Inductance

The current I flowing in a circuit generates a field $\vec{B}$ whose flow, $\phi\ (\vec{B})$, is self-chaining with the circuit itself. The relationship between $\phi\ (\vec{B})$ and I is constant and is defined **inductance**, L, or **self-induction**.

$$\phi(\vec{B}) = L \cdot I$$

$$[L] = \frac{[B] \cdot [Lungh]^2}{[I]} = \frac{[B] \cdot [Lungh]^2 \cdot [t]}{[Q]}$$

It is measured in Henry (H) while $\phi\ (\vec{B})$ is measured in Weber (Wb).

$$1\ Wb = 1\ T \cdot 1\ m^2 \qquad 1\ H = \frac{1\ Wb}{1\ A}$$

For example, for a solenoid we have: $\phi(\vec{B}) = (\mu \cdot \frac{N}{l} \cdot I\ ) \cdot (S \cdot N\ )$

Where:
S = surface of the solenoid
l = length of the solenoid
N = number of turns

μ *: magnetic permeability of vacuum*

therefore: $L = \mu_0 \cdot \mu_r \cdot \frac{N^2}{l} \cdot S = \mu \frac{N^2}{l} \cdot S \qquad \mu_r \geq 1$

NOTE: L depends only on geometry and contained medium such as capacitance in the electrostatic field.

Faraday / Neumann – Lenz law

The induced electromotive force ϵ_i induced at the ends of a loop or any closed circuit of surface S is given by the speed with which the concatenated $\phi_s(\vec{B})$ varies over time. The direction of the induced current is such as to minimize $\frac{\Delta\phi_s}{\Delta t}$. (**Lenz's Law**)

$$\epsilon_i = -\frac{d\,\phi_s}{d\,t}\,(\vec{B})$$

$$\epsilon_i \rightarrow \oint \vec{E} \cdot d\vec{l} \neq 0, \qquad instead\ in\ the\ static\ case: \oint \vec{E} \cdot d\vec{l} = 0$$

This fundamental law couples a time-varying field variable $\vec{B}$ to an electric field that is rotational (non-conservative, circulation ≠ 0) unlike the static one. The electric field is generated necessarily otherwise it would explain the potential difference (electromotive force) induced in the circuit.

