Ionizing radiations

Ionizing radiations: radiations having a minimal energy of the carriers (particles or photons) that exceeds the ionization potential, E_p, typical of atoms and molecules, ≈ 10 eV. (1 eV = $1{,}6 \cdot 10^{-19}$ J)

Non-ionizing radiations: $E_{min} < E_p < 10$ eV

MAIN IONIZING RADIATIONS

α RAYS (α particles): they are charged (+ 2 e) and constituted by 2p e 2n. They are associated to decay of atomic nuclei.

β RAYS (β particles): they are charged (- e, β^-) or (+ e, β^+). They are made of electrons or positrons of high energy. They penetrate 100 times less than α through biological matter.

X RAYS, γ RAYS: they are photons, i.e. electromagnetic neutral radiation.

E_x ~100 eV – 10/100 KeV (X band); E_γ > 100 KeV (γ band)

Another natural radiation source comes from the unstable heavy nuclei having galactic origin. E.g.: unstable isotopes of Uranium and Thorium which decay, emetting α. Radon belongs to this group. It is found as a gas exiting some rocks, and can be inspired and successively can decay into lungs. It constitutes the main component of the so-called natural radiation. Also ^{40}K is important that decays into ^{39}K: it enters our food chain.

Radioactive decays: decay law

α e β radiations are produced by nuclear transformation processes. If *N*, the number nuclei taht can decay, is large, within the *Δt* time slot the number of decaying nuclei will be:

$$\Delta N(t) = -\lambda N(t) \cdot \Delta t \qquad (1)$$

ΔN(*t*) is the negative variation of *N* nuclei in the initial state, *N*(*t*).

λ is constant, only depends on the nature of the decay.

It is a purely statistic process. Each nucleous in independent of the other in its decay. Let us integrate equation (1) by variable separation.

$$\frac{\Delta N}{N} = -\lambda \cdot \Delta t$$

$$\int_{N_0}^{N(t)} \frac{dN}{N} = -\lambda \int_0^t dt'$$

$$\ln\left(\frac{N(t)}{N_0}\right) = -\lambda t \qquad N_0 = N(t=0)$$

$$N(t) = N_0 e^{-\lambda t}$$

λ = probability that the decay will take place within Δ*t*. [*λ*] = [*t*]$^{-1}$

Lifetime and halflife

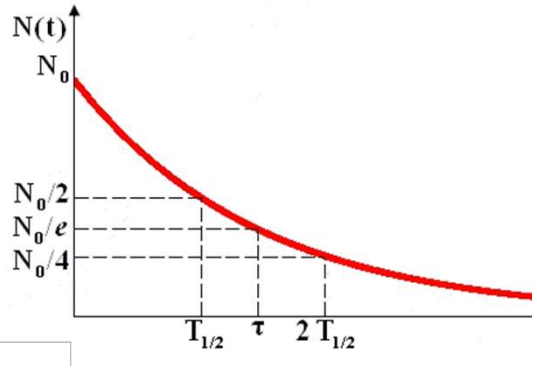
The process is characterized by its halflife, $t_{1/2}$. It is defined as:

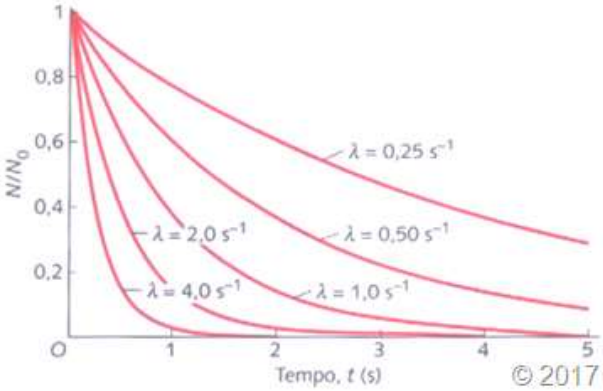
$$N\left(t_{1/2}\right) = \frac{N_0}{2} = N_0 e^{\left(-\lambda\, t_{1/2}\right)} \rightarrow t_{1/2} = \frac{1}{\lambda} ln2$$

