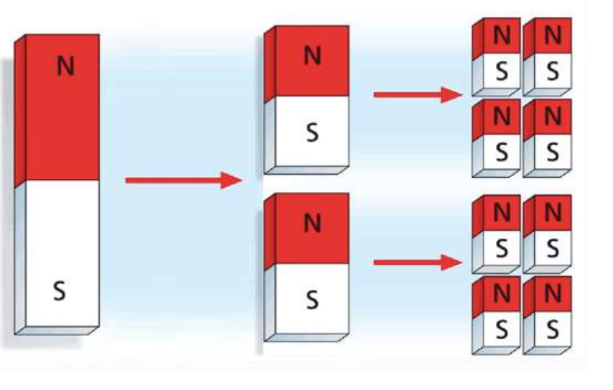
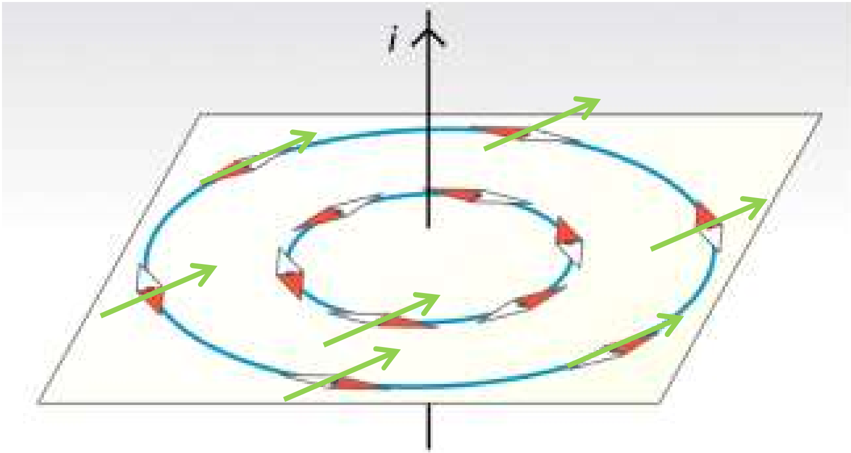
Magnetism

The materials that we are going to study, on the occasion of **Magnetism**, have a phenomenology similar to that of electrified materials, however without the need to be rubbed to accumulate charge, but which still exhibit an attraction or repulsion at a distance due to their nature. .
The substantial difference with the *electrostatic dipole* is that while for the latter it is possible to separate the two charges, the positive one from the negative one, in the case we are going to study (we will no longer have pole + and pole -, but north and south), that we will call *magnetic dipole*, going to cut the bodies / dipoles we will never be able to isolate the north pole from the south pole and with each cut we will always have new magnetic dipoles, only gradually smaller. Furthermore, as well as interacting with each other, these objects also interact with electric currents and this is the aspect that will lead us to consider magnetism in some way connected to electricity in electromagnetic theory, of which we will mention the microscopic aspects only at a phenomenological level.

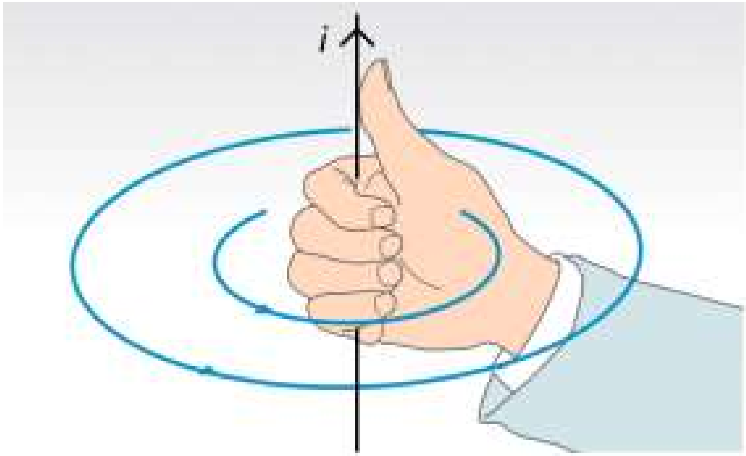


Oersted Experiment

In Oersted's experiment these small pieces of magnetic material (iron oxides), which we will call magnetic dipoles, are arranged along a circle. At the center and perpendicular to this circle passes a conductor that can be traversed by current if necessary (closed circuit). Before the circuit closes, the magnetic dipoles tend to align themselves with the Earth's magnetic field, therefore parallel to each other (green arrows in figure "a"), because on the planet it is as if there were a "dipolone" (a large magnet) as large as the diameter of the planet itself an almost uniform magnetic field aligning on the earth's surface. Thanks to this Earth's magnetic field, sailors have been able to orient themselves for millennia following the direction indicated by the compass. When the circuit is closed, the dipoles align tangent to the circumference with a certain direction (white-red dipoles in figure «a»); if the direction of the current in the circuit changes, the dipoles circulate in the opposite direction according to the rule of the right hand (if the thumb from palm to fingertip represents the direction of the current, the dipoles line up like the four remaining fingers, figure "b") .



a Le linee del campo sono circonferenze concentriche, disposte su piani perpendicolari al filo.



b Regola della mano destra: con il pollice rivolto nel verso della corrente, le dita piegate indicano il verso del campo.

Ampére's law

The passage of the current I in the circuit, therefore, is associated with a magnetic field, which is indicated by $\vec{B}$ which interacts with the magnetic dipoles.

