# Geometric optics

The geometric optics is an approximation of the optics valid whenever the light on its path encounters obstacles much larger than the wavelength. Then we neglect its wave nature. In this case we can treat light as a set of rays that propagate rectilinearly, if the medium is optically homogeneous.

## *LAWS OF GEOMETRIC OPTICS*

1- The propagation of rays in an isotropic medium (a medium that has the same properties in all points) occurs in a straight line.

2- The path of a light ray between 2 points is always the one that requires the least time (in an isotropic medium this leads to a straight path).

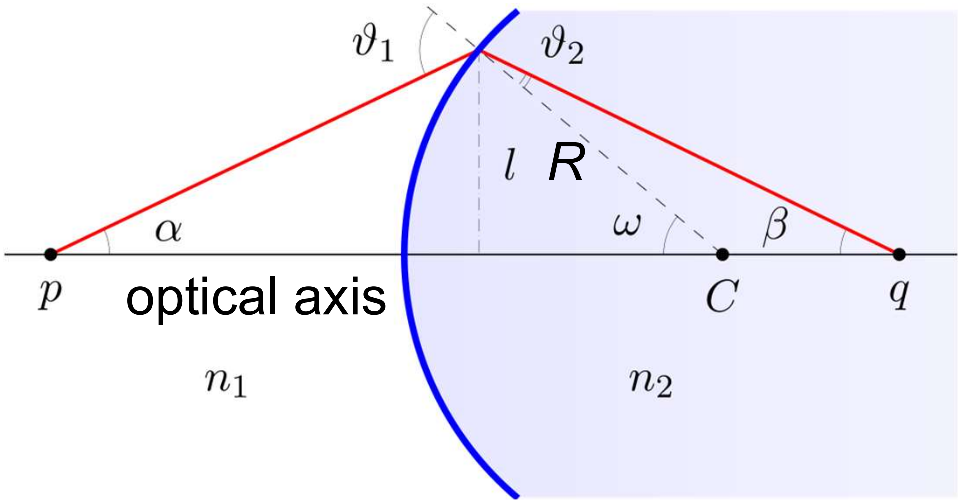
3- The trajectory followed by a light beam is the same regardless of the direction of propagation.

4- The laws of reflection and refraction are valid.

# The diopter

Diopter is defined as the surface that separates 2 transparent media with different refractive index. This surface can be flat or curved. If it is a spherical surface, it is a spherical diopter.

The spherical diopter is stigmatic, that is, the rays that depart from a point source converge in one direction at a single other point (image) in the other medium.
The vertex of the spherical cap determines the optical axis, C is the center of curvature.




It can be shown that in the ideal case (if the angle $\alpha$ between the ray and the optical axis is not too large - paraxial approximation -, if the refractive indices do not depend on the wavelength) all the rays starting from p converge at point q, and q is the image point of the light source p.

This therefore occurs in the approximation of:

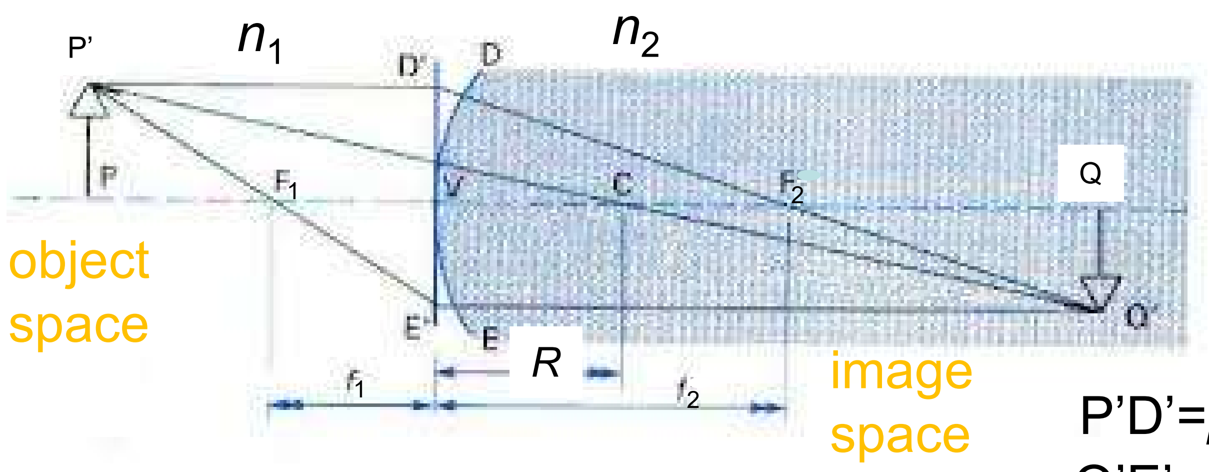
1- Paraxial rays (they form small angles with the optical axis)
2- Amplitude of the spherical cap is small compared to the radius of curvature.

It happens that the rays converge not in a point but in a segment, and the phenomenon of astigmatism occurs.

The ideal diopter is stigmatic and it is valid that:

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

p = distance of object - source from the interface
q = image distance from the interface
R = radius of curvature (> 0 when the cap has the convexity towards the object)




We have a diopter, a source that can be extended, the optical axis, the object space, the image space.

P'D'=*p*
Q'E'=*q*

C = center of curvature, any ray passing through C does not undergo deviations from 1 to 2.
$F_1$ = first focus, all rays starting from $F_1$ go to infinity in the image space (parallel to the optical axis).
$F_2$ = second focus, all the rays coming from infinity in the object space converge in $F_2$ in the image space.
V = vertex of the cap
$f_1$, $f_2$ = focal distances

It can be shown that : $$\frac{f_1}{f_2} = \frac{n_1}{n_2}$$

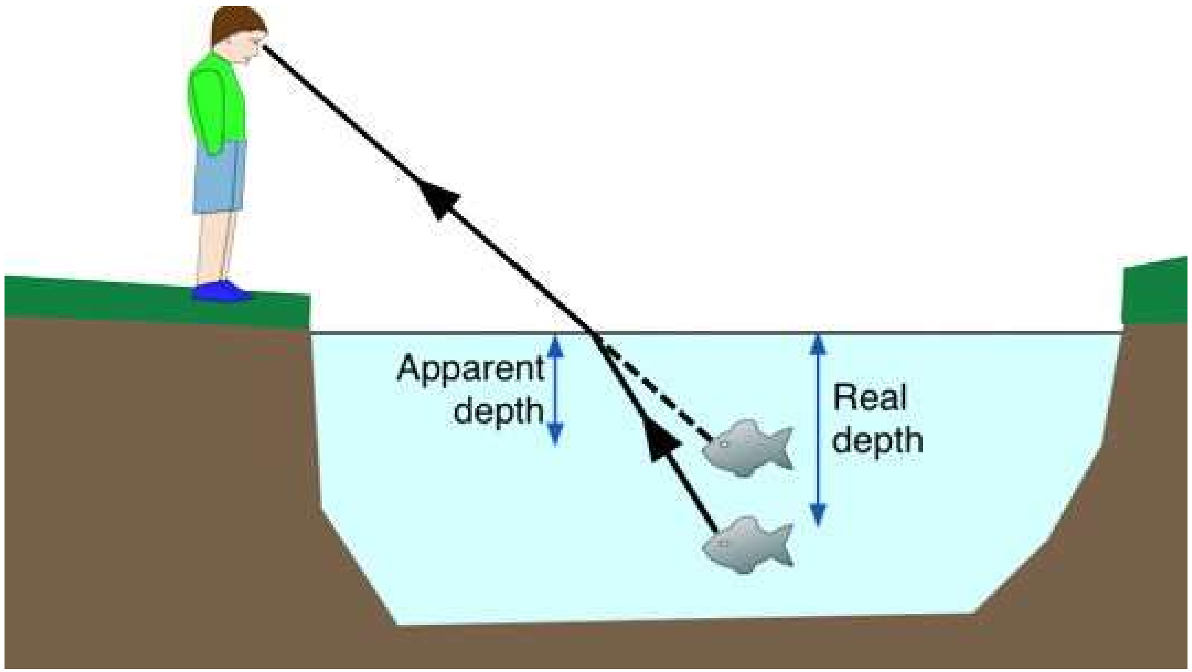
When R = ∞ (flat diopter) we have that: $$\frac{n_1}{p} + \frac{n_2}{q} = 0$$