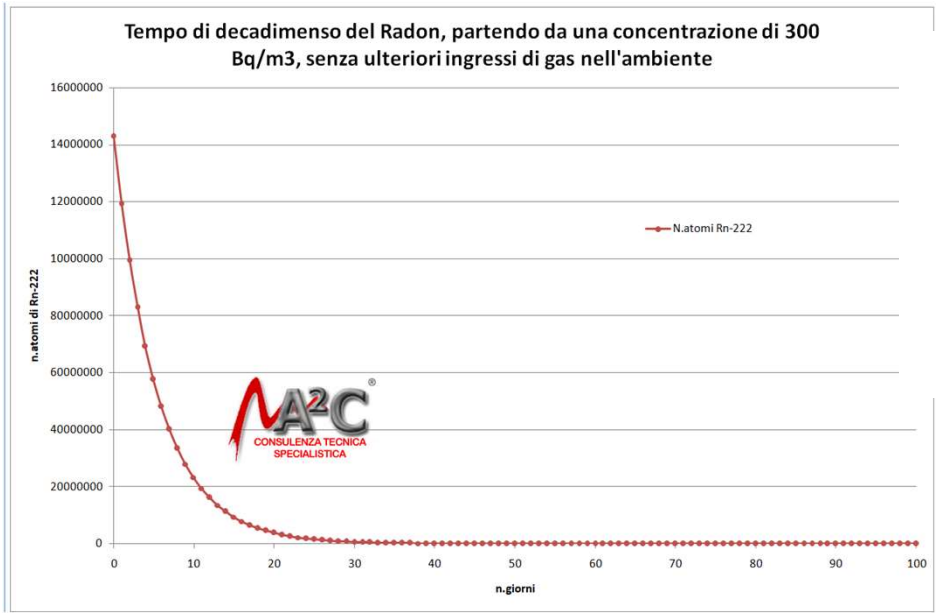
Lifetime, τ

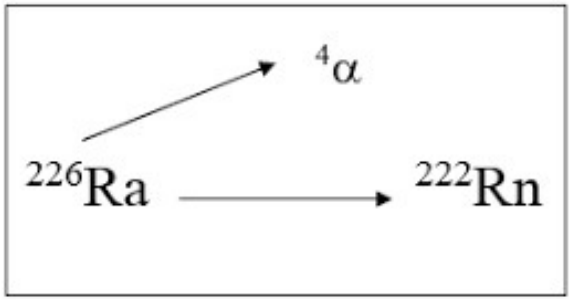
$$\tau = \frac{1}{\lambda} = \frac{t_{1/2}}{\ln 2}$$

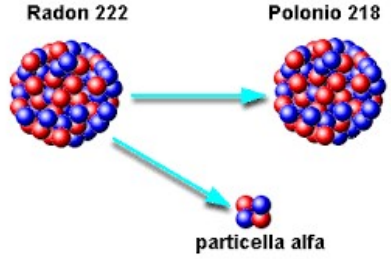
τ = average time before the nucleous decays.











Activity

The activity of a radioactive source is the number of its decays per unit time.

$$A\,(t) = \lambda\, N\,(t) = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$$

$$A_0 = \lambda\, N_0$$

A (t) is measured as the number of decays per second. Its unit is the Bequerel (Bq). 1Bq = 1 decay./s

We use also the Curie (Ci). 1 Ci is the activity of 1g of ^{226}Ra that decays by α with $t_{1/2}$ = 1620 years. 1Ci a large quantity of radioactivity. Also mCi or μCi are used. The α sources of ^{241}Am have A ~ 30 Bq (used for smoke detection). The legal limit for the natural radioactivity is set to 200 Bq/m^3 as the year average and to 300 Bq/m^3 into strategic buildings. For a PET (Positron Emission Tomography) we inject into the patient radioactive material having A ~ mCi ≈ 10^9 Bq. Radioisotopes for anti-tumor therapy have A ~ 10^{14} Bq. A of the human body is A ≈ 100 Bq/Kg.

1 Ci = 3,7 · 10^{10} Bq

If there are several decay processes occurring at the same, λ_1, λ_2, λ_3.., we can write:

$$\lambda_{TOT} = \lambda_1 + \lambda_2 + \lambda_3 \ldots.$$

given the statistic nature of independent decay processes.

Biological halflife

$t_{1/2\,B}$ = biological halflife, required by the organim to eliminate half of the radioactive substance, through purely biological processes.