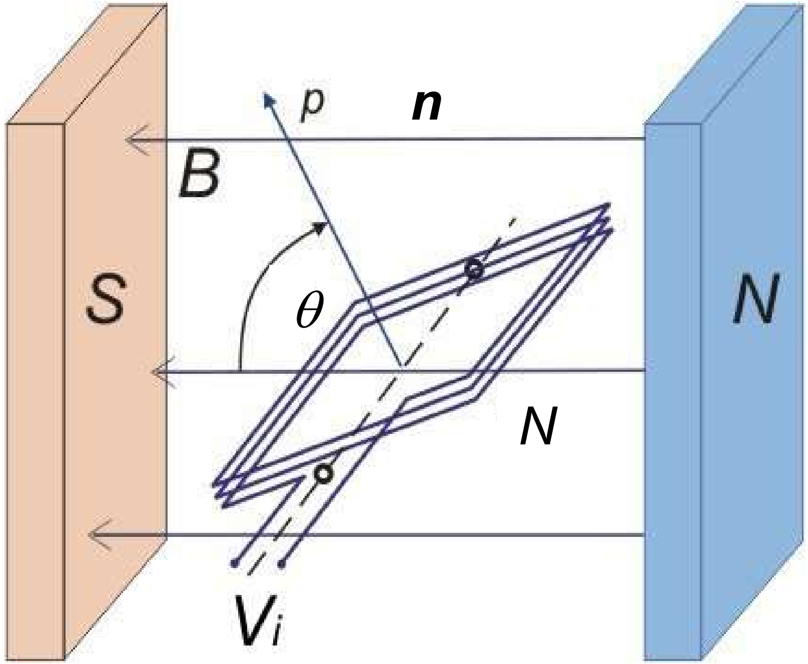
Applications of Faraday / Neumann's law

SINUSOIDAL CURRENT GENERATOR

$$\phi_s(\vec{B}) = BS\cos\theta\,(t) = BS\cos\omega t$$

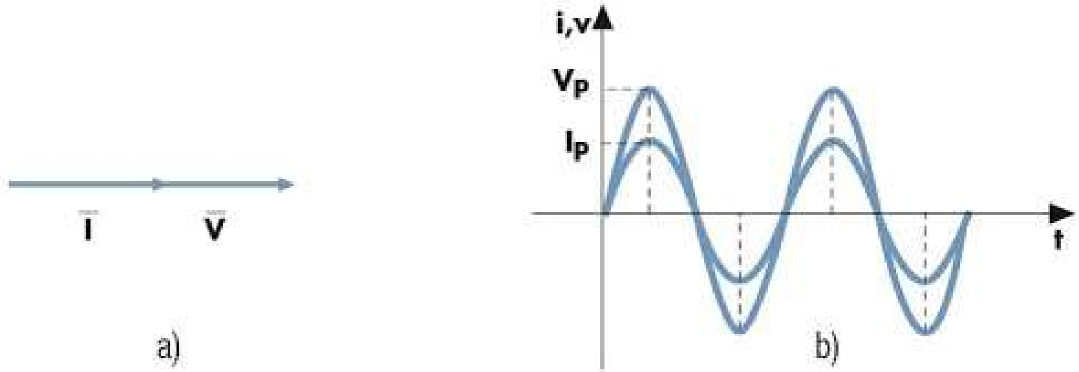
$$\epsilon_i = -\frac{d\,\phi_s(\vec{B})}{d\,t} = -\frac{d}{d\,t}BS\cos\omega t$$
$$= \omega BS\cdot\sin\omega t\ \ (alternating\ voltage)$$

$$I = \frac{\epsilon_i}{R} = \frac{\omega\,B\,S}{R}\sin\,(\omega t)$$



The induced power is given by:

$$P = \epsilon_i\cdot I = (\omega\,BS\cdot\sin\omega t)\cdot(\frac{\omega\,BS}{R}\cdot\sin\omega t) = \frac{\omega^2B^2S^2}{R}\,sin^2\omega t = \frac{\epsilon^2{}_{max}}{R}\,sin^2\,\omega t$$



This is the operating mechanism of the alternator which in power plants produces the alternating current which is then used in city networks. To keep the coil rotating, it is necessary to spend mechanical energy from the outside (turbines driven by waterfalls) or high-pressure steam (thermoelectric power plants).

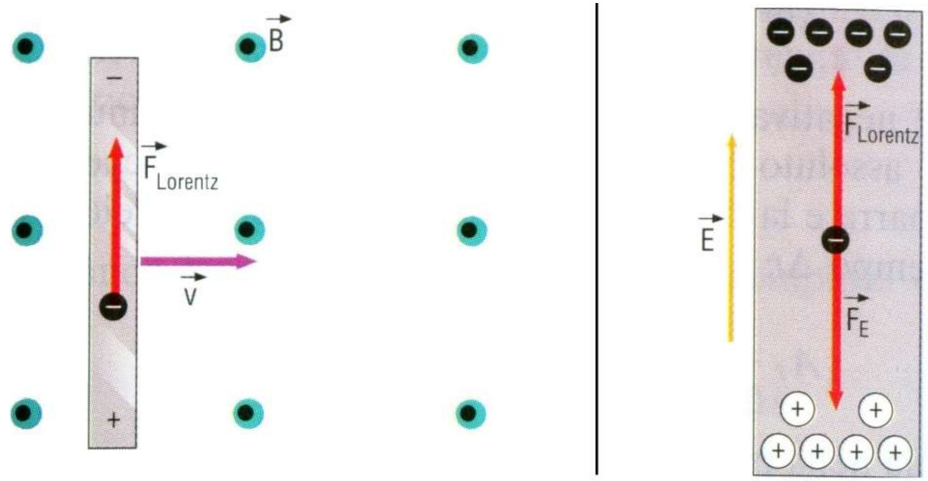
Applications of Faraday / Neumann's law

MOVING CONDUCTOR IN A UNIFORM MAGNETIC FIELD

The magnetic force $\vec{F}_m = q\vec{v}\ x\ \vec{B}$ pushes the charge carriers from one end to the other end of the conductor. The separation of charges generates a field $\vec{E}$ in the conductor and, therefore, a ΔV (a ϵ) at its ends. In equilibrium:

$$q\ v\ B + q\ E = 0 \rightarrow |E| = v\ B, \qquad \epsilon = |E| \cdot l$$

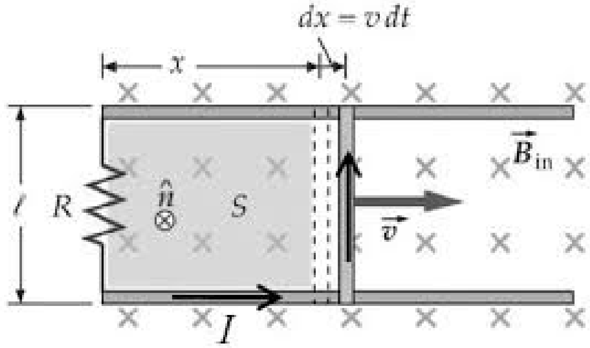
l=length of conductor $\qquad \epsilon = v\ B\ l$



If we "close" the moving conductor bar on a conductor track, current I will flow, given by:

$$I = \frac{\epsilon}{R} = \frac{B\ v\ l}{R}$$

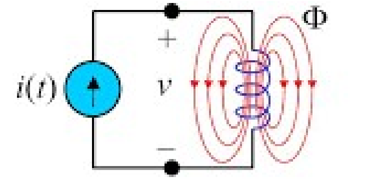
To establish the direction of the current we use Lenz's law.



Thus with $\overrightarrow{B_{\text{in}}}$ entering the sheet, the induced current flows counterclockwise so that the magnetic field generated by it has lines coming out of the sheet inside the circuit so as to oppose the increase of the total flux $\phi_S(\vec{B})$.

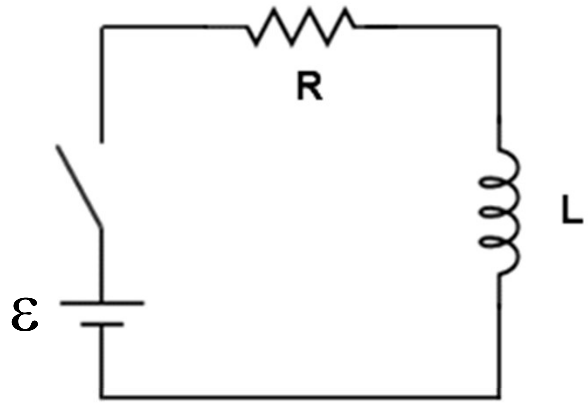
Self-inductance

The magnetic field flux is self-chaining with the circuit where the current that generates it flows. Therefore, it is always born in e.m.f. induced, ε_L, associated with that circuit.
In the presence of a external potential, ε , as in the circuit in the figure, the circuit equation becomes (the Kirchoff's law):