So, for example, the apparent depth of an object underwater is different from the real one. For the underwater depth of the image, we have: $$q = -\frac{n_2}{n_1}\, p$$

The "-" means that q corresponds to a virtual image where the refracted rays do not really (physically) converge. If we look at objects in water from the air then:

$$q \approx -\frac{1}{1{,}33}\, p = -\,0{,}75\, p$$

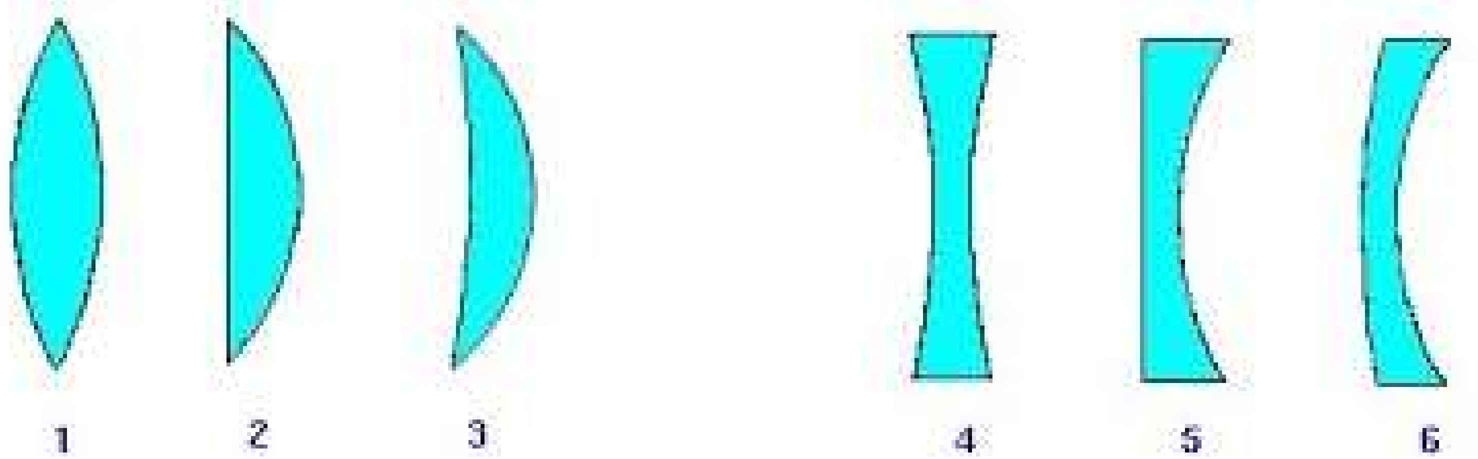
Objects appear closer than where they really are and also magnified.



# Thin lenses

Two spherical diopters facing and very close together form a thin lens, the thickness of which is negligible compared to the diameter and the radii of curvature of the spherical surfaces that surround it.

Thin lenses can be convergent (make parallel rays converge) or divergent (make parallel rays diverge). The biconvex (both convex surfaces)(1), the plano-convex (2) and the concave-convex (3) ones are converging lenses where the radius of the convexity, which is the second surface, is smaller (and therefore the curvature greater) than that of the concavity. The biconcave (4), the plano-concave (5) and the convex concave (6) are diverging.
All the nomenclature already introduced for the diopter and the approximation of paraxial rays applies.



# Law of conjugate points

For a thin lens the formula of conjugate points applies

$$\frac{1}{p}+\frac{1}{q}=\left(\frac{n_2}{n_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

If we put (so-called lens constructor equation):

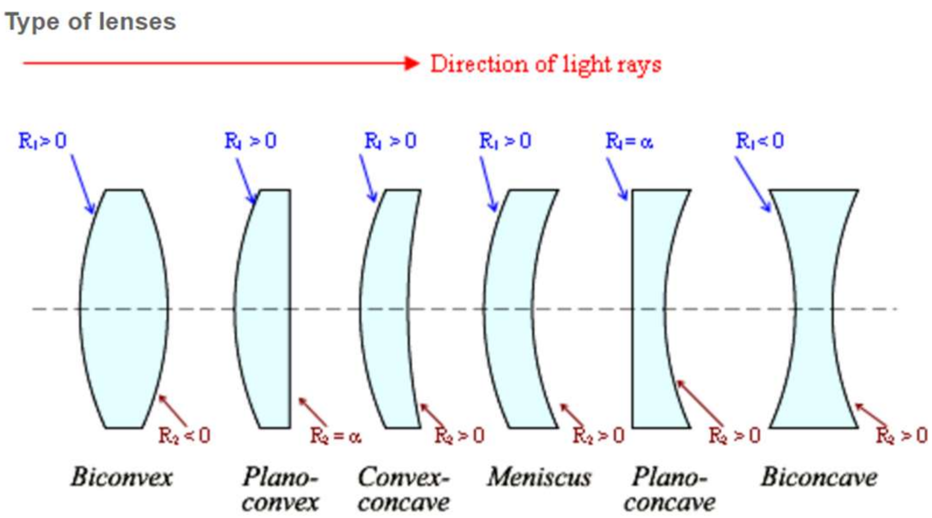
$$\frac{1}{f}=\left(\frac{n_2}{n_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

we have the **LAW OF CONJUGATED POINTS**:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

*f* = focal distance of the thin lens ( >0 for a converging lens forming real images, instead <0 for a diverging lens forming virtual images)

**CHARACTERISTICS OF THIN LENS TYPES IN AIR (as seen in the lens constructor equation, f depends on the medium in which we are)**



# Dioptric power (Optical power) and image formation

Dioptric power or power of a lens, D, is defined:

$$D = \frac{1}{f}$$

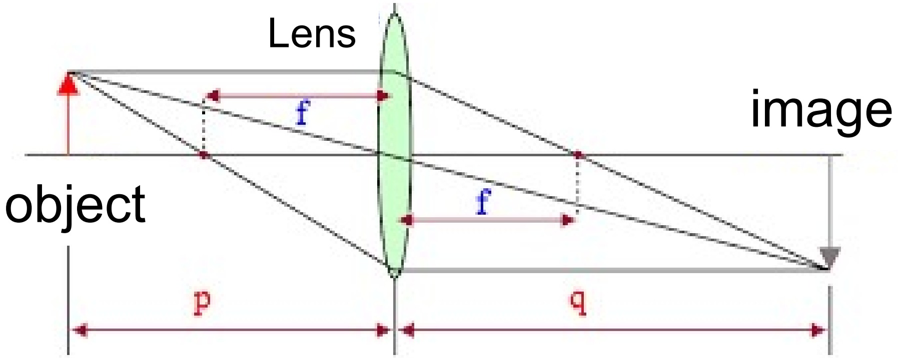
1 diopter = 1 $m^{-1}$

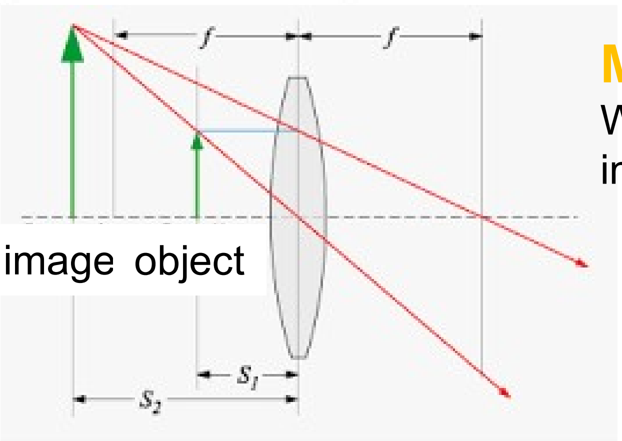
It is measured in diopters.