Thus, we can define a new parameter, that is the quantity of oriented dipoles per unit of length that are aligned along the closed path. Experience tells us that this quantity is proportional to the current I, concatenated to the circuit, through a new fundamental constant of proportionality μ_0 which depends on the medium and in the case in question is relative to the vacuum and is defined as magnetic permeability in vacuum.

along a closed path γ:

$$\oint_{\gamma} \vec{B} \cdot d\vec{l} = \mu_0 \cdot I$$

Ampére's law

Therefore, the source of this type of action at a distance on the magnetic dipoles and, therefore, the source of the magnetic field is certainly the electric current I

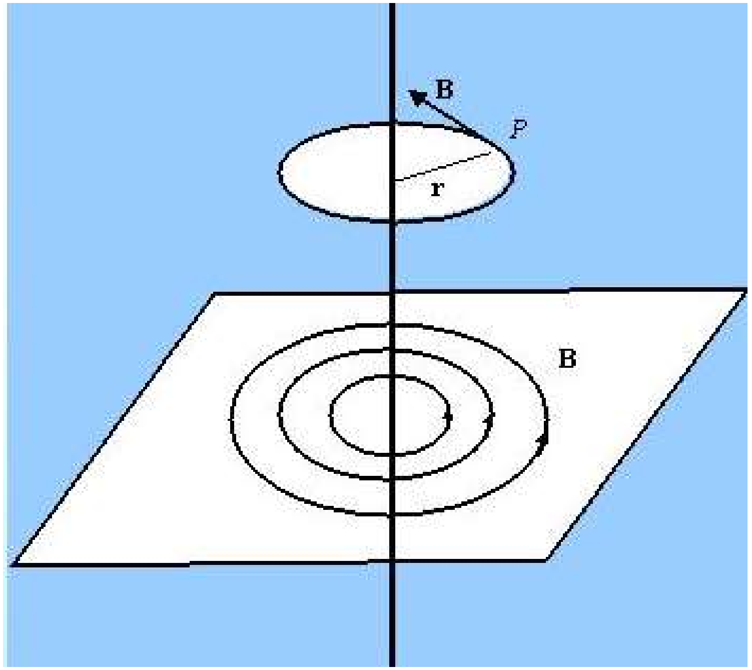
μ_0 (*magnetic permeability of vacuum*) = 4 π 10^{-7} T · m / A

Through this fundamental law, known the magnetic permeability, we can measure the magnetic field B which is measured in Tesla (T).
From Ampère's law we can deduce that the magnetic field has a fundamental characteristic that distinguishes it from the electric field, which has open lines of force, namely the fact that its lines of force are closed. In fact, the dipoles always position themselves tangent to a closed path, instead the lines of forces of E orient themselves according to half-lines that are born or die. The reason is due to the fact that it is not possible to isolate the magnetic pole and therefore there is no isolated magnetic charge (monopole) from which to generate the lines of force.

Magnetic field of an indefinite straight wire

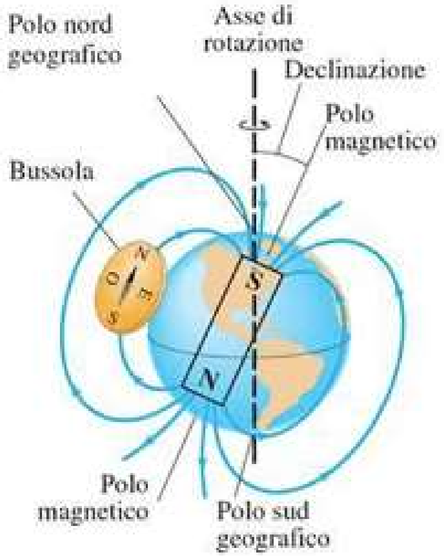
$$B = \frac{\mu_0 i}{2\pi R}$$

$$C_B = B \oint_l dl = B(2\pi r)$$



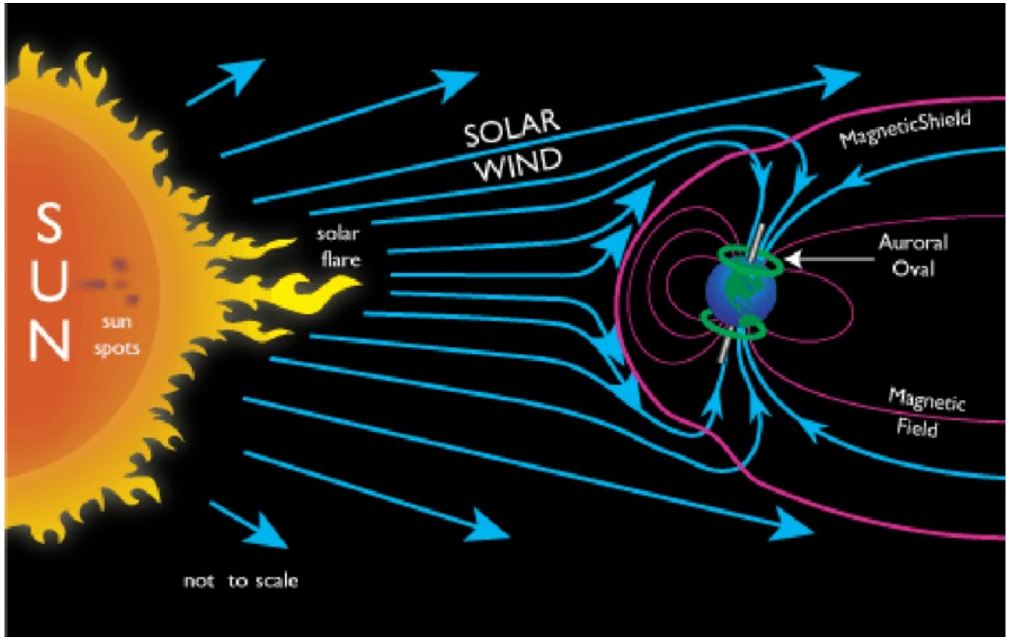
The lines of force of **B** are all concentric circles floor by floor. The current flowing in the wire is such that, in steady state, there is no charge accumulation. The total net charge is 0 and, therefore, the electrostatic field is only that of the battery (external field). In non-steady state (non-continuous currents) the scenario will be different.

