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***Cooperation in business - an application of
game theory***

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Abstract

In this paper we analyze if the cooperation will yield a better outcome than competition. It is shown that indeed cooperation helps achieving a higher result, but it depends especially on how much the firms are valuing the future compared to the present. This is the most important factor that has to be taken in consideration. If the situation that will be analyzed is not clear enough, other factors than the previously mentioned one have to be taken in consideration. There are other factors that influence cooperation and they are discussed into more detail in the paper. One other important aspect that influences the efficiency of the cooperation is the setting around which the players are competing; it can be competition in prices, quantities or at different stages of a R&D game.

1. Introduction

The general description of the business environment is as being competitive. The reason behind this description is that the aim of a business is to increase their profits by any means, while still taking in consideration the business ethics.

Franklin D. Roosevelt stated that '*Competition has been shown to be useful up to a certain point and no further, but cooperation, which is the thing we must strive for today, begins where competition leaves off.*'

The idea developed in this paper is that even though competition is the main state in which the business environment is found; through cooperation higher outcomes can be achieved. This will be proven by using game theory, which is one of the most effective tools that can be used when having to make a decision by thinking strategically. The game theory concepts related to cooperation in business will be presented theoretical by describing the actual concepts, but as well as practical by applying them in real-life scenarios.

Cooperation in business has been previously analyzed by a consistent number of researchers. Their analysis includes different cases and scenarios. Books in game theory have summarized the main findings related to cooperation in business while analyzing especially the cases of competition in prices or quantities. Therefore, the work of Jean Tirole '*The Theory of Industrial Organization*' and the work of Robert Gibbons '*A primer in Game Theory*' have been used in the current paper as the most important source of theoretical background. Moreover, there is a wide variety of academic articles which deal with different scenarios of cooperation in business. In the academic articles different assumptions are made in order to analyze different potential scenarios that can be encountered in the market, such as assumptions about costs or the type of information. For example, in the previous research it has been found that people, in different life situation, not necessarily in business, do not automatically cooperate; rather they have the incentive of competing (Berg 2010). Another research shows that collusion among firms competing in an oligopoly may be sustained in a static equilibrium (Shaffer 1995), while others have studied and showed that mergers between

firms that are competing in prices are advantageous, whereas mergers between firms competing through quantities have the opposite effect (Deneckere & Davidson 1985).

As it can be seen, the applications of game theory are appealing and interesting for many researchers, the beauty of it being that it can be applied in many fields, such as politics, biology or economics, just to name a few. More recently, Bruce Bueno de Mesquita, an academic at New York University has developed a computer software that can forecast human behavior. The software uses game theory and it predicted the events on the political stage of different countries (*The Economist*, September 3rd 2011).

In order to accomplish the purpose of the study, the paper is structured as follows. First, in the *research questions*, the questions that have to be answered are clearly presented. Second, the *methodology* of the paper is described, which includes the theoretical models that have been used as a tool and the reason behind the choice of models. The next step is stating with certainty the limits imposed in the study, in the section *limitations*. The *main body* of the paper represents the actual study analysis made in a hermeneutical way, where all the models that have been presented in the methodology part are used in order to accomplish the goal set by answering the questions stated in the *research questions* part. In order to show that cooperation is a better choice than competition, we have considered the cases when the firms compete in quantities, Cournot model, prices, Bertrand model, and a final case when the firms compete through R&D. After outlining directions for *further research*, we present the *conclusion* of the study through summarizing the findings.

The scope of this paper is to put together part of the research already existing in a manner which will have as a result the answer to the research questions.

2. Research questions

While looking at the business world that surrounds us, we can observe competition more often than cooperation. We are aware of the influence of competition on the economy and all its advantages, but what would be the case if the firms will try to cooperate, instead of competing? What will be the factors that would trigger cooperation? What will be the factors influencing cooperation?

Therefore, one could wonder how does cooperation in business work, namely how can the firms cooperate and what are its implications?

3. Methodology

We begin by describing the classic game of Prisoner's Dilemma in order to introduce the specific terms used throughout the study. Being an easy to understand example of a game, it was the perfect choice in order to introduce the specific terms used in the field of game theory and as well to show how the incentive of competing is part of the human nature, even in the case when it is known that cooperation yields a better off result.