For example +2 diopters equals f = ½ m = 0,5 m

NOTE: The characteristics of a lens also depend on the refractive indices of the considered media. For example, a biconvex lens in air is positive or convergent (f> 0), but in water it can be divergent if the n of its material is less than 1.33.

## IMAGE FORMATION BY RAYS: CONVERGENT LENS

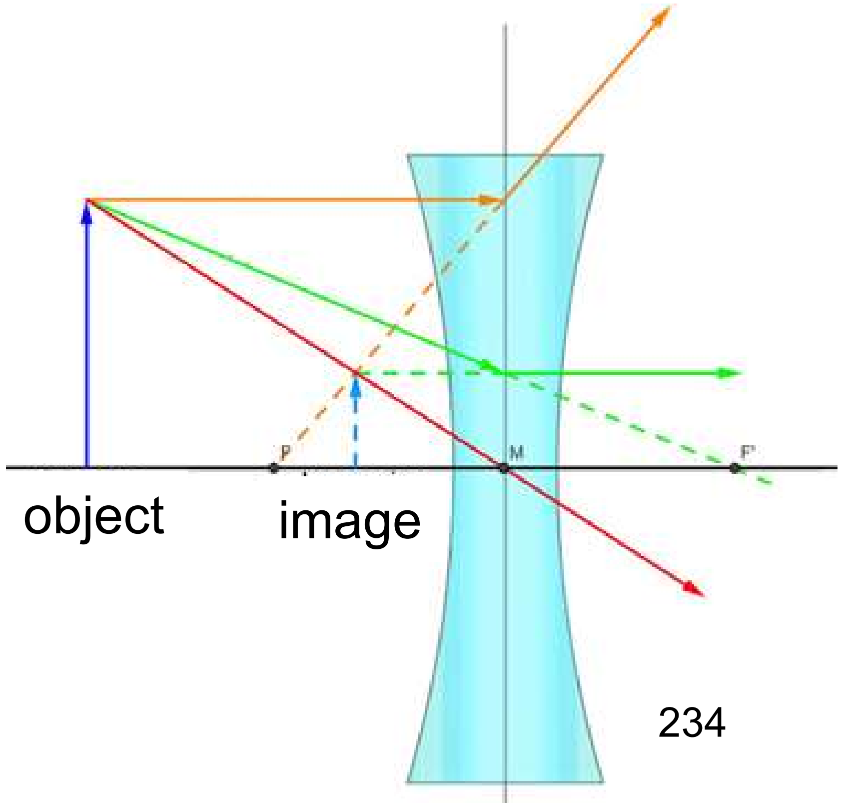



## Magnifying glass

When p <f then q <0 → left image is virtual.

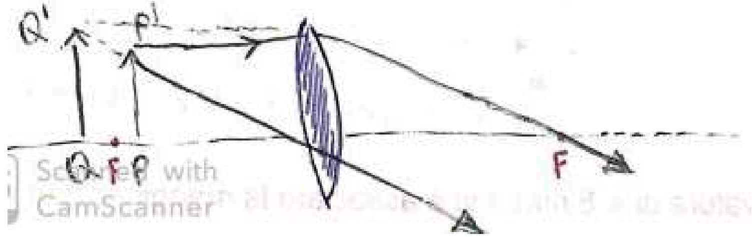
## IMAGE FORMATION BY RAYS: DIVERGENT LENS