Earth's magnetic field



We have defined the Earth, in magnetic terms, as a big dipole whose north pole is located in the Arctic, north of Canada and the south pole is located in the Antarctic area. These poles do not coincide with the geographic north and south, but are close together at an angle of 11.5 °. The Earth's magnetic field is measured in Gauss, a submultiple of Tesla (1 G = 10^{-4} T) and measures 0.5 Gauss (G). The Earth's magnetic field is changing direction quite rapidly and appears to be migrating towards Siberia with a view to polarity reversal.

The magnetic field on Earth exists because there is a huge liquid ferrous core whose rigid and solidified exterior is set in rotation. The presence of the magnetic field shields harmful charges from ionizing radiation arriving from the sun. The uncharged particles are shielded from the atmosphere.



Magnetic force on electric charges

If we place an electric charge q that moves with speed $\vec{v}$ in the magnetic field $\vec{B}$, a force of magnetic origin acts on it, also called Lorentz force.

$$\overrightarrow{f_m} = q\vec{v} \times \vec{B}$$

The product in question among the vectors is a vector (in general it has 3 components), it is a **vector product**, not a scalar. The scalar product between vectors gives us a scalar, a number, therefore, which can be positive or negative. The vector product, on the other hand, gives a vector.

VECTOR PRODUCT NOTE

$$\vec{c} = \vec{a} \times \vec{b}$$

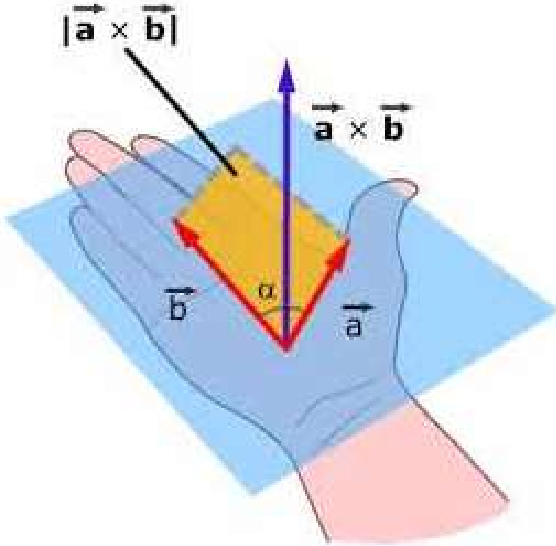
If we do $\vec{a} \times \vec{b}$, with angle α between the two vectors we will have:
$\vec{c} \perp \vec{a}, \vec{b}$ (direction of $\vec{c}$ given by the right hand rule)

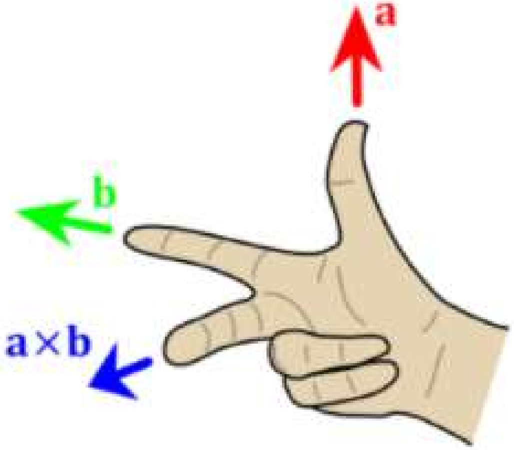
Right hand rule: If the thumb is the first vector and the index is the second vector, the middle finger denotes the direction and the direction of the third vector c. If you invert the order of the two vectors, yes the thumb instead of the index, α changes sign and goes to - α; in the scalar product we have cos (α) = cos (- α), while in the vector product we have sin (α) = - sin (- α). Hence, the scalar product is commutative, while the vector one is anticommutative.

$$\vec{a} \text{ x } \vec{b} = -\vec{b} \text{ x } \vec{a}$$

The vector product is 0 when the two vectors are parallel or antiparallel and is maximum when the two vectors are perpendicular (⊥). The vector product is used in physics whenever we talk about moments of vector quantities and in the magnetic force.

Since $\overrightarrow{f_m} \perp \vec{v}$ (as for centrifugal force), the magnetic force never does work on a charge; because the infinitesimal work done by any force is $\boldsymbol{F}\cdot \boldsymbol{ds}$ (scalar product), but the infinitesimal displacement is dictated by speed in the sense that it is parallel and concordant with it; therefore the infinitesimal displacement is ⊥ to the force and therefore the work is 0.