$$v = N\frac{d\Phi}{dt} = N\frac{d\Phi}{di}\frac{di}{dt} = L\cdot\frac{di}{dt}$$

$$\epsilon + \epsilon_L = IR = \epsilon - L\,\frac{dI}{dt}$$

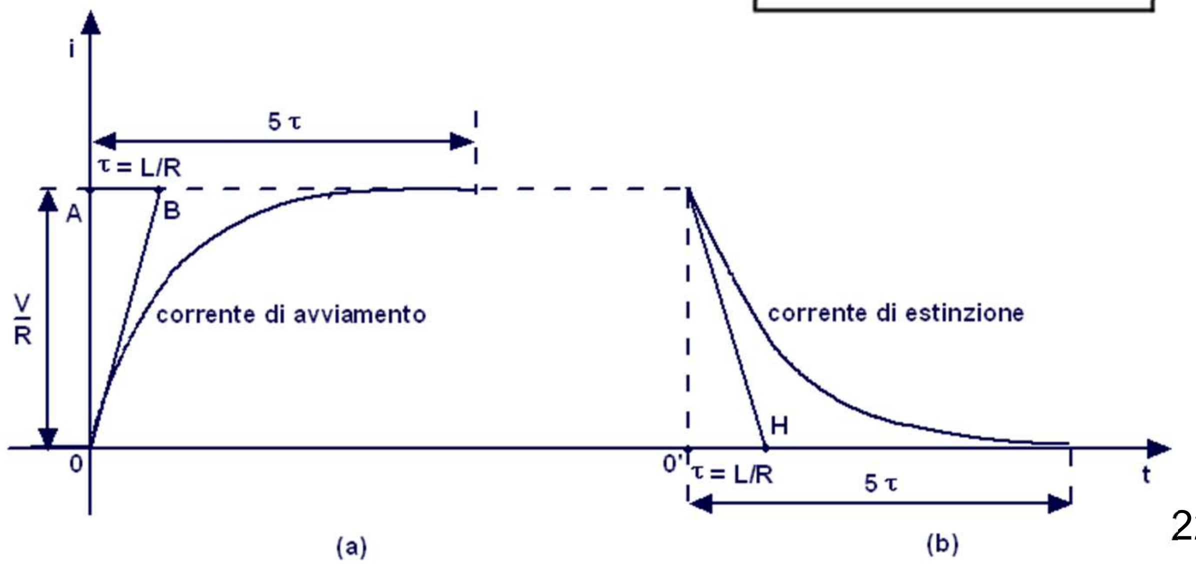


Which is a linear inhomogeneous first order differential equation with the unknown I (t). It is easy to show that the current has a trend over time as in the graph:

$$I\,(t) = \frac{\epsilon}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\tau = \frac{L}{R}$$

time constant of the circuit



Magnetic field energy

When we have an emf generator, ϵ, which supports a current I and therefore delivers a power ϵ · I, the power is:

$$\epsilon\, I = I \cdot IR + I \cdot L\, \frac{dI}{dt} = (I^2 R) + \left(L\, I \cdot \frac{dI}{dt} \right)$$

power stored in the magnetic field of L

$$I \cdot L \cdot \frac{dI}{dt} = \frac{1}{2} \frac{d}{dt} (L\, I^2)$$

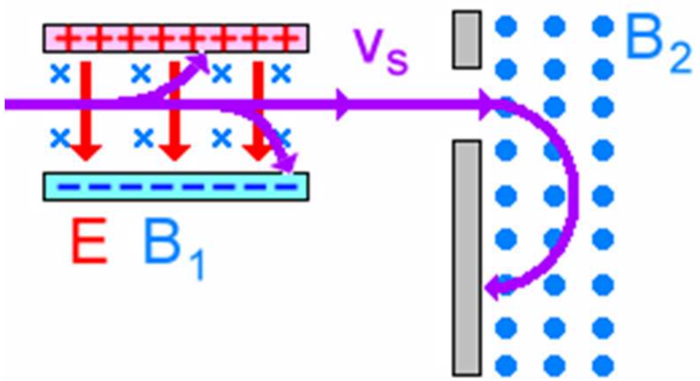
$$= \frac{d}{dt} \text{(derivative } in\ time) \left(\frac{1}{2} L\, I^2 \right) (magnetic\ energy)$$

MAGNETIC ENERGY:

$$U_B = \frac{1}{2} L\, I^2$$

This is the energy that is deposited in the inductance.
It is analogous to that in a charged capacitor $U_C = \frac{1}{2} C\, \Delta V^2$
This is magnetic power, not dissipated (as in the resistor), but stored.

Mass spectrometry



$$[1] \quad qE = qvB_1 \Rightarrow v_s = \frac{E}{B_1}$$

$$[2] \quad E = \frac{V}{d} \Rightarrow v = \frac{V}{dB_1}$$

$$[3] \quad r = \frac{mv}{qB_2}$$

$$[4] \quad r = \frac{mV}{qB_1B_2d}$$

$$[5] \quad m = \frac{qB_1B_2d}{V}r$$

r = radius of curvature of the circular path, which is measured in the spectrometer. It allows to measure for certain molecules the mass-to-charge ratio, m / q, or the mass, known the charge. Therefore, this technique gives the identity card to chemical agents. It is one of the great techniques of qualitative chemistry, but physics always remains quantitative

Electromagnetic waves

Electromagnetic waves are transverse, which means that what oscillates is perpendicular to the direction of propagation. They are waves in which the electric field and the magnetic field oscillate; if there is an oscillating electric field there is also a magnetic field. For example, for a wave that propagates along the x axis, we can write the electrical component and the magnetic component, always perpendicular to the electric field:

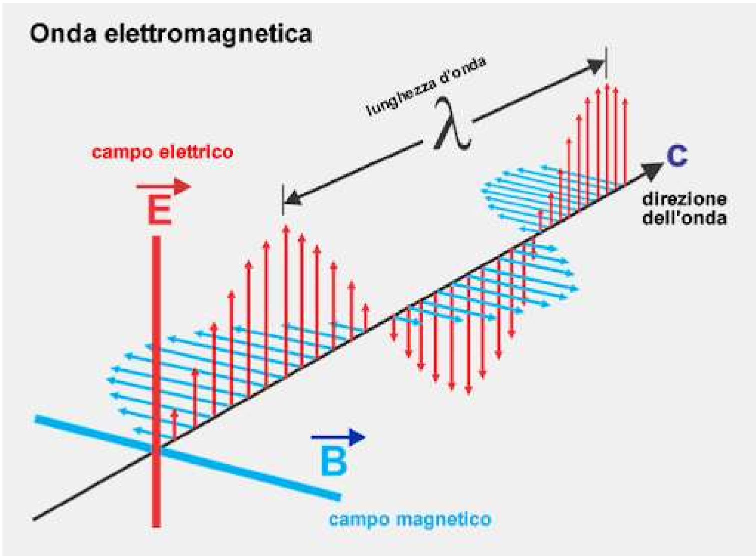
$$E\,(x,t) = E_0 \sin\left[2\,\pi\,\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]$$

$$B\,(x,t) = B_0 \sin\left[2\,\pi\,\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]$$

In vacuum the propagation speed is:

$$c = \frac{1}{\sqrt{\epsilon_0\,\mu_0}}\ equal\ for\ every\ \lambda$$

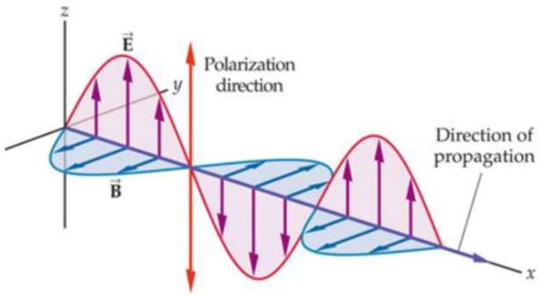
$\hat{n}$ (direction of propagation), $\vec{E}$, $\vec{B}$, constitute a right-handed orthogonal triad. (plane electromagnetic wave that propagates).
It is also easy to see that $E_0 = c \cdot B_0$.



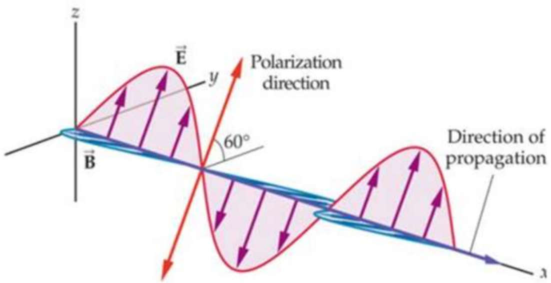
Polarization

Polarization of light is defined as the direction of oscillation of $\vec{E}$:

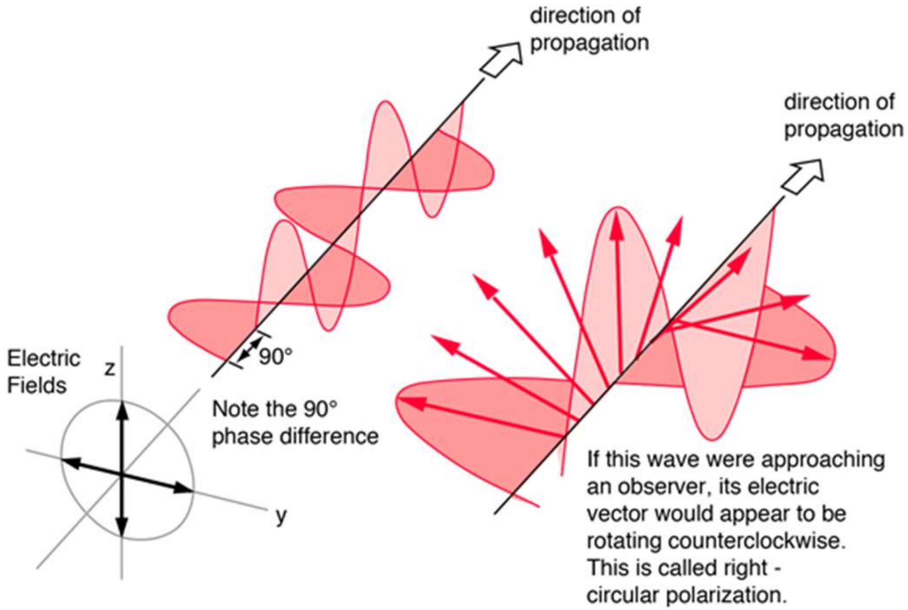
Linear polarization: the directions of oscillation of the electric and magnetic fields are constant. They're perpendicular to each other and to the direction of propagation.
Circular polarization: The perpendicular directions of electric and magnetic fields rotate with constant angular velocity. (right, left, clockwise, anticlockwise)
Elliptical polarization: The arrows of the electric and magnetic field vectors describe an ellipse in a period.
Non-polarized light: its polarization is random.
Polarizers: serve to change the polarization of the electromagnetic field (typically in the visible region of the spectrum).