# Linear magnification

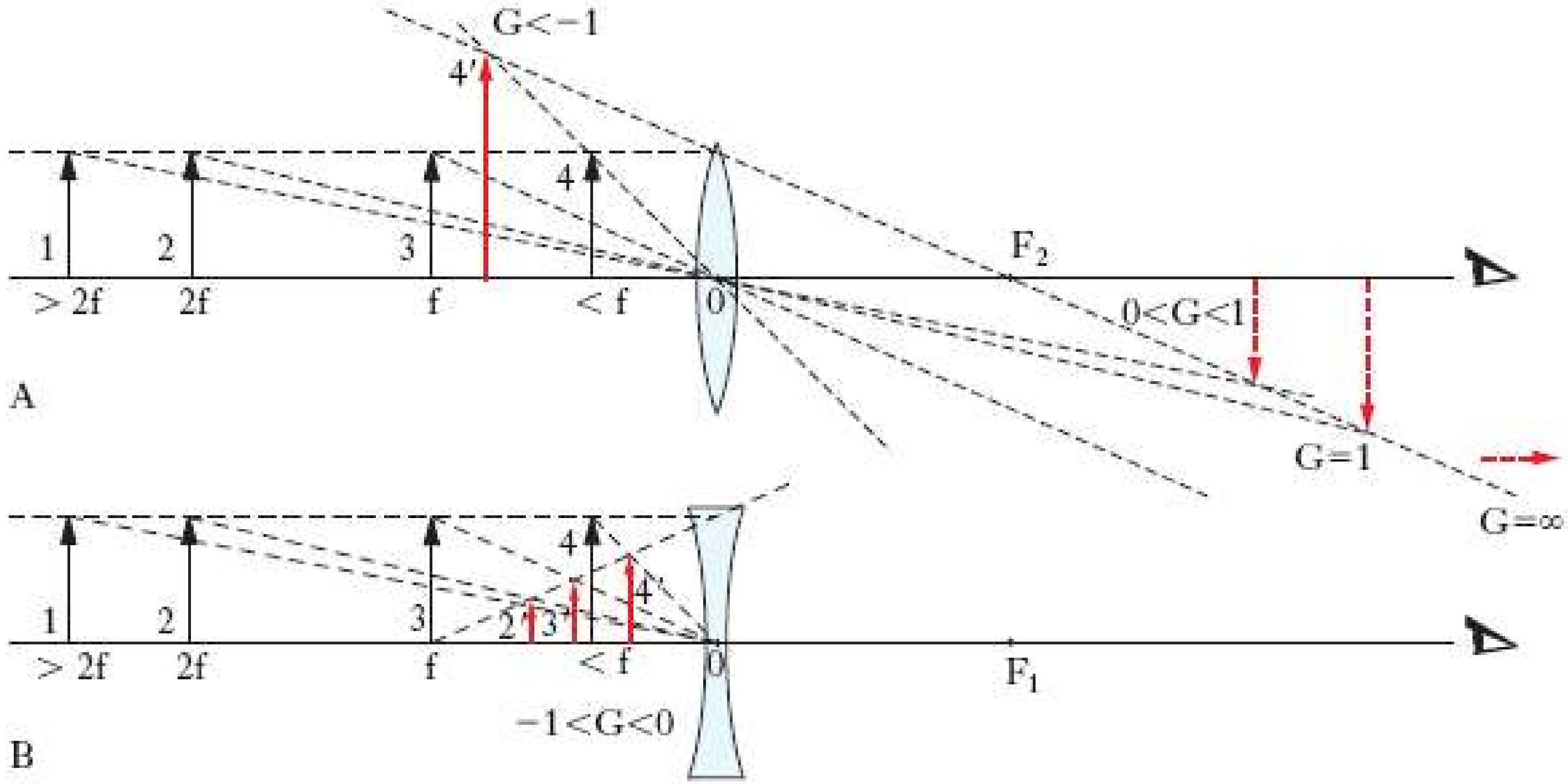
**LINEAR MAGNIFICATION G:**

$$G = \frac{QQ'}{pp'} = \frac{q}{p} = \frac{q-f}{f} = \frac{f}{p-f}$$



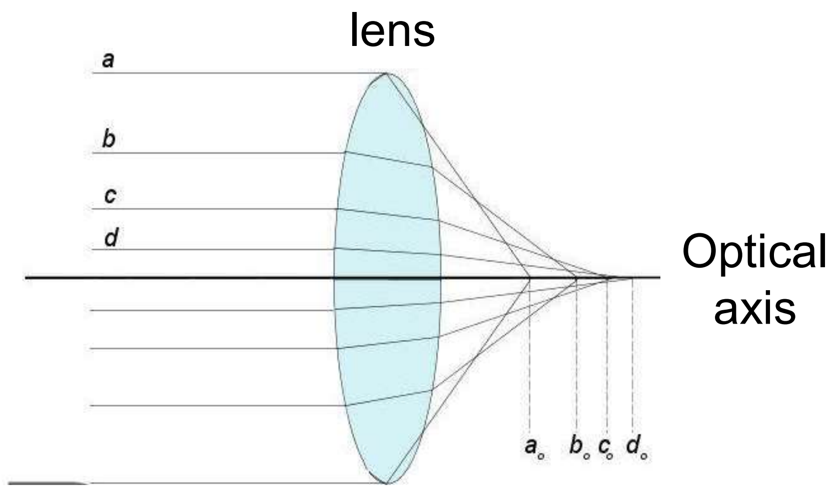

Optical axis

For the magnifying glass obviously G <0 and | G | > 1 since | p - f | <f.



# Aberrations: spherical and chromatic

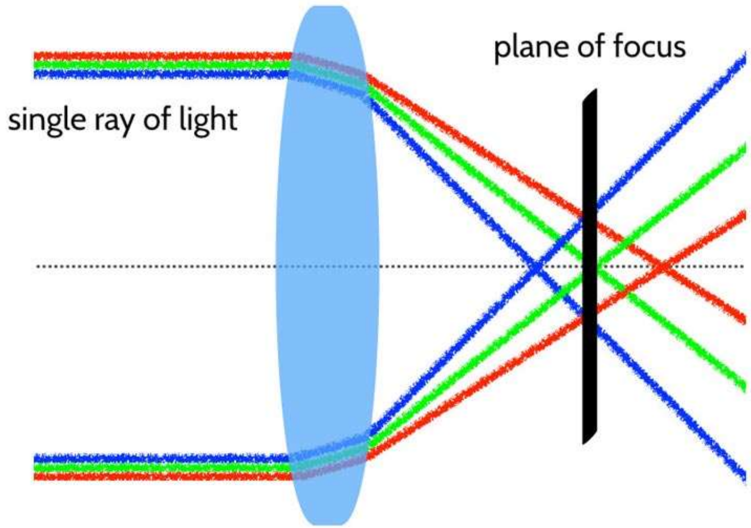
**Spherical aberration**: when the rays are not paraxial. The rays at a large angle to the optical axis are focused at different points. This creates a blurred image: a point is imagined in a segment. It is minimized by modifying the curvature of the surfaces or by shielding the outermost part of the lens.




**Chromatic aberration:** n depends on λ, so $f$ depends on λ.

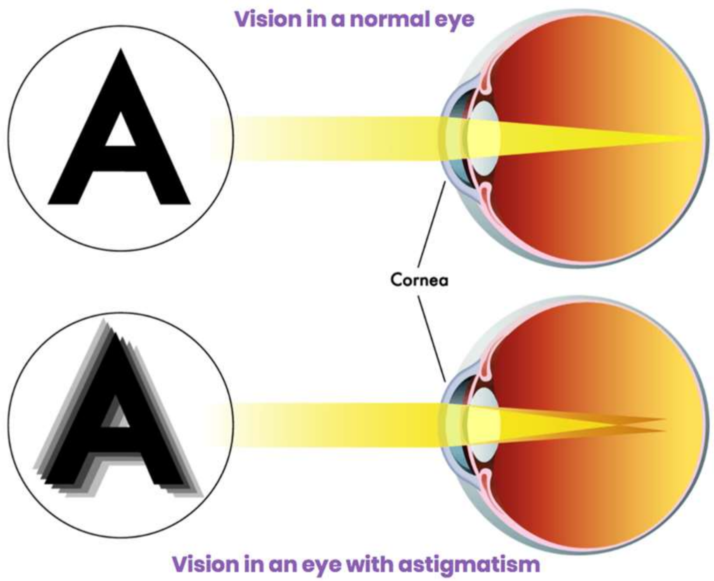
$$\frac{1}{f} = \left(\frac{n_2(\lambda)}{n_1(\lambda)} - 1\right) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \rightarrow f = f(\lambda)$$

A white source will result in images in different points placed at different distances q from the lens for each different color. The edges of the lens images will appear more or less iridescent. By coupling different lenses, converging and diverging, of different materials with suitable refractive indices, one compensates at the various frequencies and minimizes the defect (*achromatic lenses*).

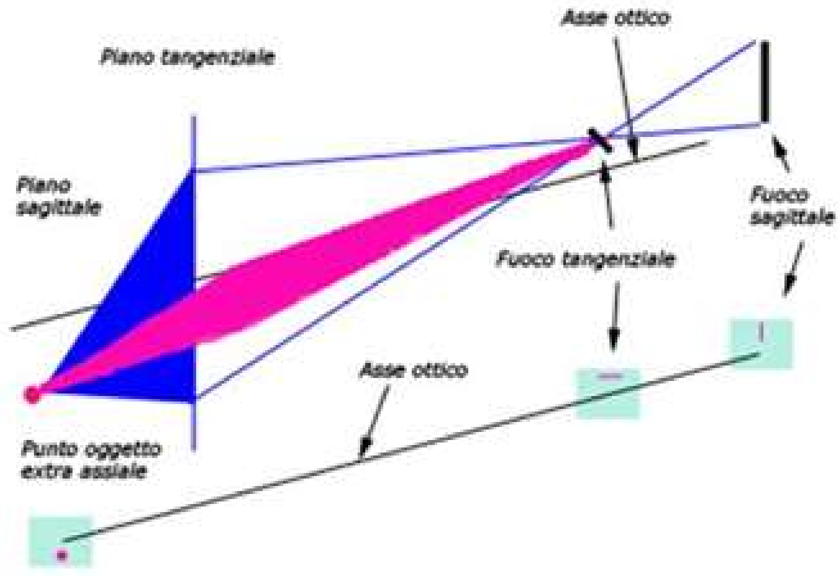


# Astigmatism

Astigmatism (from the Greek ἀ-, ἀν- a, an, lack, deprivation; and στιγμή stigmé, point, that is "non-point") is, in general, an aberration that occurs when the optical system (also the eye) is unable to form a point image of a point object. In general, it is due to morphological changes in the optical system, which does not have a constant radius of curvature. In the eye it is due to morphological changes of the cornea and lens.