We can define λ_B as:

$$\frac{1}{\lambda_B} = \frac{t_{1/2B}}{\ln 2}$$

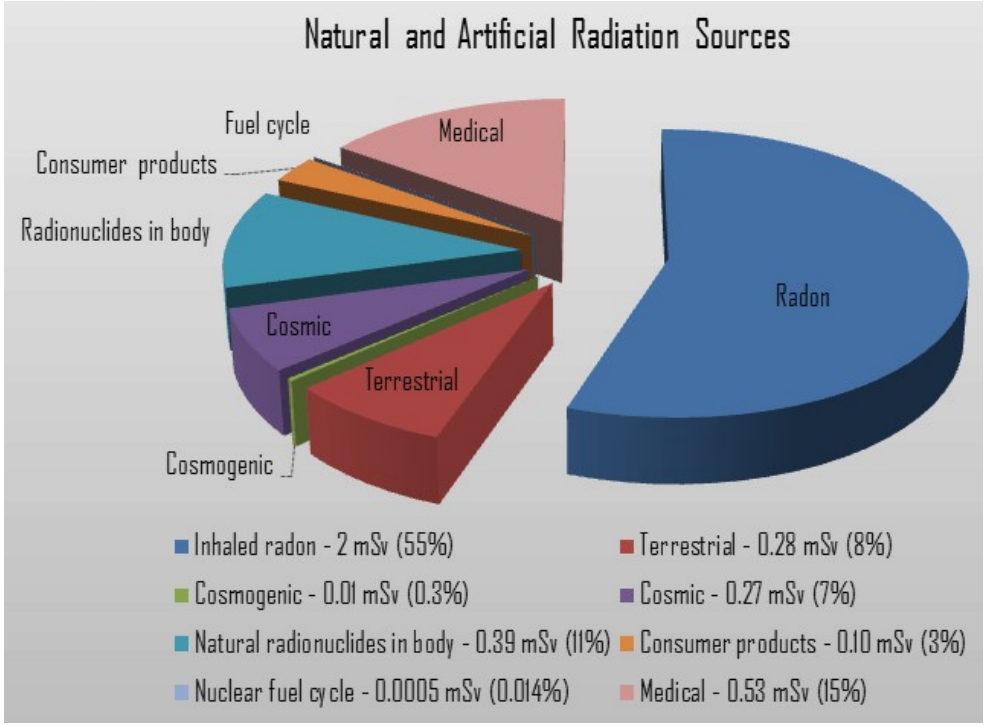
$$\lambda_{TOT} = \lambda + \lambda_B$$

Thus:

$$\frac{1}{t_{1/2TOT}} = \frac{1}{t_{1/2}} + \frac{1}{t_{1/2B}}$$

Radionuclides used in medicine

	Half-life
americium - 241	433 years
cesium - 137	30.1 years
cobalt - 58	71.3 days
iodine - 124	4.17 days
iodine - 130	12.4 hours
iodine - 131	8.041 days
iron - 52	8.3 hours
manganese - 52	5.63 days
molybdenum - 99	66.02 hours
technetium – 99m	6.02 hours



Alpha decay

It is due to a nucleous which decays to assume a less energetic configuration, while emitting a ^{4}He (α particle). The force responsible for alpha decay is the **strong nuclear**. The total electric charge and mass number keep constant over the process. The scheme is:

$$(Z, N, A) \rightarrow (Z-2, N-2, A-4) + \alpha$$

The residual nucleous is different from the initial one with an atomic number i.e. number of protons, *Z*, that decreases by 2 and mass number, *A*, by 4. A typical example is:

$$^{238}\text{U} \rightarrow ^{234}\text{Th} + \alpha$$

It is spontaneous since favorable from the energetic point of view

$$\text{m}(^{234}\text{Th}) + \text{m}(\alpha) < \text{m}(^{238}\text{U})$$

A quantity of energy is, then, liberated into the process:

$$\Delta E = \Delta m \cdot c^2 = 4{,}274\ MeV \qquad \Delta m = \text{m}(^{238}\text{U}) - \text{m}(^{234}\text{Th}) - \text{m}(\alpha)$$

This excess energy is the total kinetic ebergy acquired by the fragments, K_{TOT}

By imposing momentum conservation over the process:

$$\frac{K_{fr1}}{K_{fr2}} = \frac{m_2}{m_1}$$

Then the kinetic energy, going to the α particle, is:

$$K_\alpha = \frac{A-4}{A}\Delta E = 4{,}20\ MeV \qquad K_\alpha \approx \Delta E \text{ when } A \rightarrow \infty$$

α decays are typical of the heavy elements, beyond Pb. The sons-nuclei can further decay, when still heavy and unstable, so to generate series of radioactive decay. Energies carried by α are in the range of ~ MeV with $t_{1/2} \sim 10^9$ years. (long, $t_{1/2} \sim 4{,}5 \cdot 10^9$ years in case of ^{238}U).