After the reader has been familiarized with the terms which will be used, the business model taken in consideration as subject to discussion is introduced. On this model there have been applied the two types of competition, in prices and in quantities, as described by Bertrand and Cournot respectively. In each of the two cases of competition, there are considered three scenarios, namely pure competition or cooperation between the firms and the collusion-deviation case which is applied during a period of time.

Additionally, the model of D'Aspremont and Jacquemin presented in '*Cooperative and Noncooperative R&D in Duopoly with spillovers*' has also been interpreted and analyzed since it is one of the basic applications of competition in R&D.

In each of the cases discussed, a relevant real-life example has been revised as an application of the theory, except for the R&D section, where the existence of one real-life example has only been mentioned.

Therefore, *Real Case 1 - Cigarette Advertising on Television* is an application of Prisoner's Dilemma, while in *Real Case 2 – OPEC* is used to exemplify Cournot and Bertrand competition. As it will be seen in the analysis, since in the OPEC case the good produced is oil, which is a substitute good, the Cournot competition analysis is more relevant than the Bertrand competition. Furthermore, throughout the theoretical presentation, several potential real-life examples are used. It has to be kept in mind that the latter ones are fictional.

4. Limitations

The limitations of the paper have been established through the assumption made, by restricting the model.

The cases of incomplete information, when the firms are not certain about the other participants' in the market costs, have not been included in the analysis. The assumption that the model is one of complete information has been made in the beginning.

5. Cooperation vs. competition

While cooperation and competition are both good for the business, by having different impacts on it, the question is how you know when the perfect moment to compete is and when it is preferable to cooperate.

As many communication theorists have stated, among them Argyris and Schön (1996), people do not choose consciously how to react in an unfamiliar or threatening situation, they instinctively choose to respond defensively, engaging in a behavior that favors competition rather than cooperation (Berg 2010). Therefore, in a market, competition is the general environment that it is found, rather than cooperation.

Thus, the question that has to be answered is; what are the factors that can trigger cooperation? If we recall what Adam Smith states in *The Wealth of Nations*: 'It is not from the benevolence of the butcher, the brewer, or the baker that we expect our dinner, but from their regard to their own interest. We address ourselves, not to their humanity but to their self-love...', we have the perfect description of the market environment. The butcher, the brewer and the baker have the products we need and we have the money to buy them. The exchange, goods for money is only reasonable to think of. It is only the situation when any of the participants sees their interest and act in a way to obtain what they want (Sen 1993). What Adam Smith is describing is the relationship between consumers and producers but we will look beyond the obvious understanding of the statement described earlier. What generates the exchange of goods for money is actually the mutual interest in what the other participant in the trade has. Picturing the situation

in this way, describes exactly what the simplified version of the business environment should be. A trade can only take place if the participants have an interest in what the other participant has; whereas when thinking about cooperation, the situation is similar, the most important factor that motivates and facilitates the cooperation being the mutual interest. The difference that arises between the two situations is that in the cooperation scenario, the two participants, who later will be called players, have the mutual interest of increasing their profits.

We will assume that the most important condition, that through cooperation, higher profits will be achieved, is fulfilled. If there are two firms competing on the market by producing homogenous goods, cooperation is possible (as it will be shown in more detail further in the paper) by making an agreement whether to collude in prices or quantities. Even though they have made an agreement to collude, there is still subject to discussion if it will be the case or not; if the players would respect the agreement they have settled on or they will choose to deviate. When thinking about this decision, the players have to keep in mind that what makes cooperation possible to emerge is the fact that the players might meet again. This is important since the decisions that will be taken at the future encounter depend on the past experiences the players had together. That is the reason why the players have to be careful on the impact of their actions on the other participants in the game. On the other hand, when thinking about cooperation, one might think about how much they value the future compared to the present. First, the 'player' has to think that the value of the payoff is greater in the present than in the future and second is that there is also the possibility that the two players might not interact again for different reasons.