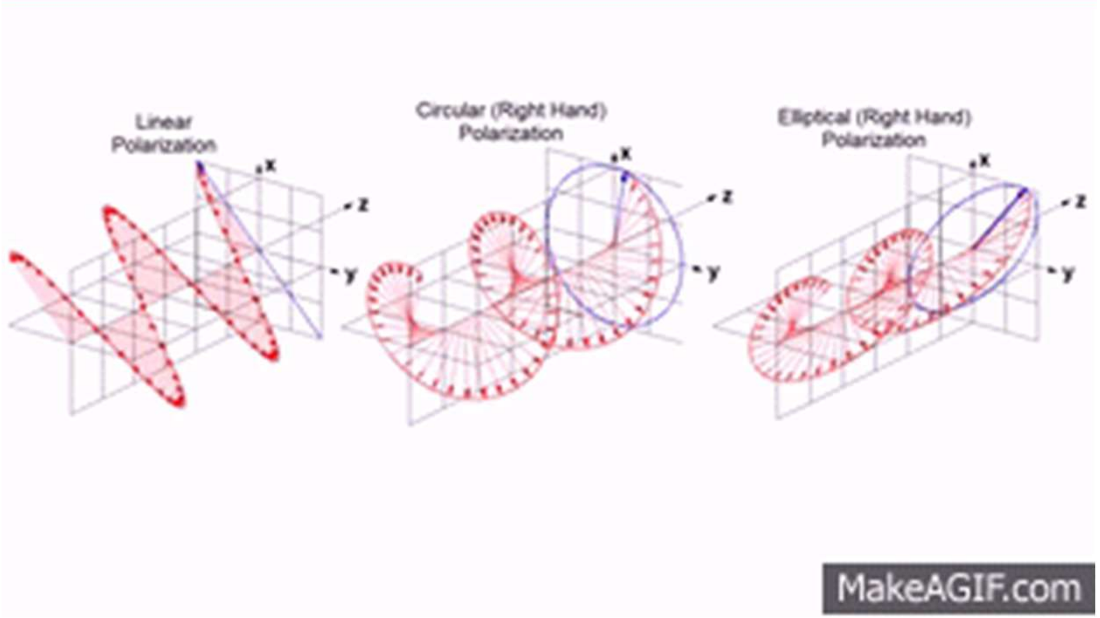
This wave **is polarized** in z -direction