Beta decay

It is a process basically other than α. It is due to the **weak nuclear** force. The net effect is to transform within the nucleous n → p or p → n with A (mass number) = const.

Typical examples are:

^{14}C (Z = 6, N = 8) → ^{14}N (Z = 7, N = 7)

^{11}C (Z = 6, N = 5) → ^{11}B (Z = 5, N = 6)

Processes will be spontaneous if favorbale from the energetic point of view. In the former example we have production of β^-, in the latter of β^+. The scheme of the former process is:

$$(Z, N, A) \rightarrow (Z + 1, N - 1, A) + e^- + \bar{\nu}_e$$

where $\bar{\nu}_e$ is the electron anti-neutrin needed to explain the energy spectrum of the exiting β, otherwise with no $\bar{\nu}_e$ a they should have a unique and fixed value for their kinetic energy. On the contrary, experimens demonstrate that we have a continuous spectrum within an energy window.

$\bar{\nu}_e$ practically does not interact with matter. It is proven that:

$$< K(e^-) > = \frac{1}{3}\Delta E$$

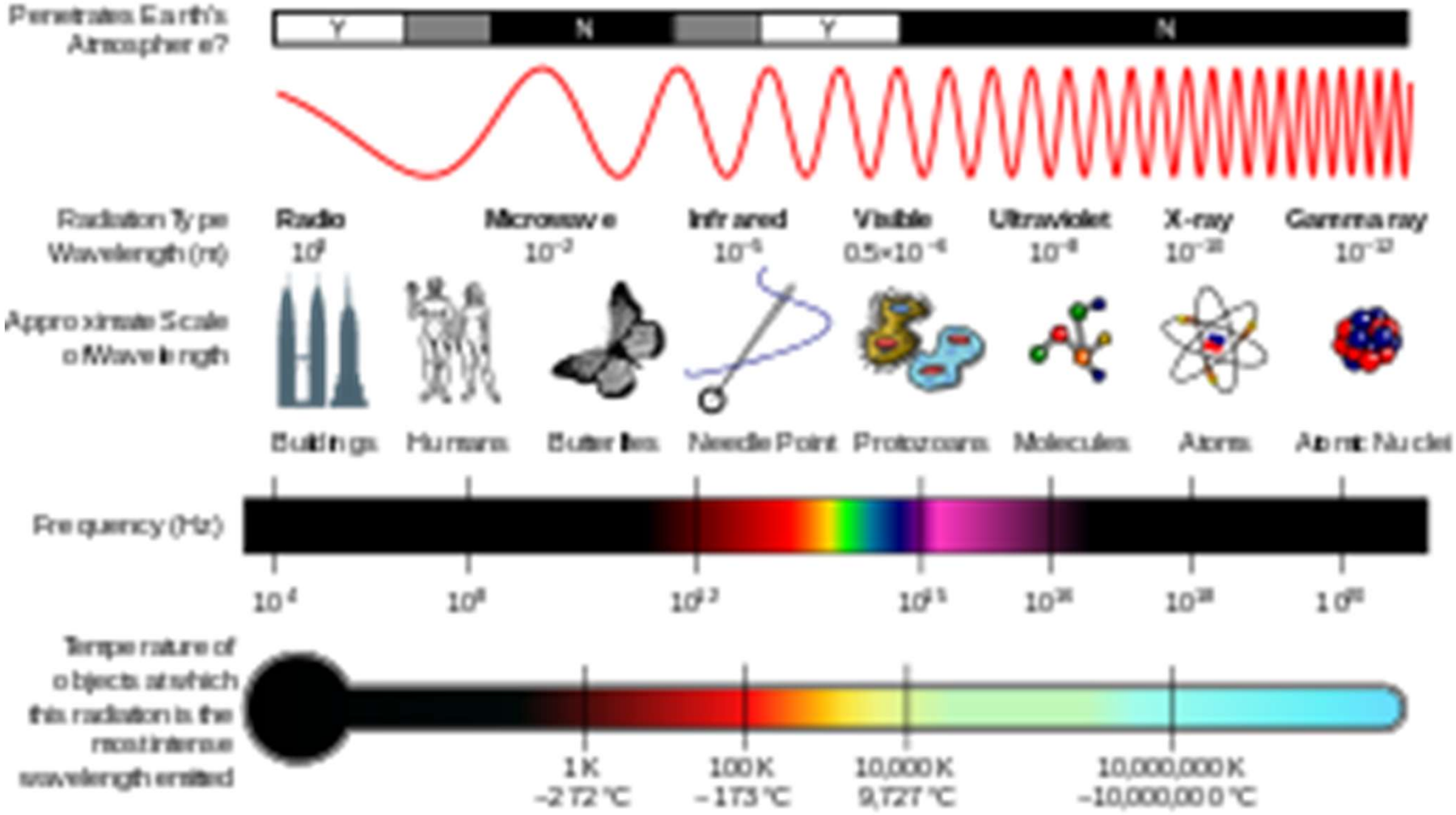
For β^+ we have:

$$(Z, N, A) \rightarrow (Z - 1, N + 1, A) + e^+ + \nu_e$$

From the β^- decay of ^{14}C we have β^- emission with E_{max} = 156 keV and $t_{1/2}$ = 5730 years. From the β^+ decay of ^{12}C we have β^+ emission with E_{max} = 1,98 MeV and $t_{1/2}$ = 20 mins (they are used for PET).

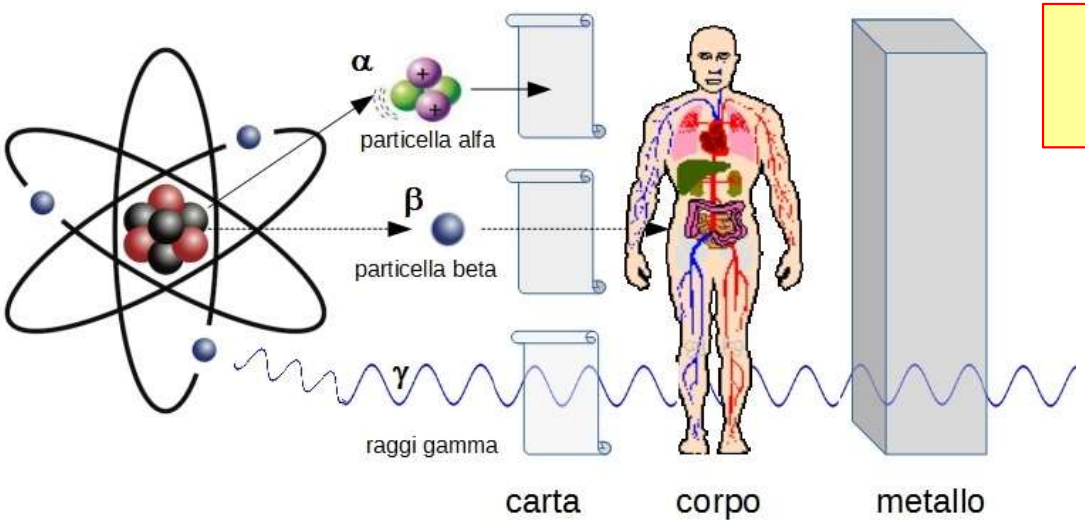
Gamma rays

γ rays are due to transitions between nuclear energetic levels. Their photons have energy in the order of MeV. For the X rays we have 10 – 100 keV photon energies. γ are absorbed very quickly by the surrounding environment.



Interactions with the biological matter and dosimetry

The three different kinds of ionizing radiations have different penetration depth into matter as they have a different probability to be absorbed.



$$D\ (dose) = \frac{Total\ absorbed\ energy}{mass}$$

Into the I.S. *D* in J/Kg (Gray, Gy). We use also rad (Radiation Absorbed Dose). 1Gy = 100 rad.

The interaction type and impact with biological matter strongly depend on the radiation type. α particles strongly interact. α with 5 MeV are fully absorbed in 3.5 cm of air, 0.2 mm of soft tissue or 21 μm of Al. We can define the LET (Linear Energy Transfer). For β the behavior is similar, but LET is much smaller and the path much more spread out. β of 1 MeV are absorbed in 420 cm of air or 0.5 cm of soft tissue and 0,15 cm of Al.

Photons are either absorbed or not. The beam intensity decreases with the travelled thickness.