Keeping these reasons in mind, we have to think on what we expect from a cooperation partner. The most important action we would require from a partner is to respect the agreement made. Testing the Red-Blue Exercise, a variation of the Prisoner's Dilemma (which will be presented in the next section), has as purpose to raise awareness of the factors involved in distinguishing between situations appropriate for competition or cooperation. A result that has been found is that the factors that are important for differentiating cooperation from competition are: fair-play, trust and ethics (Berg 2010). All of these three traits are not independent one from the other, instead they are related. A fair-play behavior of a participant on the market is created by respecting the rules and

will earn the trust of the partner, which creates a business environment where ethics play an important part. Another way of seeing the relation between the three characteristics is that business ethics and fair-play create a ‘climate of trust’ (Coomans 2005). Some believe that a basic sense of trust and fair-play is a cornerstone of an economic system even though at the same time, as exposed by Milton Friedman, the only ethical responsibility of a business is that of making profits within the law (Kelly 2002).

The term of fair-play was taken over from sports and it simply means ‘respecting the rules of the game’ (Coomas 2005). When analyzing the results of the Red-Blue Exercise, it has been observed that when people feel in danger of being unfairly treated, not only will they have the tendency of preventing the other participants from obtaining more than what they should get, but also will try themselves to cheat and obtain a higher outcome than initially expected (Berg 2010).

We have mentioned that a fair-play behavior can be achieved by respecting the rules. The rules that have to be respected are component of the ethics behavior. As mentioned by Nicholas G. Moore in his speech about Business Ethics at Bentley College in 1998, ‘ethics are a hard-core value’ because everybody has to have principles, values and standards in order for people to be able to rely on them (Moore 1998).

Respecting the rules by acting in a fair-play manner by applying business ethics will win the trust of the partners. Some argue that ‘trust is of paramount importance to drive economic agents toward mutually satisfactory, fair and ethically compliant behavior’ (Castaldo et all. 2010).

D. Gambetta looks at trust in the context of cooperation and defines it in relation to distrust: ‘trust (or, symmetrically, distrust) is a particular level of subjective probability’ and it varies from distrust, when probability of trust = 0, to blind trust, when probability of trust = 1. Gambetta also explains that ‘trust would seem to be one of those states that cannot be induced at will, with respect either to one’s self or to others’ and that it should be acknowledged as ‘a by-product of familiarity and friendship, both of which imply that those involved have some knowledge of each other and some respect for each other’s welfare’ (Coomans 2005).

As discussed, social qualities are important in business as well, but the most important factor while considering cooperation still remains the economic one. In the next sections we will analyze a few cases where cooperation can be implemented. We will start with the discussion of the basic model of Prisoner's Dilemma, after we will look into Cournot and Bertrand competition, while in the end and lastly we will discuss the case of cooperation in R&D.

6. Prisoner's Dilemma and definition of terms

6.1. Prisoner's Dilemma

A good and easy model to explain why cooperation is important and how it can yield higher outcomes and rewards is the famous game of The Prisoner's Dilemma.

The setting is as follows: two suspects (which in our game will be seen as two *players*) are arrested and accused of a crime. The police do not have enough evidence unless at least one of the suspects confesses. The suspects are kept in separate cells and the consequences are explained to them. If neither suspect confesses then both of them will be sentenced to one month in jail for minor offense (which in our game, a pay-off of -1 would be considered relevant). If one of them confesses, then the one who confesses will be released, whereas the other one will be convicted to 7 months in prison (this will be translated in our game as 0 for the one that will be released and -7 for the one convicted). If both confess then both will be sentenced to 5 months in prison (yielding a pay-off of -5 for each) (Gibbons 3). The game which results will be:

		Player 2	
		Cooperate	Confess
Player 1	Cooperate	-1,-1	-7,0
	Confess	0,-7	-5,-5

The Prisoner's Dilemma

Figure 1

This is the *normal-form representation* of a game with *complete information* and *simultaneous actions*; it specifies the *players of the game*, namely Player 1 and Player 2, the *strategies* of each player (the options each player has when confronted with a decision-making situation), Cooperate (with the other suspect by not confessing) or Confess, and the *payoffs* (outcomes) received in each of the four situations, (-1,-1) for (Cooperate, Cooperate), (-7,0) for (Cooperate, Confess), (0,-7) for (Confess, Cooperate) and (-5,-5) for (Confess, Confess).