Magnetic force on electric currents

When the magnetic force is considered relative to all the charges that run through a wire crossed by current, we will have $\vec{F}_m$ (with a capital letter as it acts on all the charges of the wire) ⊥ $\vec{v}, \vec{B}$. A vector $\vec{l}$ can be defined with modulus given by the length of the wire (or of the section of wire considered) and direction and direction of the current, (which are determined by the velocity vector of the charge carriers).
It generalizes to a section of wire l crossed by current I

$$\overrightarrow{F_m} = I\,\vec{l} \times \vec{B} \qquad\qquad |\vec{l}| = l$$

In the case of an infinitesimal section:

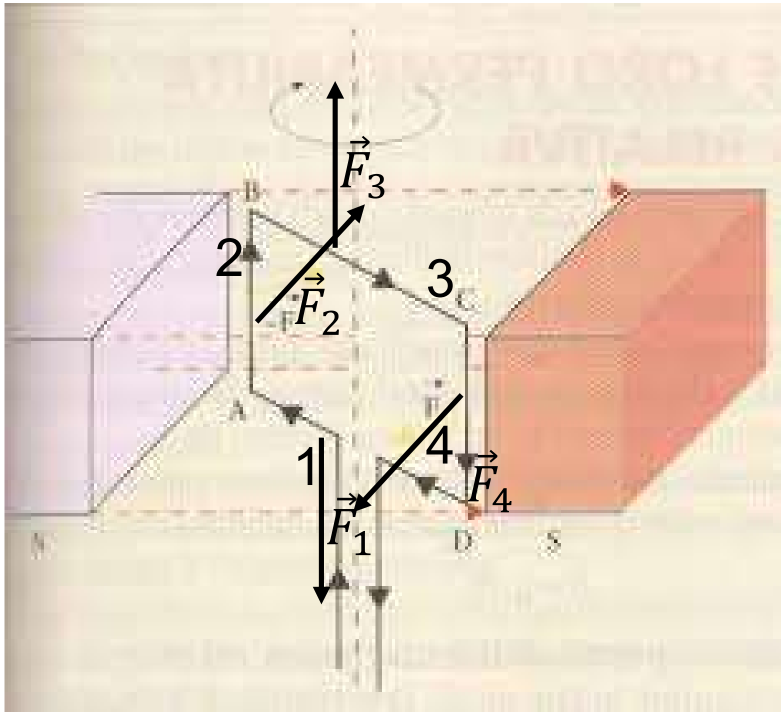
$$d\vec{F}_m = I\,d\vec{l} \times \vec{B}$$

The infinitesimal formulation helps us for curved path circuits. In this case we consider an infinite straight piece of circuit (ie of current) and then we add, with an integral, the contribution to the strength of all these infinite infinitesimal pieces.

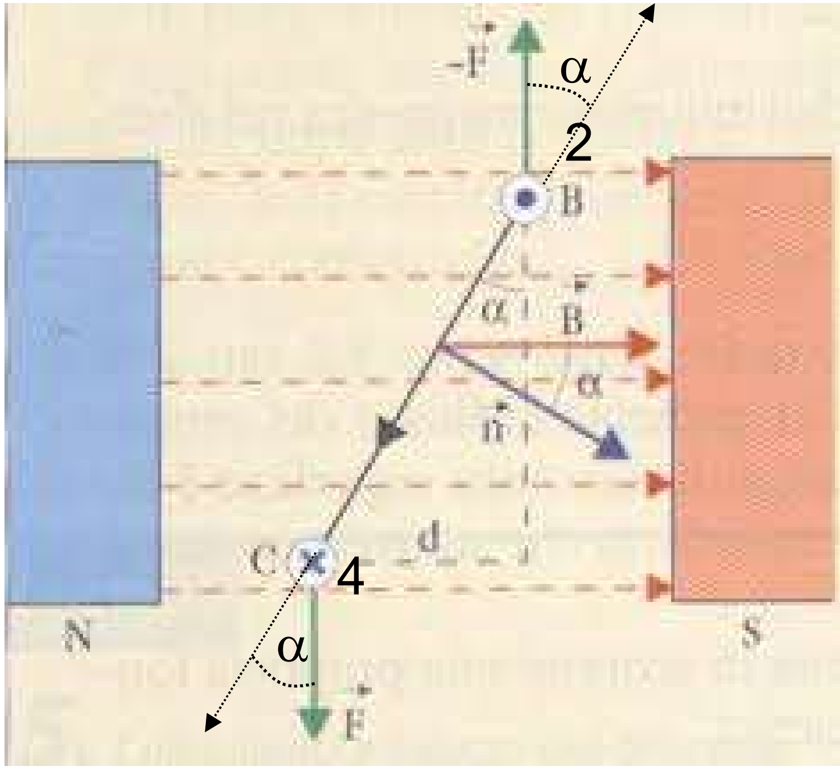
At this point, let's see the action of external magnetic fields on electric currents. Let's put a current inside a magnetic field and see if it is true that current-carrying circuits placed in an external magnetic field are affected by some action. A quadrangular circuit (loop) is inserted between the poles in the region where there is a magnetic field (with uniform field lines). We have 4 sides and we can calculate 4 forces of the type $\vec{F}_m$ each associated with a side.

From the right hand rule we can derive the direction of each force and the direction of speed. $\vec{F}_3$ is a force equal and opposite to $\vec{F}_1$ because $\vec{B}$ remains the same but $\vec{v}$ is equal and opposite to the one flowing in $\vec{F}_1$.

$$\vec{F}_1 = -\vec{F}_3$$
$$\vec{F}_2 = -\vec{F}_4$$



$\vec{F}_1$ and $\vec{F}_3$ are opposite, they do not rotate the coil, nor do they distort it (the coil is rigid), therefore they will cancel each other out by adding up the different forces. $\vec{F}_2$ and $\vec{F}_4$ also will cancel each other in the sum of the forces.