γ and X rays absorption: Lambert-Beer law

$I = I_0 e^{-\mu x}$ I = beam intensity, μ is related to the half-life thickness $x_{1/2}$.

$$x_{1/2} = \frac{\ln 2}{\mu}$$

$$I\left(x_{1/2}\right) = \frac{I_0}{2}$$

For instance in Pb:

X (100 KeV) → $x_{1/2}$ = 0,3 mm
γ (1 MeV) → $x_{1/2}$ = 8 mm
Le e.m. radiations have low LET values.

RBE: Relative Biological Efficacy
RBE (reference): 1 for X rays of 200 KeV

RBE of a radiation is the scaling factor to obtain the same fixed biological effects as the reference. Radiations with RBE = 2 means that D = 1Gy of that radiation has the same effect as D = 2 Gy of X rays with 200 KeV. We have:

RBE = 1 γ 1-1,7 β 10 P 10-20 α 20 heavy ions

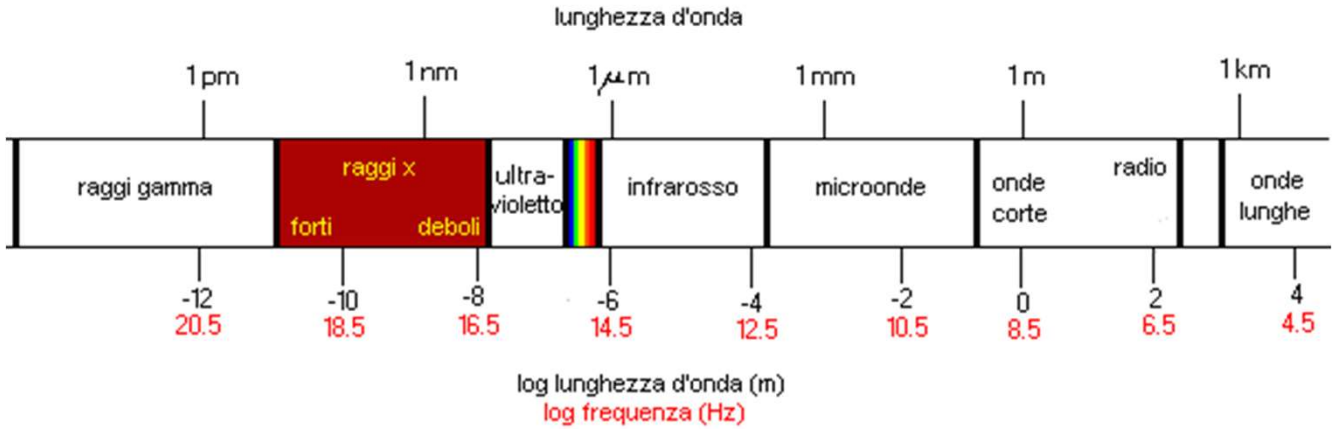
Equivalent Dose = DOSE · RBE = D · RBE

In the I.S. it is measured in Sievert (Sv). We also use the rem (Rontgen Equivalent in Man). 1Sv = 100 rem.

X rays

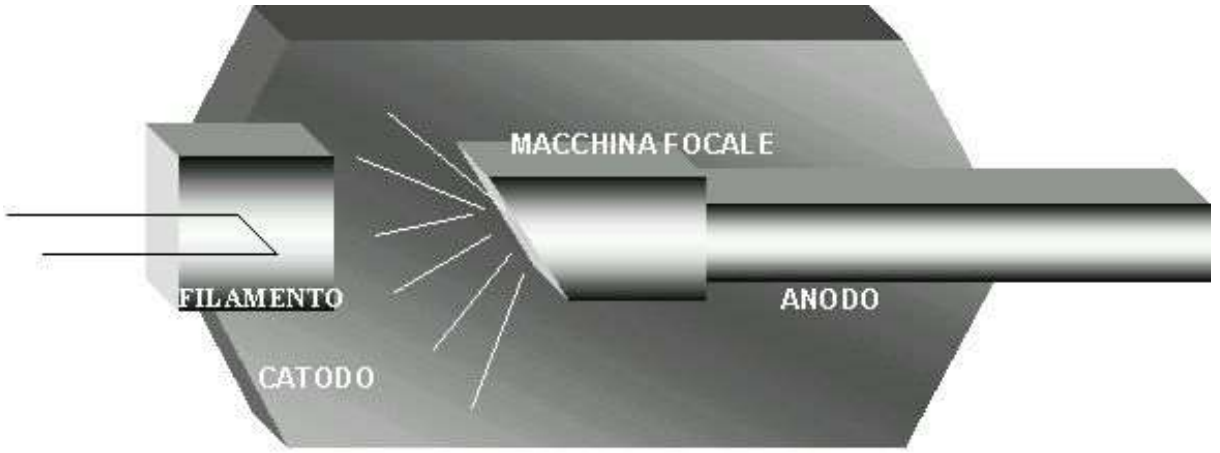
X-rays are em waves having a short wavelength (compared to visible light) and high energy per photon, $E = h\nu = h\frac{c}{\lambda}$.