The game functions as follows: each of the players chooses to Cooperate with the other player or to defect and Confess the crime. This game is a *simultaneous game*, but the simultaneity of the game does not necessarily imply that the two players take the decisions in the exact same moment, but that they take the decision without knowing what the decision of the other player will be (Gibbons 4). Given the situation, we can analyze the options of Player 2 as a start. Player 2, before making his decision to Cooperate or to Confess, he has to anticipate the action of Player 1. If he assumes that Player 1 will Cooperate then he finds himself in the first column and he will have to choose between Cooperate, which will give him a payoff of -1 or Confess, which will give him a payoff of 0. Before proceeding with the analysis we have to assume that this is a game with *complete information*, meaning that the players have the same information about each other and that both of them are *rational players*. Thus, any rational player, a player that weights its options before taking a decision, would choose to defect since it gets him a higher payoff than Cooperating. In the case when Player 2 assumes that Player 1 will choose to Confess, then Player 2 has to look at the second column in the table and has to choose between Cooperating, which yields a payoff of -7, or to Confess as well, which will get him a payoff of -5. Any rational player would choose, again, to Confess. Therefore, Player 2 should choose to Confess in any of the situations. Moreover, since Prisoner's Dilemma is a *symmetric game*, which means that when analyzing the situation in order to take a decision, the two players would think alike and will have the same payoffs. This being said, the analysis made for Player 2 applies as well for Player 1, therefore, Player 1 will choose to Confess as well, in any situation (Axelrod 9). Confess is a *dominant strategy* (a strategy that regardless the actions of the other player, this particular strategy earns the player a larger payoff) for each of the players.

The result we have reached, (Confess, Confess) is a unique *Nash-equilibrium* of the game. A Nash-equilibrium of a game is a set of strategies that gives the players an outcome that will make them not to have any incentive to unilaterally change their actions (Gibbons 8).

Looking back at Figure 1, it is easily observed that if *mutual cooperation* would take place, both players would be better off. Unfortunately, the desire of a bigger outcome, that being when one of the players cooperates and the other one confesses is greater. This might be seen as the greediness of the human nature. The greediness appears in this type of game, *a one-shot game*, but also in the *finite repeated* version of The Prisoner's Dilemma. We will now show that in the latter, the players would still have no incentive to cooperate because they know that the game will end at one point.

We will now make the assumption that the players know that the game they are facing is a ten-shot game. We will solve this game by using *backward induction*. Imagine we are in the last stage of the game, the 10th stage. We know that this is the final stage of the game and that this is the last encounter between the two players. Consequently, we will treat this stage of the game as a one-shot Prisoner's Dilemma game which, as shown before, yields a unique Nash- equilibrium of (Confess, Confess). Going one step behind, in the 9th stage of the game, the players will again choose the Nash-equilibrium of (Confess, Confess) since they will know that they will anyway defect from the supposed cooperation at the next stage. The same equilibrium will be found at the previous stages for the same reasons. Therefore we are in the situation that the players will choose (Confess, Confess) at each of the 10 stages of the game, thus having no incentive to cooperate.

The results would take a different turn if the game is played an *indefinite number* of times and the players are sufficiently patient. In this case, cooperation can emerge since the players cannot be sure when the last interaction will take place or if there will be a last interaction. The issue will then become the discovery of the precise conditions that are necessary and sufficient for cooperation to emerge.

In this case we will have to compare the cumulated payoffs if the two players cooperate as one option, or if one of the players deviates when the other one expects him to cooperate as the second option. The case when one of the players deviates is a *trigger*

strategy, which means that player i will cooperate until the other player fails to cooperate. At this point, player i will punish the other player by not cooperating anymore (Gibbons, 91). The more applicable to business environment is the *Tit – for – Tat strategy*. The difference between these two strategies is that the latter is more forgiving in the sense that it punishes the transgression enough to make it unprofitable, but the deviating player is welcomed to come back to the cooperation situation (Binmore 368).

When analyzing the indefinite number of encounters scenario, the *discount factor* (δ) is a key tool which captures that the present is more valuable than the future.

