Angolo formato da due vettori

$$\cos\theta = \frac{\langle \vec{v},\, \vec{u}\rangle}{\|\vec{v}\|\|\vec{u}\|}, \qquad \vec{v},\vec{u}\in\mathbb{V}^n,\ \vec{v},\vec{u}\neq\vec{0}$$

che permette di calcolare l'angolo tra due vettori non nulli di cui siano note le componenti.

Per esempio, se $\vec{v}=(\sqrt{3},3)$ e $\vec{u}=(-2\sqrt{3},6)$, si ha $\langle \vec{v},\, \vec{u}\rangle = 12$, $\|\vec{v}\| = 2\sqrt{3}$, $\|\vec{u}\| = 4\sqrt{3}$, e quindi $\cos\theta = \frac{1}{2}$ da cui si ricava $\theta=\pi/3$.

$$(\sqrt{3},3)\cdot(-2\sqrt{3},6) = -2\sqrt{3}\cdot\sqrt{3}+3\cdot 6 = -2\cdot 3+18 = -6+18 = 12$$

$$\|\vec{v}\| = \|(\sqrt{3},3)\| = \sqrt{(\sqrt{3})^2+3^2} = \sqrt{3+9} = \sqrt{12} = \sqrt{4\cdot 3} = 2\sqrt{3}$$

$$\frac{\langle \vec{v},\vec{u}\rangle}{\|\vec{v}\|\cdot\|\vec{u}\|} = \frac{12}{2\sqrt{3}\cdot 4\sqrt{3}} = \frac{12}{24} = \frac{1}{2}$$

Assegnati i seguenti vettori $\mathbf{u}$, $\mathbf{v} \in \mathbf{R}^3$, calcolare il modulo di $\mathbf{u}$, il versore di $\mathbf{u}$, il vettore $\mathbf{u}+3\mathbf{v}$, il prodotto scalare $\mathbf{u}\cdot\mathbf{v}$ e l'angolo $\widehat{\mathbf{uv}}$:

$\mathbf{u}=(8,-2,2)$ $\quad \mathbf{v}=\left(\frac{1}{2},\frac{3}{2},-\frac{1}{2}\right)$

Risposta: $|\mathbf{u}|=6\sqrt{2}$, $\frac{\mathbf{u}}{|\mathbf{u}|}=\left(\frac{2\sqrt{2}}{3},-\frac{1}{3\sqrt{2}},\frac{1}{3\sqrt{2}}\right)$, $\mathbf{u}+3\mathbf{v}=\left(\frac{19}{2},\frac{5}{2},\frac{1}{2}\right)$, $\mathbf{u}\cdot\mathbf{v}=0$, $\widehat{\mathbf{uv}}=\frac{\pi}{2}$

$$\vec{u}=(8,-2,2) \qquad \vec{v}=\left(\frac{1}{2},\frac{3}{2},-\frac{1}{2}\right)$$

$$\|\vec{u}\|=\sqrt{8^2+(-2)^2+2^2}=\sqrt{64+4+4}=\sqrt{64+8}$$

$$=\sqrt{72}=\sqrt{36\cdot 2}=6\sqrt{2}$$

$$\frac{u}{\|\vec{u}\|}=\frac{1}{6\sqrt{2}}(8,-2,2)=\left(\frac{8}{6\sqrt{2}},-\frac{2}{6\sqrt{2}},\frac{2}{6\sqrt{2}}\right)=\left(\frac{4}{3\sqrt{2}},-\frac{1}{3\sqrt{2}},\frac{1}{3\sqrt{2}}\right)$$

$$4=2\cdot 2=2\cdot\sqrt{2}\cdot\sqrt{2} \qquad \frac{4}{3\sqrt{2}}=\frac{2\sqrt{2}\cancel{\sqrt{2}}}{3\cancel{\sqrt{2}}}=\frac{2\sqrt{2}}{3}$$

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$$\vec{u}+3\vec{v} = (8,-2,2)+3\left(\frac{1}{2},\frac{3}{2},-\frac{1}{2}\right)$$

$$=\left(8+\frac{3}{2}, -2+\frac{9}{2}, 2-\frac{3}{2}\right) = \left(\frac{19}{2},\frac{5}{2},\frac{1}{2}\right)$$