We project $\boldsymbol{M}_2$ and $\boldsymbol{M}_4$ directly along the rotation axis and we have:

$$M_2 = F_2 \cdot \frac{l}{2} \sin\alpha$$

$$M_4 = F_4 \cdot \frac{l}{2} \sin\alpha$$

Both moments are upward and in agreement. What changes sign is the arm $\left(\frac{l}{2}\right)$ besides the force.

$\hat{n}$ is the vector of unit modulus ⊥ at the coil, consistent with the rule of the right hand: if the 4 fingers of the right hand are oriented like the current in the coil from the thumb to the $\hat{n}$

$\hat{n}$ forms an angle α with the lines of force of the magnetic field, therefore with the orientation of the coil.

However, if all the forces cancel each other out two by two, the coil turns anyway because while $\vec{F}_1$ and $\vec{F}_3$ are opposite, but are exerted along the same line, $\vec{F}_2$ and $\vec{F}_4$ constitute a pair of unbalanced forces because they do not act along the same line. $\vec{F}_2$ is exercised in the center of side 2 (homogeneous sides) and $\vec{F}_4$ is directed from the center of side 4 downwards in the slide; this will lead to an overall moment of unbalanced forces. The moment M2 of $\vec{F}_2$ will add instead of subtracting from that M4 of $\vec{F}_4$ and, therefore, as for any pair of forces, we will have that the total moment, due to $\vec{F}_2$ and $\vec{F}_4$, is directed like the rotation axis (axial moment) and rotates the coil perpendicular to $\vec{B}$.

$$M_{TOT} = (F_2 + F_4)\frac{l}{2}\sin\alpha = 2 \cdot I\, l\, B \cdot \frac{l}{2}\sin\alpha = B \cdot I \cdot l^2 \sin\alpha$$

l^2 is the area A of the coil. So if we define a new vector $\vec{\mu}$ representing the <u>magnetic moment</u> of the loop crossed by current and defined as:

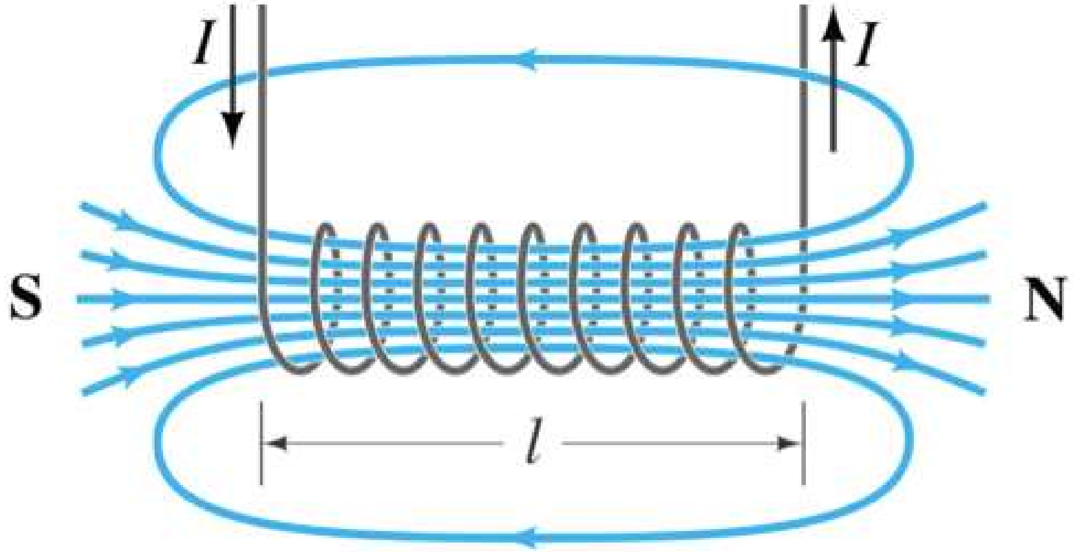
$$\vec{\mu} = I\,\vec{A}$$ with $\boldsymbol{A} = A\,\hat{n}$

We then obtain in vector form a result which is completely general and does not depend on the particular shape of the coil.

$$\vec{M} = \vec{\mu} \times \vec{B}$$

There is, therefore, a sort of reciprocity. If you take magnetic dipoles and place them close to a current, there is an action by the current on the dipoles. If, however, I put a current between the dipoles, there is an action on the current by the magnetic field of the dipoles.

Magnetic field of a solenoid



$$B = \mu_0 \frac{N}{l} I = \mu_0 \, n \, I \; (inside)$$

$$= 0 \; (outside)$$

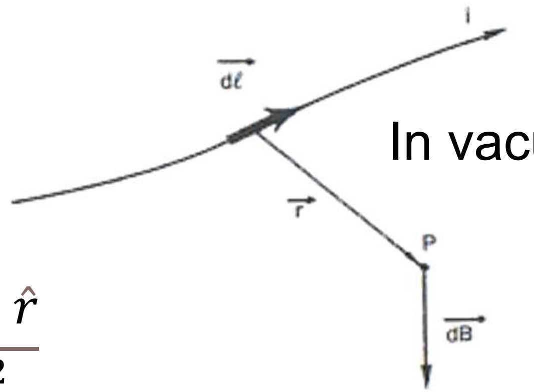
n = density, number of turns N total per unit of length $\left(\frac{N}{l}\right)$ where l is the length of the solenoid.