If both players cooperate for an indefinitely number of encounters in the Prisoner's Dilemma game showed in Figure 1, each of them will have -1 payoff at each of the stages of the game ((Cooperate, Cooperate) strategy in the game showed in Figure 1), then the cumulated payoff for each of them will be:

$$(-1) + (-1) \delta + (-1) \delta^2 + (-1) \delta^3 + \dots = (-1)/(1-\delta)$$

If player 1 deviates when player 2 expects him to cooperate, the payoff for player 1 will be 0 in the first stage of the game ((Confess, Cooperate) strategy in the game showed in Figure 1) and -5 in the rest of the stages of the game ((Confess, Confess) strategy in the game showed in Figure 1). Thus, the cumulated payoff for player 1 when deviating (since this game is a symmetric game, this payoff is the same for player 2 when deviating) is:

$$0 + (-5) \delta + (-5) \delta^2 + (-5) \delta^3 + (-5) \delta^4 + \dots = (-5) \delta/(1-\delta)$$

In order for the cooperation to be worth it $(-1)/(1-\delta) \geq (-5) \delta/(1-\delta)$ has to occur. After doing the computations, we reach the conclusion that as long as $\delta \geq 1/5$, cooperation is better off. This means that if the players value the future $1/5$ or more than the present, cooperation makes sense and it is achievable.

6.2. Real Case 1 - Cigarette Advertising on Television

A good applied Prisoner's Dilemma is the case of Cigarette Advertising on Television in the 1960's – 1970's in the US. The cigarette market in the 1960's was an

oligopolistic market where competition among the tobacco companies, the four biggest ones being American Brands, Reynolds, Philip Morris, and Liggett & Myers, was conducted mainly in terms of advertising and not by product price since the products sold, the cigarettes, were substitutes for the consumers (Doron 1979). In 1964 the first official warning that smoking cigarettes might be dangerous for public health was issued. On January 1, 1970 the agreement that on each pack of cigarettes will have a warning label and the advertisements on television would stop went into effect. In the period before the companies could choose to whether to advertise on television or not. Advertising on television meant that actors, celebrities or ex-athletes would appear in TV commercials, which made them the image of the brand and was a powerful marketing tool since people could identify themselves with the celebrities shown in the commercials (Gardner 68).

In order to put this real-life situation in a normal-form game we will consider the strategic interaction between two of the main competitors on the market, and for the sake of anonymity we will refer to them as Company 1 and Company 2 (Gardner 68).

		Company 2	
		Do not advertise on television	Advertise on television
Company 1	Do not advertise on television	50, 50	20, 60
	Advertise on television	60, 20	27, 27

Cigarette Television Advertising Case

Figure 2

The normal-form of the game shows the profits the two companies had in the possible scenarios. The profits are in 1970 \$ million. As it can be seen in Figure 2, and as mentioned before, advertising on television was a useful marketing tool if only one of the companies would use this type of advertising. In this case, the profits of the firm using this tool would increase 20%, when compared to the case of not using advertising on TV, and will decrease 60% for the company not using TV advertising. If both of the

companies choose to advertise on television, their profits will decrease to \$ 27 million, almost 50%. The reason behind this is that all the commercials on TV were having the tendency of canceling each other out by not attracting more consumers while still spending the money on advertising.

The unique Nash-equilibrium is (Advertise, Advertise) and it is a typical example of Prisoner's Dilemma where the players are prisoners of their own strategies until something changes the game. In the tobacco case the intervention was external, namely the US government who prohibited the TV commercials for this specific industry. The surprising result of this case was that the companies saved \$ 63 million dollars of their costs by not advertising and still increased their profits by \$ 91 million dollars. This last result is what had happened in real-life, when taking all the participants on the market in consideration (Gardner 69).

7. The model

Historically, economists have put a great emphasis on studying the market interaction in extreme cases of business interaction, namely monopoly and perfect competition with many numbers of firms. A reason for their choice might be that one does not have to worry about the strategic interaction in the market in any of these cases: in monopoly, by definition, since there is only one firm on the market, whereas in the perfect competition, one might see it as unreasonable to track all the activities of all the many competitors on the market. But when looking at real life scenarios, the most possible is that the market interaction can be resumed as an interaction between few firms. On an even smaller scale, we could say that there are two main grocery stores in your town or two large car rentals. Taking the situation even more locally, we can say that there are two bagel shops in your neighborhood or two big supermarkets. Therefore, we could say that in most of the situations one could see the market as a duopoly or a 'local duopoly'. (Dutta 75) In the case of a duopoly, we can be certain that the two firms interacting will pay attention to its competitor's actions and will try to anticipate each of the moves or they might even try to cooperate by operating as a cartel. Consequently, in this paper, the model we will consider will be a duopoly. The model is simple and is stated as:

Two firms competing in a duopoly. The two firms are producing homogeneous goods and having the same cost of production c . For the moment, the only difference between them can be considered the price at which they are sold or the quantity that the two firms will produce.