Typically, this radiation corresponds to the window of the em spectrum within $0.1\text{Å} \leq \lambda \leq 100\text{Å}$ ($1\text{Å}=10^{-10}\text{m}$). Correspondingly, as for the photon energy X rays extends within the $124\ \text{keV} \leq E \leq 0.12\ \text{keV}$ band. X-ray wavelengths most commonly used (for material analysis and medical diagnostics) fall within the $0.7\text{Å} \leq \lambda \leq 2.5\text{Å}$. As for the visible light radiation emission corresponds to an excited atom that returns to the ground state, also for X-rays the emitted photon energy corresponds to the energy gap between two atomic electron levels; the only difference is that the involved levels for X-rays are much closer to the nucleous, where electrons are tightly bound.



Generation of X rays: the Coolidge machine

X-ray radiation is emitted when matter is irradiated by a high energy electron beam (primary radiation) or other X-rays (secondary X-rays). An electron beam generator is made by a tube containing a metallic filament with large atomic number and low ionization potential (e.g. Tungsten). A cw high current flows through the filament (cathode) under high vacuum. Due to Joule effect the filament becomes really hot until reaching incandescence. In such condition the thermoionic effect takes place: electrons get such a high kinetic energy (high temperature) to escape from the metal. To reach such a regime the filament is supplied by a cw tension delivering a power of about 3kW. The vacuum into the tube reaches a pressure of 10^{-6}mmHg. By applying a strong cw electric field to the tube ($30\text{kV} \leq \Delta V \leq 60\text{kV}$) the electron beam is accelerated and focused onto a target (anode). To finally obtain X-rays this second target have a high atomic number, e.g. Molibdenum. As a consequence of the irradiation of the Molibdenum disk with the accelerated electron beam we have X-ray production.



X-ray Generator layout

X ray generation mechanisms

Several phenomena can take place into the anode disk. If the primary beam is constituted by accelerated electrons we can have X-ray emission because of bremsstrahlung (breaking) of the fast electrons into the material. If the primary beam is contituted by X-rays we can have absorption due to photo-electric effect and/or scattering of the primary X-rays. The transmitted beam is the portion of the incident beam that travels through the anode with no interaction. The outcoming radiation will have lower energy, being the outcoming intensity:

$$I_{out} \propto (1/d) \cdot Z$$

where *d* is the crossed thickness of the anode and *Z* the anode atomic number. The overall outcoming beam will have, in general, a longer wavelength than the incident, since an incident X photon can be absorbed and liberate a material electron with high kinetic energy; this fast electron can be braked into the material (again bremsstrahlung) and contextually emits a softer X photon (secondary X rays), having a longer wavelength.
Scattering occurs when the incident X photon interacts with a target electron. If the collision is elastic the scattered radiation will have the same wavelength as the incident one; if the collision is inelastic the outcoming radiation will have a different wavelength.

X-rays are used in radiography, CT scan and radiotherapy

Problem solving

LIQUIDS - A wooden cube with side L = 20 cm and density $\rho_c = 0.65 \times 10^3$ kg/m^3 floats partially immersed in water. Determine the height h of the emerged fraction of the cube.

a) 7 cm b) 10 cm c) 12 cm d) 14 cm
e) 19 cm

SOLUTION…

At equilibrium we have $m_c g = F_{A,}$ where the Archimedes force is given by $F_A = \rho_{H2O} \cdot L^2 \cdot L \cdot (1 - h) \cdot g$.

Solving for *h*, we obtain: $h = L \cdot \left(1 - \frac{\rho_c}{\rho_{H2O}}\right) = 7\ cm$

Problem solving

OPTICS - An object is located 1 m underwater (n_{H2O} = 1.33) and is observed from the air. How deep into water does the object seem to be?

a) 2.25 m b) 1.50 m **c) 0.75 m** d) 3.0 m e) 3.75 m

SOLUTION….