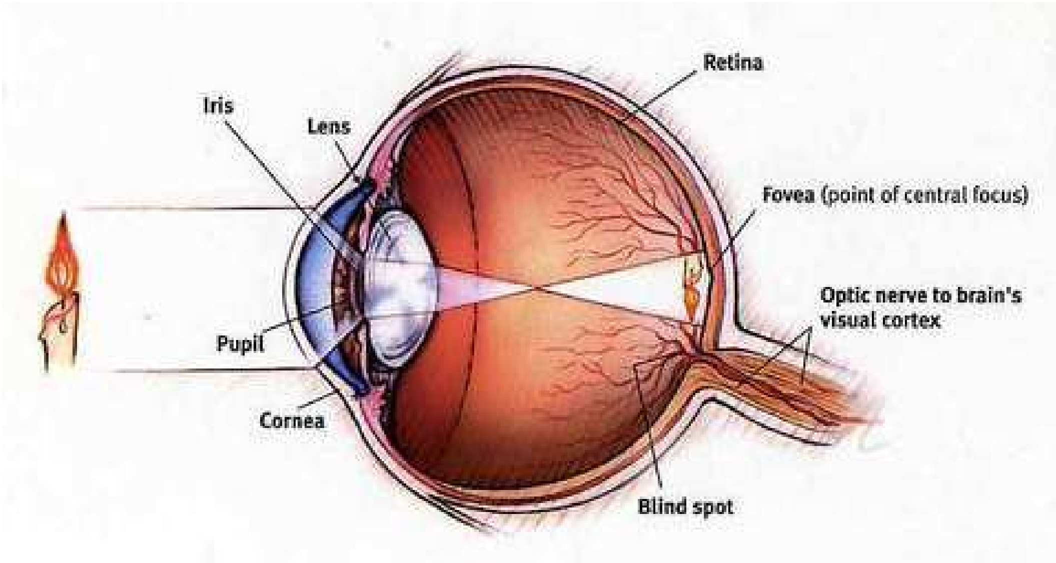

**Extra-axial astigmatism:** The point source does not lie on the optical axis and its rays are not all paraxial. The rays arriving from the source have a rotation symmetry different from that of rotation around the optical axis of the lens. The image corresponds to 2 orthogonal segments separated by a small distance.



# The human eye - anatomy

**Cornea (n = 1.33):** it has a radius of curvature of ≈ 8 mm and is associated with most of the dioptric power of the eye.

**Iris:** is the colored part of the eye; it is an elastic membrane with a central opening called the pupil. The contraction of the membrane regulates the size of the pupil and the light intensity.




**Crystalline (n = 1.44):** is a biconvex lens with variable curvature on the anterior surface regulated by the contraction of the ciliary muscles. The focal length of the lens is therefore variable.

**Vitreous humor (n = 1.33):** gelatinous substance.

**Retina:** photosensitive nervous tissue on which images are formed starting from photoreceptors, cones and rods. The images are transformed into action potentials (electrodynamic), sent to the brain for processing via the optic nerve.

**Macula:** formally called **macula lutea**, it is the central area of the human eye, the most sensitive to light stimuli. It has an oval shape and is located near the center of the retina. The macula in humans has a diameter of about 5.5 mm and is divided into fovea and foveola. The fovea is the area with the highest concentration of cones.

# Reduced eye model

The eye is assumed to be a single lens of variable "f". In relaxed conditions f = 17 mm, equal to the average distance between the center of the lens (of the eye or the lens) and the retina. In these relaxed eye conditions we have a far vision, p = ∞, and therefore, for anatomical reasons, the dioptric power is in the far vision conditions:

$$\frac{1}{f} = \frac{1}{0{,}017\ m} = 59\ \text{diopters}$$

Near point: The minimum distance that can be focused in the average normal adult (emmetrope) is 25 cm from the eye. In these conditions of near vision there is a dioptric power of:

$$\frac{1}{f} = \frac{1}{0{,}25\ m} + \frac{1}{0{,}017\ m} = 63\ \text{diopters}$$

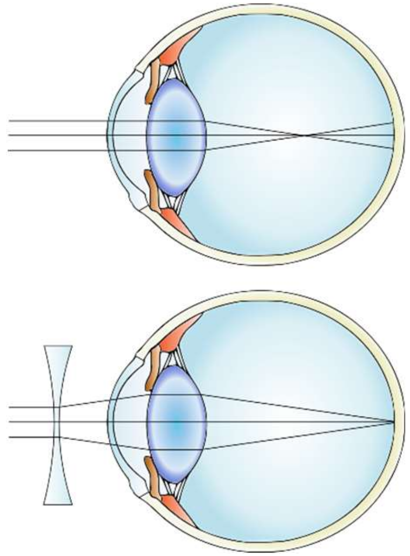
In the normal, emmetropic eye, the eye operates continuously for p = 25 cm - ∞ (~ 6 m).

# Defects of vision

**THE MAIN DEFECTS ARE DUE TO PROBLEMS OF REFRACTION OR ELASTICITY OF THE CRYSTALLINE OR STILL ANATOMICAL DEFECTS**

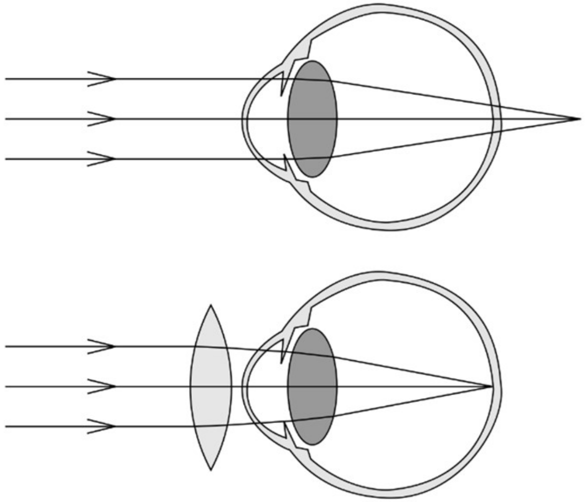
***MYOPIA***

The dioptric power is excessive compared to the size of the eye. The remote point (sometimes even the next) are focused before the retina. It is corrected by placing a diverging lens in front of the eye. The corrective image formed is virtual, straight and reduced.



***HYPERMETROPIA***

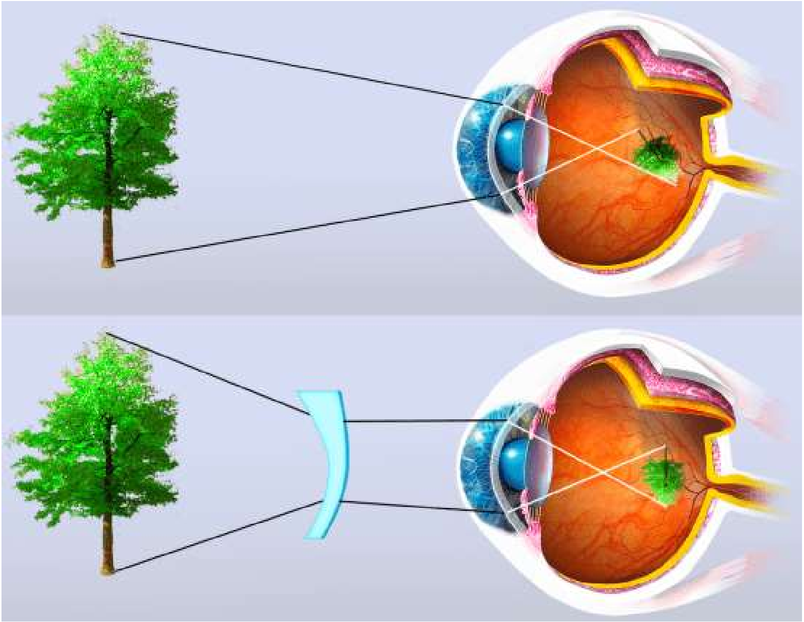
The size of the eye is too small compared to its dioptric power. The remote point is focused behind the retina. It is corrected by placing a converging lens in front of the eye. The corrective image is virtual, straight and enlarged.