When competing in prices, we will use the Bertrand model to solve the game, while when the two firms will compete in the quantities they produce, we will use the Cournot model.

8. Cournot Duopoly

8.1. Traditional Cournot analysis

As mentioned before, we start by setting the scene. In the world we have pictured, there is a duopoly, two firms producing two homogeneous goods and having the same cost of production c , which is the basic Cournot model. At first, we will consider a one-stage game, where the firms compete when choosing their quantities simultaneously. Let q_1 and q_2 be the quantities produced by firm 1 and 2, respectively. Let $P(Q) = a - Q$ be the market-clearing price when the aggregate quantity on the market is $Q = q_1 + q_2$ (more precisely, $P(Q) = a - Q$ for $Q < a$, and $P(Q) = 0$ for $Q \geq a$). We assume there are no fixed costs and the marginal cost is constant at c ($c < a$), therefore the total cost for a firm i to produce the quantity q_i is $C_i(q_i) = cq_i$ (Gibbons 15).

Since in this particular case the firms are competing in quantities, this makes the model a Cournot game. Here the players, namely the two firms existing on the market, can differ one from the other through the strategies they choose, that being the different quantities they might produce. We define each firm's strategy space as $S_i = [0, \infty)$, the nonnegative real numbers, in which case a typical strategy s_i is a quantity choice $q_i \geq 0$. One can as well argue that extremely large quantities are not feasible solutions and should not be included in the firm's strategy space because $P(Q) = 0$ if $Q \geq a$, however neither firm will produce any quantity that confirms the equation $q_i > a$ (Gibbons 15).

In order to prepare the game for its normal form we have to define the payoffs for the strategies taken into account. In this case, we will consider that the payoffs are each firm's profit. The profit for firm 1 will be $\Pi^1(q_1, q_2) = q_1 P(Q) - C_1(q_1)$. Similarly, for

firm 2, the profit function will be $\Pi^2(q_1, q_2) = q_2P(Q) - C_2(q_2)$; in the general form of $\Pi^i(q_i, q_j^*) = q_iP(q_i + q_j^*) - C_i(q_i)$.

In the Cournot duopoly the quantity pair (q_1^*, q_2^*) is the Nash- equilibrium of the game, the set of strategies that would yield the highest payoff for both of the firms. (q_1^*, q_2^*) will be the set of strategies that will solve both of the following equations:

$$\max_{q_1} \Pi^1 = \max_{q_1} (q_1P(Q) - C_1(q_1)) \quad \text{and}$$

$$\max_{q_2} \Pi^2 = \max_{q_2} (q_2P(Q) - C_2(q_2)).$$

Assuming that $q_1^* < a - c$ and $q_2^* < a - c$ (as it will be shown to be true), the first-order condition for firm 1's and firm 2's respectively optimization problem is both necessary and sufficient, it yields: $q_1^* = (\frac{1}{2})(a - c - q_2^*)$ and $q_2^* = (\frac{1}{2})(a - c - q_1^*)$. By solving this specific pair of equations, we will find that $q_1^* = q_2^* = (1/3)(a - c)$.

The most important goal in a business is increasing profits therefore, after doing the calculation we find that $\Pi_1^* = \Pi_2^* = (a - c)^2/9$ (later in the paper, in the collusion-deviation part for Cournot duopoly Π_1^* , Π_2^* will be referred to as Π_N), profit reached by having a price of $P(Q) = (a/3) + (2c/3)$.