The magnetic field inside the solenoid is constant and uniform (we are neglecting the edge effects, i.e. the fact that the solenoid has finite length). It plays the same role that the capacitor plays in electrostatics with the electric field, and is proportional to the current that passes through. It is 0 outside the solenoid.

Biot-Savart law / Laplace formula

That of Biot - Savart, then generalized to the so-called Ampère formula, is the law that allows us to find the magnetic field produced by a current of any shape at a certain point. We take a piece of wire through which the current passes (it can also be curved like the one in the figure), we calculate the magnetic field produced by this piece of circuit at point P. To do this we take infinitesimal pieces by adding them, using the superposition principle. I take the piece $d\vec{l}$ which is at distance r from the point P. At the point P, the piece of current will produce an infinitesimal field $d\vec{B}$. What Biot-Savart law tells us is that this infinitesimal field at point P is equal to a constant for a term very similar to Coulomb's law. The source here is not the electric charge, but a "piece" of current $(I\, d\vec{l})$. This field depends on the distance of the source from the point P and this dependence is of the type $1/r^2$, as in the electrical case. For the magnetic field, the difference regarding the vector nature should be emphasized: while in one case we have the radial electric field along the joining charge - point, the magnetic field is perpendicular to r because it is proportional to the vector product between $d\vec{l}$ and $\hat{r}$ (Unit vector) . For this reason the magnetic field will be perpendicular to the currents.

$$d\vec{B}\,(P) = \frac{\mu_0}{4\,\pi}\frac{I\,d\vec{l}}{r^3}\,\mathrm{x}\,\vec{r}\left(\frac{r\cdot\hat{r}}{r\cdot r^2}\right) = \frac{\mu_0}{4\,\pi}\frac{I\,d\,\vec{l}\,\mathrm{x}\,\hat{r}}{r^2}$$

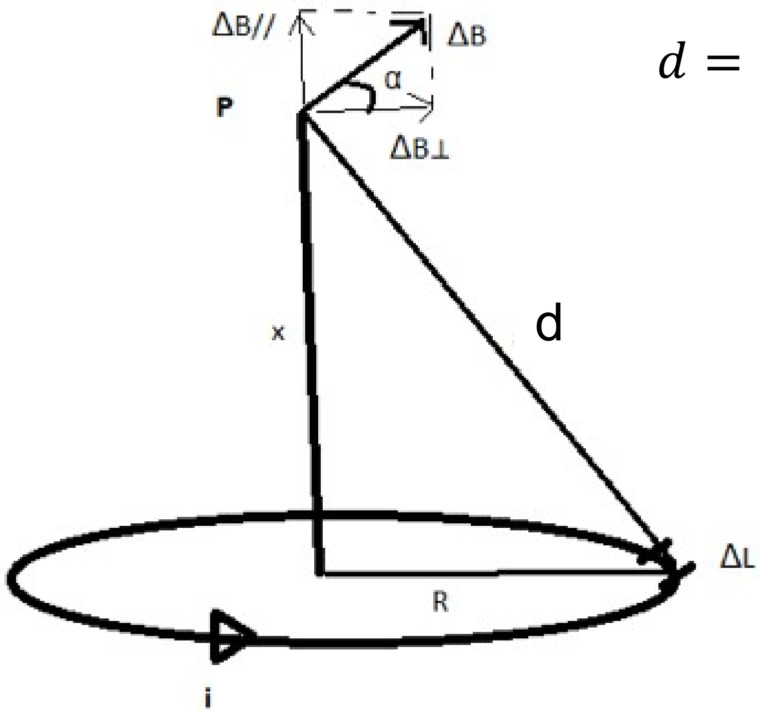


In vacuum

Analogy with the electric case $$d\vec{E}\,(P) = \frac{1}{4\,\pi\,\epsilon}\frac{dq\ \hat{r}}{r^2}$$

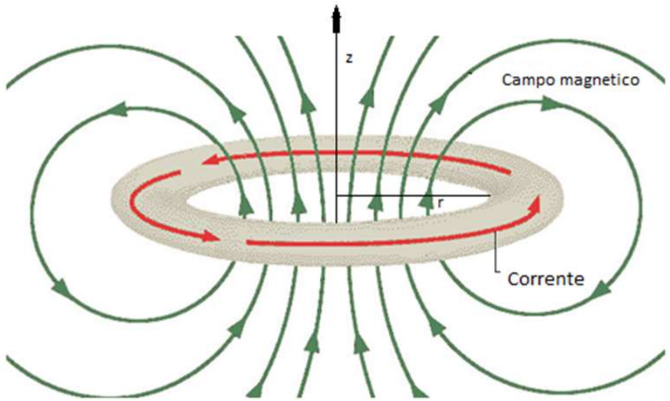
Magnetic field on the axis of a coil crossed by current