# Defects of vision (2)

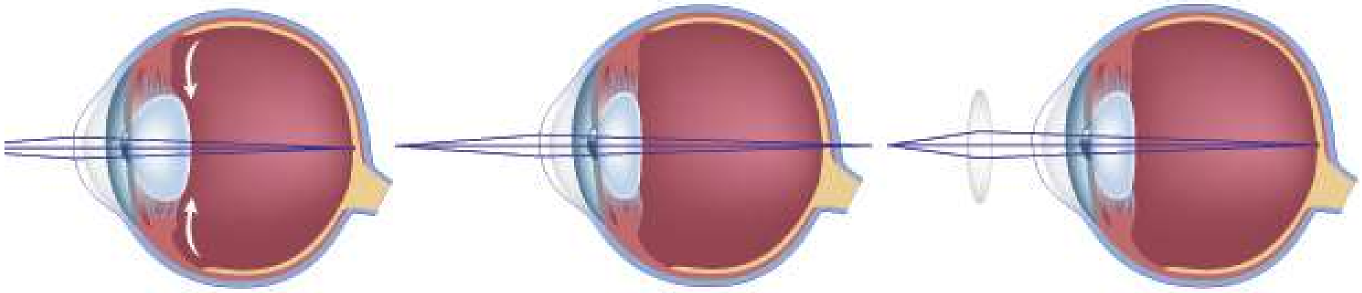
*ASTIGMATISM*

It is due to the imperfect sphericity of the cornea (refractive-anatomical defect). The radius of curvature along one axis is different from the radius of curvature along another axis, perpendicular to the first. It is treated with cylindrical lenses that influence the path of the light rays on one plane only, without affecting the refraction of other planes.



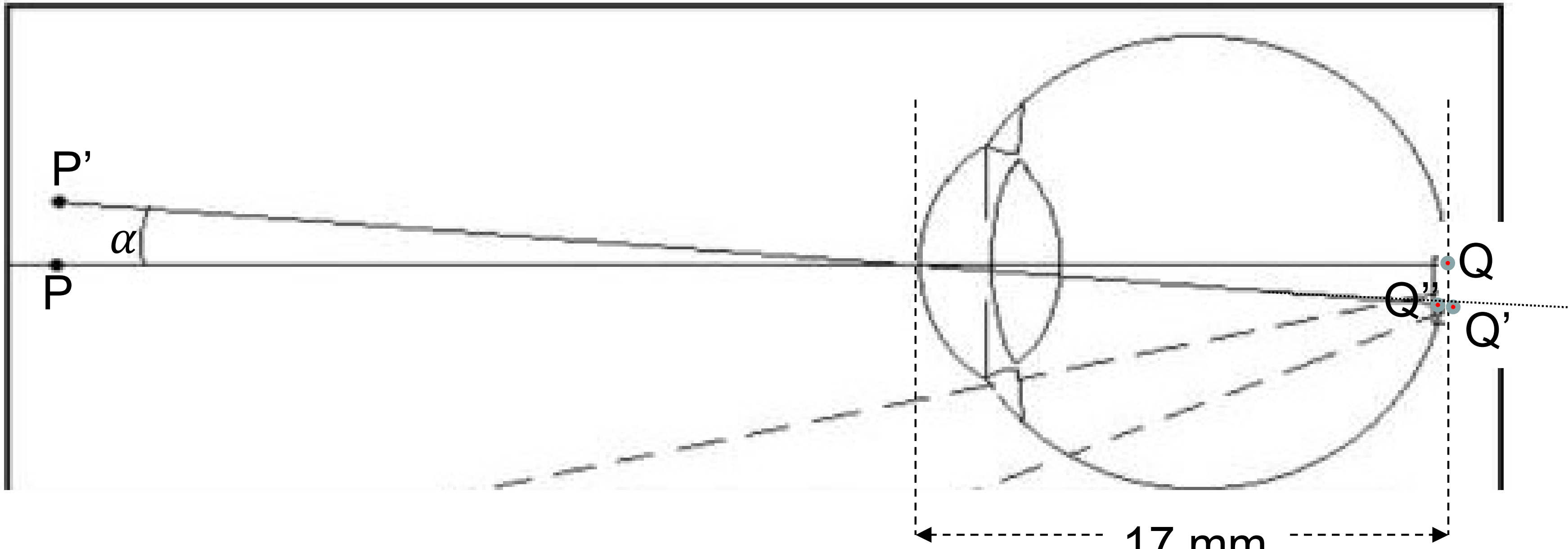
*PRESBYOPIA*

It is due to the aging of the ciliary muscles and the lens, which loses its elasticity and is unable to bend enough. It is treated with converging lenses such as hyperopia.



# Mechanism of vision: visual acuity

**Visual acuity:** 2 point sources P and P 'are seen as distinct when they illuminate non-contiguous receptors.




The diameter of the cones on the fovea is 1.5 μm. There the density of cones is highest. In order not to hit 2 contiguous cones, let's set QQ '≈ 4 μm. (QQ '' ≈ QQ '). From the trigonometry of right triangles we have:

$$\sin\alpha = \frac{QQ'}{0{,}017\ m} = \frac{4\cdot\ 10^{-6}m}{0{,}017\ m} \rightarrow\ \ \alpha\ \approx 3\cdot\ 10^{-4}\ rad.$$

From diffraction derives another limit that is touched when the distance between photoreceptors becomes comparable to the λ of light. This limit is smaller, in fact it results:

$$\alpha_{diffr.} \lesssim 1\cdot\ 10^{-4}rad. < 3\cdot\ 10^{-4}rad.$$

The limiting factor comes to be the structural detail of the retina.

# Mechanism of vision: color perception

The cones photoreceptors detecting the colors and are mainly in the fovea around the optical axis. The rods are distributed around the optical axis. The rods are evenly distributed on the retina (less on the fovea) and are 1000 times more sensitive than the cones. They have a single pigment, rhodopsin, which has an absorption peak at 500nm (blue - green). In these conditions we see objects with poor lighting (scotopic vision. Side vision is linked to the rods - "corner of the eye"). The lateral view in achromatic way allows to capture moving objects. At 15 ° from the optic axis on the nasal side there is a small blind area due to the entry of the optic nerve. We are not aware of it because the brain integrates in reconstructing images. The cones have 3 pigments:

**X** = generic mixed color

A, B, C = basic colors (usually blue, green, red)

$$X = aA + bB + cC$$

a,b,c = coefficients relating to the intensities of colors A, B, C to constitute X.

**Vector representation of colors** : X = (a,b,c)

# Problem solving from the book: E.M. field

**Example 24.4 Calculate Microwave Intensities and Fields**

On its highest power setting, a certain microwave oven projects 1.00 kW of microwaves onto a 30.0 by 40.0 cm area. (a) What is the intensity in $W/m^2$? (b) Calculate the peak electric field strength $E_0$ in these waves. (c) What is the peak magnetic field strength $B_0$?

**Strategy**

In part (a), we can find intensity from its definition as power per unit area. Once the intensity is known, we can use the equations below to find the field strengths asked for in parts (b) and (c).

**Solution for (a)**

Entering the given power into the definition of intensity, and noting the area is 0.300 by 0.400 m, yields

$$I = \frac{P}{A} = \frac{1.00\text{ kW}}{0.300\text{ m} \times 0.400\text{ m}}. \tag{24.21}$$

Here $I = I_{\text{ave}}$, so that

$$I_{\text{ave}} = \frac{1000\text{ W}}{0.120\text{ m}^2} = 8.33\times10^3\text{ W/m}^2. \tag{24.22}$$

Note that the peak intensity is twice the average:

$$I_0 = 2I_{\text{ave}} = 1.67\times10^4\text{ W/m}^2. \tag{24.23}$$

**Solution for (b)**

To find $E_0$, we can rearrange the first equation given above for $I_{\text{ave}}$ to give

$$E_0 = \left(\frac{2I_{\text{ave}}}{c\varepsilon_0}\right)^{1/2}. \tag{24.24}$$

Entering known values gives

$$\begin{aligned} E_0 &= \sqrt{\frac{2(8.33\times10^3\text{ W/m}^2)}{(3.00\times10^8\text{ m/s})(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}} \\ &= 2.51\times10^3\text{ V/m}. \end{aligned} \tag{24.25}$$

**Solution for (c)**

Perhaps the easiest way to find magnetic field strength, now that the electric field strength is known, is to use the relationship given by

$$B_0 = \frac{E_0}{c}. \tag{24.26}$$

Entering known values gives

$$\begin{aligned} B_0 &= \frac{2.51\times10^3\text{ V/m}}{3.0\times10^8\text{ m/s}} \\ &= 8.35\times10^{-6}\text{ T}. \end{aligned} \tag{24.27}$$

**Discussion**

As before, a relatively strong electric field is accompanied by a relatively weak magnetic field in an electromagnetic wave, since $B = E/c$, and $c$ is a large number.