The refractive index of air can be taken as 1. We can consider this case as a plane diopter (a spherical diopter having infinite radius) for which we can write:

$\frac{n_{H2O}}{p} + \frac{n_{air}}{q} = 0$. Then, solving for *q* we obtain:

$q = -\frac{p}{n_{H2O}} = -0.75\, m$, where the minus sign ahead is due to the fact that the image is virtual.

Problem solving

WAVES - A typical red blood cell has a radius of 5×10^{-6} m. Doppler displacement, measured by flowmeters, is based on the reflection by red blood cells. Flowmeters devices use ultrasonic frequencies. If the frequency of the source is 1×10^{7} Hz, how many red blood cells will be included in a wavelength of sound (v_{sound} in the blood approximately equal to that in water equal to 1498 m/s)?

a) 1500 b) 0.15 c) 150 **d) 15**
e) 1.5

SOLUTION…..

We readily get $N_{red\ blood\ cells} = \frac{\lambda}{2r} = \frac{v_{sound}}{f \cdot 2r} = 15$

Problem solving

ELECTRICITY - A thin and flat membrane separates a layer of positive ions on the outside of the cell from a layer of negative ions inside. If the electric field due to these charges is 10^7 N/C, find the surface charge density on the side of the membrane in $\mu C \cdot m^{-2}$, knowing that the two sides are in modulus identically charged.

a) 88.5 b) 107 c) 442 d) 44.2 e) 56.3

SOLUTION……

The electric field into the flat membrane between the two ion layers can be written as $E=\sigma/\varepsilon_0$, as it was the field into a capacitor. Then, we have for σ:

$$\sigma = E \cdot \varepsilon_0 \approx 89\ \mu C$$

Problem solving

RADIATIONS - 200 X-photons pass through 10 cm of wood (μ = 0.1609 cm^{-1}). How many X photons survive after the wooden tablet?

a) 400 103 b) 64 c) 100 **d) 40**
e) 400

SOLUTION…….

From the Lambert-Beer law for the radiation absorption we can write, by expressing the intensity in terms of number of X photons:

$N(x) = N_0 e^{-\mu \cdot x}$. Then, knowing μ = 0.1609 cm^{-1}, we obtain N(10 cm) = 40,01 $\approx$ 40.

Problem solving

THERMODYNAMICS. A mercury thermometer is made of a glass capillary having 0.1 mm diamater and a volume of 1 cm^3. What length is necessary for the capillary to measure an expansion corresponding to a temperature variation of 100 °C? Linear thermal expansion coefficient of Hg = 1.82×10^{-4} $(°C)^{-1}$ (neglect glass thermal expansion).

a) 180 b) 60 c) 600 d) 360 **e) 230**

$$l = l_0 \cdot \lambda \cdot \Delta T$$

THERMODYNAMICS. 10 kg of water at 0 °C are mixed with 10 kg of water at 100 °C. The entropy variation of the system is in cal/K (neglect heat exchange with external):

a) -168 b) -720 c) 1000 d) 480 **e) 240**

$$\Delta S = 10kg \cdot 1000cal/kg \cdot K \left[ln\left(\frac{323}{373}\right) + ln\left(\frac{323}{273}\right) \right]$$

Problem solving

THERMODYNAMICS. Calculate the heat in kcal necessary to convert 1 kg of ice at 0 ° C in water vapor at 100 ° C [L_f = 3.33 × 10^5 J/kg; L_v = 2.26 × 10^6 J/kg; 1 cal = 4.186 J, c_s^{water} = 1,00 cal/(g ·° C)]?

a) 360 b) 640 c) 180 **d) 720** e) 620

$$\Delta Q=\Delta Q_{fus}+\Delta Q_{heating}+\Delta Q_{evap}=L_f \cdot m+m_c \cdot \Delta T+L_v \cdot m=719.4\ kcal$$

THERMODYNAMICS. A human having a body surface 1.9 m^2 wears a dress with thickness of 0.8 cm. The dress tissue thermal conductivity is k = 10^{-5} kcal·s^{-1}·m^{-1}·K^{-1}. If the skin temperature is 33.5 ° C and the external temperature is 4 ° C, calculate the heat given to the external 1h.

a) 252.3 kcal b) 252.3 Jc) 1056.12 kcal d) 1056.12 J e) 252.3 cal

$$\Delta Q = \frac{k \cdot S \cdot \Delta T \cdot \Delta t}{d} = 252.225\ kcal$$