We intend to calculate the magnetic field at point P on the axis. The distance of P from a certain point on the circumference is by the Pythagorean theorem:



$$d = \sqrt{R^2 + x^2}$$

To find **B**, we must add up all the contributions of the current circuit.
However, since it is a curved path, it is necessary to consider the different infinitesimal pieces crossed by current that contribute to giving the magnetic field of the point P and to implement the superposition principle. Each infinitesimal piece will give a magnetic field Δ***B*** with one component ⊥ and one \\ for the right hand rule. Then applying the Laplace formula we find:



$$B = \frac{\mu_0}{2} \cdot \frac{I\, R^2}{\sqrt{(R^2 + x^2)^3}}$$

***B** is proportional to the magnetic moment of the coil, in fact the area of the coil will be π · R^2, so the moment of the coil will be I · π R^2. In general this is what can be proved for any point P on the axis.*

For x = 0 at the center of the loop we have:

$$B(x = 0) = \frac{\mu_0 I}{2R}$$

At a very long distance from the coil, x ≫ R:

$$B(x \to \infty) = \frac{\mu_0 I R^2}{2 x^3}$$

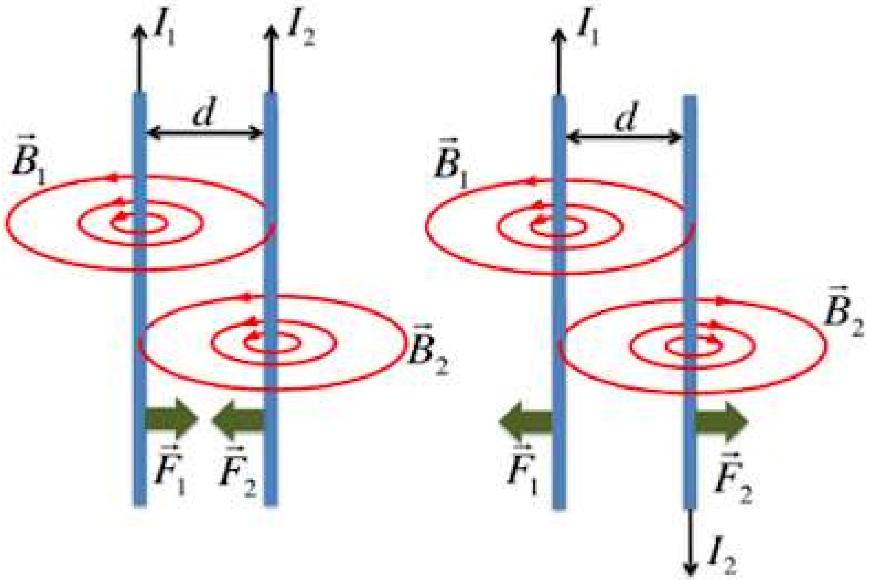
The magnetic field at great distances decays as the cube of the distance x from the coil. x becomes much larger than **R** until **R** can be neglected, so in the end, only x remains in the denominator as x^3. Furthermore, the field of the loop is analogous to the field of a dipole, being in fact direct proportional to the magnetic moment of the loop. Note, therefore, the difference with the electric case where there is the monopole (the isolated charge) which goes as $1/R^2$, while also the electric dipole can be shown to go as $1/R^3$, as in the case of the loop field which goes like $1/x^3$. In particular, since the field lines of **B** are closed, there is no magnetic monopole, the Gauss theorem for B says that:

$$\oiint \vec{B} \cdot \hat{n}\, dS = 0$$

The magnetic field flux through any closed surface is 0.

Force between currents

Let's imagine that we have two electric currents I_1 and I_2 as in the figure parallel, straight. In this case the two currents are in agreement, both facing upwards. We ask ourselves: are there any mechanical actions affected by the wires traversed by the current? Yes, they exist because one current generates the magnetic field, the other will be close to the magnetic field of the first and is therefore affected by the external field. There will be the action of 1 on 2 through the magnetic field generated by 1 on 2 and vice versa. Let's calculate these equal and opposite forces (action and reaction torque):



$$\vec{F_1} = -\vec{F_2}$$

force with which the two currents are attracted, in particular the one with which the current 2 is attracted by the current 1 is:

$$|\vec{F_2}| = I_2\, l\, B_1\,(\text{On the 2nd current}) = I_2\, l\, \frac{\mu_0\, I_1}{2\,\pi\, d} = \mu_0\, \frac{I_1 I_2}{2\,\pi\, d}\, l$$

The force / unit of length will therefore be:

$$\frac{F_2}{l} = \mu_0\, \frac{I_1 I_2}{2\,\pi\, d}$$

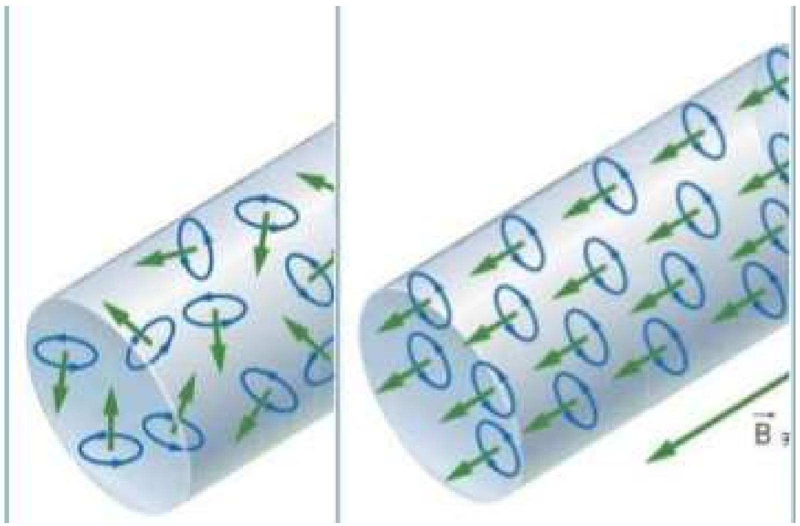
We will have that concordant currents attract and discordant currents repel each other and this is contained in the product $I_1 \cdot I_2$.