# Problem solving from the book: refraction

## Example 25.2 Determine the Index of Refraction from Refraction Data

Find the index of refraction for medium 2 in Figure 25.12(a), assuming medium 1 is air and given the incident angle is $30.0^\circ$ and the angle of refraction is $22.0^\circ$.

**Strategy**

The index of refraction for air is taken to be 1 in most cases (and up to four significant figures, it is 1.000). Thus $n_1 = 1.00$ here. From the given information, $\theta_1 = 30.0^\circ$ and $\theta_2 = 22.0^\circ$. With this information, the only unknown in Snell's law is $n_2$, so that it can be used to find this unknown.

**Solution**

Snell's law is

$$n_1 \sin \theta_1 = n_2 \sin \theta_2. \tag{25.9}$$

Rearranging to isolate $n_2$ gives

$$n_2 = n_1 \frac{\sin \theta_1}{\sin \theta_2}. \tag{25.10}$$

Entering known values,

$$\begin{aligned} n_2 &= 1.00 \frac{\sin 30.0^\circ}{\sin 22.0^\circ} = \frac{0.500}{0.375} \\ &= 1.33. \end{aligned} \tag{25.11}$$

**Discussion**

This is the index of refraction for water, and Snell could have determined it by measuring the angles and performing this calculation. He would then have found 1.33 to be the appropriate index of refraction for water in all other situations, such as when a ray passes from water to glass. Today we can verify that the index of refraction is related to the speed of light in a medium by measuring that speed directly.

**The refractive index of water strongly depends on substances like salts and/or sugars that may be solvated. 1.33 is the value for pure water**

# Problem solving from the book: refraction

## Example 25.3 A Larger Change in Direction

Suppose that in a situation like that in Example 25.2, light goes from air to diamond and that the incident angle is $30.0^\circ$. Calculate the angle of refraction $\theta_2$ in the diamond.

**Strategy**

Again the index of refraction for air is taken to be $n_1 = 1.00$, and we are given $\theta_1 = 30.0^\circ$. We can look up the index of refraction for diamond in Table 25.1, finding $n_2 = 2.419$. The only unknown in Snell's law is $\theta_2$, which we wish to determine.

**Solution**

Solving Snell's law for sin $\theta_2$ yields

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1. \quad (25.12)$$

Entering known values,

$$\sin\theta_2 = \frac{1.00}{2.419}\sin 30.0^\circ = (0.413)(0.500) = 0.207. \quad (25.13)$$

The angle is thus

$$\theta_2 = \sin^{-1}0.207 = 11.9^\circ. \quad (25.14)$$

**Discussion**

For the same $30^\circ$ angle of incidence, the angle of refraction in diamond is significantly smaller than in water ( $11.9^\circ$ rather than $22^\circ$ —see the preceding example). This means there is a larger change in direction in diamond. The cause of a large change in direction is a large change in the index of refraction (or speed). In general, the larger the change in speed, the greater the effect on the direction of the ray.

Table 25.1 Index of Refraction in Various Media

| Medium | $n$ |
|---|---|
| ***Gases at $0^\circ C$, 1 atm*** | |
| Air | 1.000293 |
| Carbon dioxide | 1.00045 |
| Hydrogen | 1.000139 |
| Oxygen | 1.000271 |
| ***Liquids at $20^\circ C$*** | |
| Benzene | 1.501 |
| Carbon disulfide | 1.628 |
| Carbon tetrachloride | 1.461 |
| Ethanol | 1.361 |
| Glycerine | 1.473 |
| Water, fresh | 1.333 |
| ***Solids at $20^\circ C$*** | |
| Diamond | 2.419 |
| Fluorite | 1.434 |
| Glass, crown | 1.52 |
| Glass, flint | 1.66 |
| Ice at $20^\circ C$ | 1.309 |
| Polystyrene | 1.49 |
| Plexiglas | 1.51 |
| Quartz, crystalline | 1.544 |
| Quartz, fused | 1.458 |
| Sodium chloride | 1.544 |
| Zircon | 1.923 |

# Problem solving from the book: total internal reflection

## Example 25.4 How Big is the Critical Angle Here?

What is the critical angle for light traveling in a polystyrene (a type of plastic) pipe surrounded by air?

**Strategy**

The index of refraction for polystyrene is found to be 1.49 in Figure 25.14, and the index of refraction of air can be taken to be 1.00, as before. Thus, the condition that the second medium (air) has an index of refraction less than the first (plastic) is satisfied, and the equation $\theta_c = \sin^{-1}(n_2 / n_1)$ can be used to find the critical angle $\theta_c$. Here, then, $n_2 = 1.00$ and $n_1 = 1.49$.

**Solution**

The critical angle is given by

$$\theta_c = \sin^{-1}(n_2 / n_1). \quad (25.18)$$

Substituting the identified values gives

$$\theta_c = \sin^{-1}(1.00 / 1.49) = \sin^{-1}(0.671) \quad (25.19)$$
$$42.2^\circ.$$

**Discussion**

This means that any ray of light inside the plastic that strikes the surface at an angle greater than $42.2^\circ$ will be totally reflected. This will make the inside surface of the clear plastic a perfect mirror for such rays without any need for the silvering used on common mirrors. Different combinations of materials have different critical angles, but any combination with $n_1 > n_2$ can produce total internal reflection. The same calculation as made here shows that the critical angle for a ray going from water to air is $48.6^\circ$, while that from diamond to air is $24.4^\circ$, and that from flint glass to crown glass is $66.3^\circ$. There is no total reflection for rays going in the other direction—for example, from air to water—since the condition that the second medium must have a smaller index of refraction is not satisfied. A number of interesting applications of total internal reflection follow.

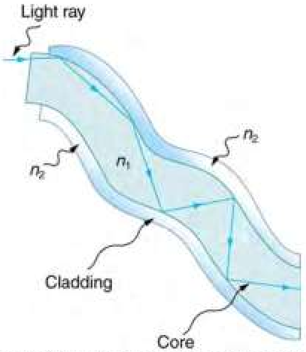



Figure 25.16 Fibers in bundles are clad by a material that has a lower index of refraction than the core to ensure total internal reflection, even when fibers are in contact with one another. This shows a single fiber with its cladding.

# Problem solving from the book: thin lenses

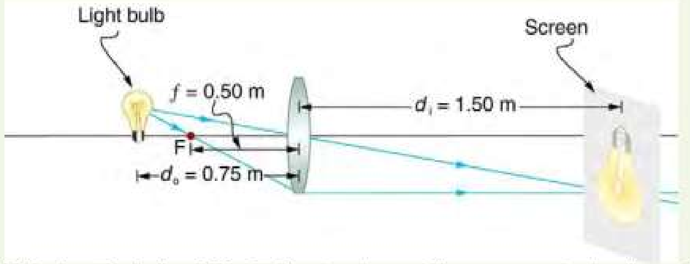



Figure 25.35 A light bulb placed 0.750 m from a lens having a 0.500 m focal length produces a real image on a poster board as discussed in the example above. Ray tracing predicts the image location and size.