This wave **is polarized** in a direction at an angle of 60° with y-axis



Elliptical polarization - Non-polarized waves



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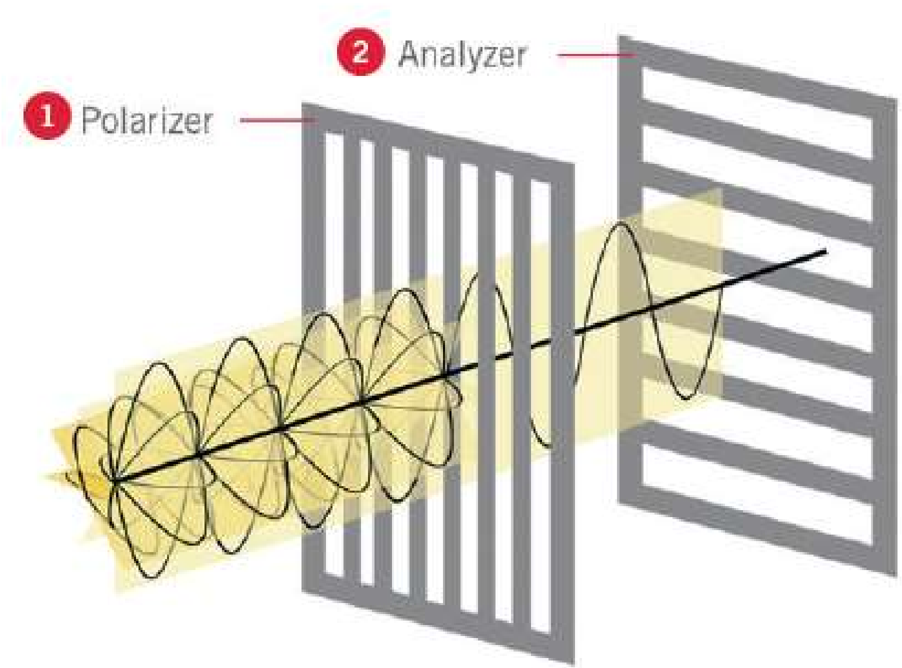
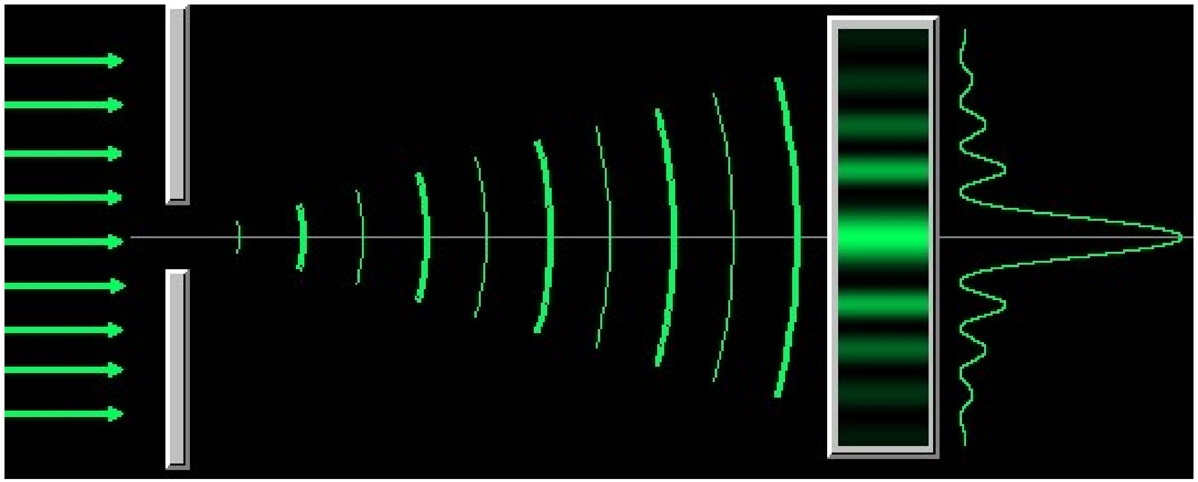
Non-polarized waves



Diffraction - Visible spectrum

Diffraction is observed whenever the light is in the presence of one or more obstacles or very small openings (of the order of magnitude of the wavelength or less). The spectrum of electromagnetic waves has all possible wavelengths, from 0 to ∞ and all possible frequencies from 0 to ∞. **The visible spectrum, light, is a small piece of this immense semi-axis, whose wavelengths are between about 400 nm and 700 nm.** In this case, the wave nature of light involves a series of intensity fringes linked to the interference of the secondary emitting sources. (Principle of Huygens)
At the opening there is a large maximum (main maximum), but there are secondary maximums that are located in very specific positions and spatially placed in correspondence with an obstruction.



Dispersion

When light propagates in a dispersive medium, the refractive index depends on λ, n = n (λ). This implies that the propagation speed v = v (λ) and that the refraction angle θ = θ (λ).

Recall that the speed of light in vacuum is c, in any other medium

$$v = \frac{c}{n}$$

As a result, the colors (different wavelengths) of a white light beam are separated. Prism