Magnetic properties of matter

Magnetism in magnetic materials originates from 2 contributions:
1- Rotation of electrons around the nucleus to form microscopic circuits (microscopic currents).
2- Rotation of electrons around their own axis (spin moment, intrinsic dipole)

Both contributions give rise to small magnetic dipoles. At room temperature, due to thermal agitation, these small dipoles are randomly oriented. In other materials, these small dipoles orient themselves more or less in the same direction and there is a magnetic medium, ie one that exhibits an intrinsic magnetic field.

$\vec{B}_{TOT} = 0$ $\qquad$ $\vec{B}_{TOT} \neq 0$



Diamagnetic materials: $\vec{B}\ intrinsic = 0$ also called magnetization $\vec{M}$ Under a field $\vec{B}_0\ external$ a $\vec{B}_m\ induced$ is born which opposes $\vec{B}_0$. In the end:

$$\vec{B} = \vec{B}_0 - \vec{B}_m$$

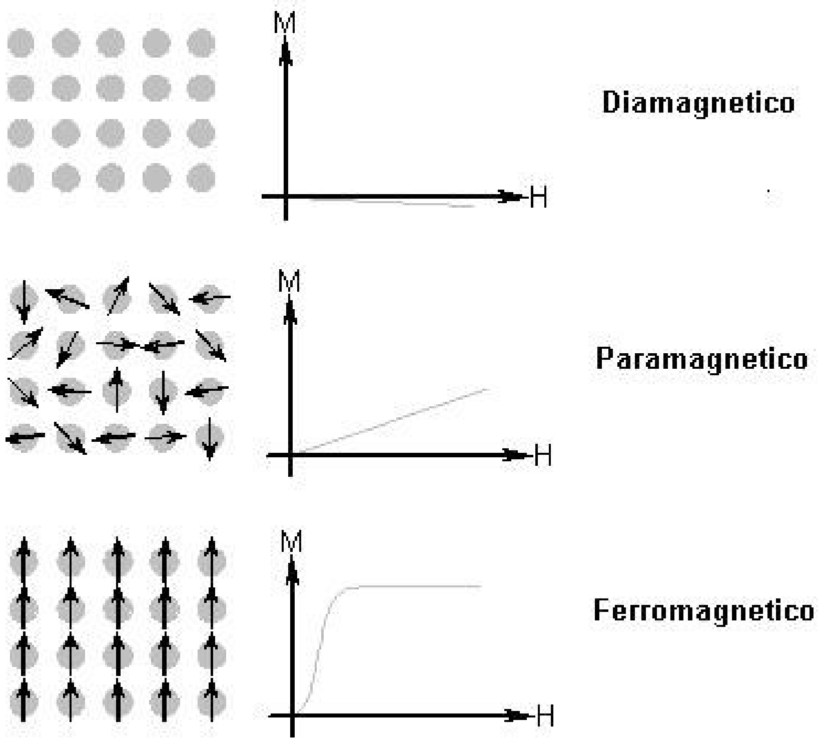
Most substances are like this.

Paramagnetism and ferromagnetism

Paramagnetic materials : $\vec{B}\ intrinsic = 0$ but there are intrinsic $\vec{\mu}$ different from 0. Under a field $\vec{B}_0\ external$ a $\vec{B}_m\ induced$ is born which is dominated by the alignment of the $\vec{\mu}$ parallel to $\vec{B}_0$. In the end:

$$\vec{B} = \vec{B}_0 + \vec{B}_m$$

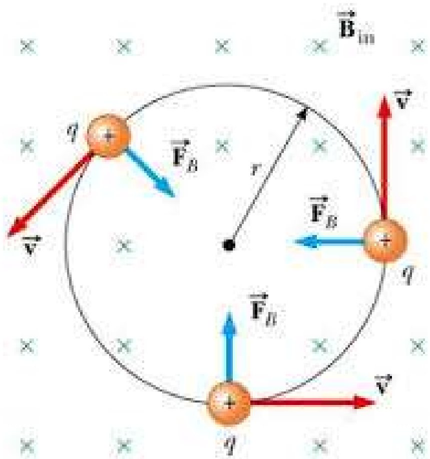
Ferromagnetic materials: The dipoles of large domains tend to arrange themselves parallel to form a field $\vec{B}_m \neq 0$ even in the absence of an external field $\vec{B}_0$.



Motion of an electric charge in a magnetic field

The magnitude Lorentz force acts (suppose the uniform magnetic field):

$$f_m = q\ v\ B$$



v remains constant in modulo and since $\vec{f}_m$ is always ⊥ to $\vec{v}$. So the motion is uniform circular; and we have:

$$m\ a_c = m\ \frac{v^2}{R} = q\ v\ B$$

$$R = \frac{m\ v}{q\ B}$$ (*R* = radius of the circumference)

If a charged particle is launched into an area of space in which there is $\vec{B}$ with $\vec{v}$ oblique with respect to the field, its motion is helical with velocity components parallel to the field, $\vec{v}_p$, and perpendicular, $\vec{v}_\perp$, with helix pitch moving as $Z\ (t) = v_p \cdot t$ and helix radius given by $R = \frac{m\ v_\perp}{q\ B}$