**Strategy and Concept**

Since the object is placed farther away from a converging lens than the focal length of the lens, this situation is analogous to those illustrated in Figure 25.33 and Figure 25.34. Ray tracing to scale should produce similar results for $d_i$. Numerical solutions for $d_i$ and $m$ can be obtained using the thin lens equations, noting that $d_o = 0.750$ m and $f = 0.500$ m.

**Solutions (Ray tracing)**

The ray tracing to scale in Figure 25.35 shows two rays from a point on the bulb's filament crossing about 1.50 m on the far side of the lens. Thus the image distance $d_i$ is about 1.50 m. Similarly, the image height based on ray tracing is greater than the object height by about a factor of 2, and the image is inverted. Thus $m$ is about –2. The minus sign indicates that the image is inverted.

The thin lens equations can be used to find $d_i$ from the given information:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}. \quad (25.28)$$

Rearranging to isolate $d_i$ gives

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}. \quad (25.29)$$

Entering known quantities gives a value for $1/d_i$:

$$\frac{1}{d_i} = \frac{1}{0.500 \text{ m}} - \frac{1}{0.750 \text{ m}} = \frac{0.667}{\text{m}}. \quad (25.30)$$

This must be inverted to find $d_i$:

$$d_i = \frac{\text{m}}{0.667} = 1.50 \text{ m}. \quad (25.31)$$

Note that another way to find $d_i$ is to rearrange the equation:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}. \quad (25.32)$$

This yields the equation for the image distance as:

$$d_i = \frac{f d_o}{d_o - f}. \quad (25.33)$$

Note that there is no inverting here.

The thin lens equations can be used to find the magnification $m$, since both $d_i$ and $d_o$ are known. Entering their values gives

$$m = -\frac{d_i}{d_o} = -\frac{1.50 \text{ m}}{0.750 \text{ m}} = -2.00. \quad (25.34)$$

**Discussion**

Note that the minus sign causes the magnification to be negative when the image is inverted. Ray tracing and the use of the thin lens equations produce consistent results. The thin lens equations give the most precise results, being limited only by the accuracy of the given information. Ray tracing is limited by the accuracy with which you can draw, but it is highly useful both conceptually and visually.

**Example 25.6 Finding the Image of a Light Bulb Filament by Ray Tracing and by the Thin Lens Equations**

A clear glass light bulb is placed 0.750 m from a convex lens having a 0.500 m focal length, as shown in Figure 25.35. Use ray tracing to get an approximate location for the image. Then use the thin lens equations to calculate (a) the location of the image and (b) its magnification. Verify that ray tracing and the thin lens equations produce consistent results.

# Problem solving from the book: diverging lenses

## Example 25.8 Image Produced by a Concave Lens

Suppose an object such as a book page is held 7.50 cm from a concave lens of focal length −10.0 cm. Such a lens could be used in eyeglasses to correct pronounced nearsightedness. What magnification is produced?

**Strategy and Concept**

This example is identical to the preceding one, except that the focal length is negative for a concave or diverging lens. The method of solution is thus the same, but the results are different in important ways.

**Solution**

To find the magnification $m$, we must first find the image distance $d_i$ using thin lens equation

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}, \tag{25.39}$$

or its alternative rearrangement

$$d_i = \frac{f d_o}{d_o - f}. \tag{25.40}$$

We are given that $f = -10.0\text{ cm}$ and $d_o = 7.50\text{ cm}$. Entering these yields a value for $1/d_i$:

$$\frac{1}{d_i} = \frac{1}{-10.0\text{ cm}} - \frac{1}{7.50\text{ cm}} = \frac{-0.2333}{\text{cm}}. \tag{25.41}$$

This must be inverted to find $d_i$:

$$d_i = -\frac{\text{cm}}{0.2333} = -4.29\text{ cm}. \tag{25.42}$$

Or

$$d_i = \frac{(7.5)(-10)}{(7.5-(-10))} = -75/17.5 = -4.29\text{ cm}. \tag{25.43}$$

Now the magnification equation can be used to find the magnification $m$, since both $d_i$ and $d_o$ are known. Entering their values gives

$$m = -\frac{d_i}{d_o} = -\frac{-4.29\text{ cm}}{7.50\text{ cm}} = 0.571. \tag{25.44}$$

**Discussion**

A number of results in this example are true of all case 3 images, as well as being consistent with Figure 25.39. Magnification is positive (as predicted), meaning the image is upright. The magnification is also less than 1, meaning the image is smaller than the object—in this case, a little over half its size. The image distance is negative, meaning the image is on the same side of the lens as the object. (The image is virtual.) The image is closer to the lens than the object, since the image distance is smaller in magnitude than the object distance. The location of the image is not obvious when you look through a concave lens. In fact, since the image is smaller than the object, you may think it is farther away. But the image is closer than the object, a fact that is useful in correcting nearsightedness, as we shall see in a later section.

# Problem solving from the book: e.m. induction

**Example 23.3 Calculating the Emf Induced in a Generator Coil**

The generator coil shown in Figure 23.20 is rotated through one-fourth of a revolution (from $\theta = 0°$ to $\theta = 90°$ ) in 15.0 ms. The 200-turn circular coil has a 5.00 cm radius and is in a uniform 1.25 T magnetic field. What is the average emf induced?

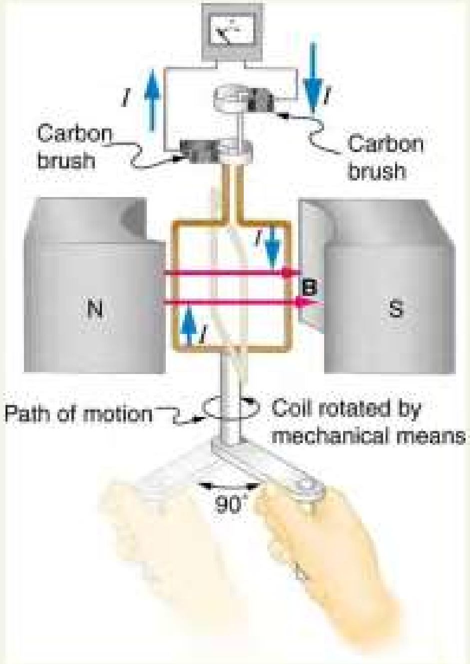



Figure 23.20 When this generator coil is rotated through one-fourth of a revolution, the magnetic flux $\Phi$ changes from its maximum to zero, inducing an emf.

**Strategy**

We use Faraday's law of induction to find the average emf induced over a time $\Delta t$:

$$\text{emf} = -N\frac{\Delta\Phi}{\Delta t}. \tag{23.11}$$

We know that $N = 200$ and $\Delta t = 15.0\text{ ms}$, and so we must determine the change in flux $\Delta\Phi$ to find emf.

**Solution**

Since the area of the loop and the magnetic field strength are constant, we see that

$$\Delta\Phi = \Delta(BA\cos\theta) = AB\Delta(\cos\theta). \tag{23.12}$$

Now, $\Delta(\cos\theta) = -1.0$, since it was given that $\theta$ goes from $0°$ to $90°$. Thus $\Delta\Phi = -AB$, and

$$\text{emf} = N\frac{AB}{\Delta t}. \tag{23.13}$$

The area of the loop is $A = \pi r^2 = (3.14...)(0.0500\text{ m})^2 = 7.85\times10^{-3}\text{ m}^2$. Entering this value gives

$$\text{emf} = 200\frac{(7.85\times10^{-3}\text{ m}^2)(1.25\text{ T})}{15.0\times10^{-3}\text{ s}} = 131\text{ V}. \tag{23.14}$$

**Discussion**

This is a practical average value, similar to the 120 V used in household power.