8.2. Cooperation in Cournot Duopoly

As a contrast on what we have computed in the previous section, we will now analyze what will happen if the two firms in the market are cooperating and acting as a cartel. In this case, the two firms will work together in order to set production targets in a way to maximize their joint profits. In this case we will have to find the quantities named 'quatas' Q_1 and Q_2 that will fulfill the requirement of maximizing the following profit function:

$$\max_{Q_1, Q_2} [a - (Q_1 + Q_2) - c](Q_1 + Q_2).$$

The difference between the cartel solution and the best response solution treated in the previous section is that in the cartel case the firms acknowledge that their profits depend on their total production. They know and put all their efforts to do good together.

Whereas, in the best response case each of the firms works and takes actions

independently and rely solely on the individual output obtained. In this case they also assume and believe that their competitor will hold on to some predicted outcome (Dutta 79).

Therefore, the two quantities produced by each firm as part of the cartel, $Q_1 + Q_2$ can be seen as a monopoly quantity Q_M that fulfills the condition $Q_1 = Q_2 = Q_M/2$. Taking this in consideration, we can rewrite the maximization of profits in the cartel situation as:

$$\max_{Q_M} (a - Q_M - c) Q_M .$$

By solving this maximization problem we will find that $Q_M = (a - c)/2$, therefore $Q_1 = Q_2 = (a - c)/4$. Since we analyze a potential business scenario, we are aware of the fact that firm are profit oriented, therefore, after calculations we find that the profit for the cartel scenario will be $\Pi_C = (a - c)^2/8$ which can be reached by having a market price of $P_C = (a + c)/2$. (Dutta 80)

8.3. Collusion – deviation in Cournot game

After looking into these two options, either to act independently or to collude and act as a cartel, we will now analyze the case when the two firms agree on collusion, but one of them deviates. We will use the same reasoning we have applied in the Prisoner's Dilemma case.

Therefore, now we will have two options that we will compare on a longer period of time, by introducing the discount factor δ , and assuming that the firms will interact an indefinite number of times.

The first option is that the two firms collude and cooperate on each of their encounters. Thus, the accumulated profit will be:

$$\Pi_C + \delta \Pi_C + \delta^2 \Pi_C + \delta^3 \Pi_C + \dots = (a - c)^2 / 8(1 - \delta)$$

If we are looking at the case when for example firm 1 expects firm 2 to cooperate and the latter does not cooperate, the accumulated profit will be:

$$\Pi_D + \delta \Pi_N + \delta^2 \Pi_N + \delta^3 \Pi_N + \dots$$

Π_D denotes the profit obtained in the first period when cooperation is expected, but it is not achieved (this particular profit can be seen as the temptation for which the firm has the intention to deviate) and Π_N denotes the profit obtain in the non-cooperative environment studied earlier for Cournot duopoly.

The deviation profit is calculated after the formula $\Pi_D = P(Q)q_2 - cq_2$, where $P(Q) = a - Q$, where Q is the total quantity on the market and it is the sum of quantity of firm 1, which will produce the collusion quantity, $Q_1 = (a - c)/4$, and the quantity produced by firm 2, which will produce the quantity aimed for the non-cooperative case, $q_2 = (a - c)/3$. Therefore, the quantity on the market in this case will be $Q = Q_1 + q_2 = 7(a - c)/12$. This will yield a market price of $P(Q) = (5a + 7c)/12$. By computing the profit, we find that $\Pi_D = 5(a - c)^2/36$. Thus, the accumulated profit for the deviation period is:

$$\Pi_D + \delta \Pi_N + \delta^2 \Pi_N + \delta^3 \Pi_N + \dots = [5(a - c)^2]/36 + [\delta(a - c)^2]/9(1 - \delta)$$

In order for the two firms to value and have the intention to cooperate, the next condition has to be fulfilled:

$$\begin{aligned} \Pi_C + \delta \Pi_C + \delta^2 \Pi_C + \delta^3 \Pi_C + \dots &\geq \Pi_D + \delta \Pi_N + \delta^2 \Pi_N + \delta^3 \Pi_N + \dots \\ (a - c)^2/8(1 - \delta) &\geq [5(a - c)^2]/36 + [\delta(a - c)^2]/9(1 - \delta) \end{aligned}$$

After solving the equation, we reach the conclusion that if $\delta \geq 1/2$, cooperation is better off, therefore, if the firms value the future $1/2$ or more than the present, cooperation is preferred and it can be achieved.

Summarizing, if the 2 firms act independently they will reach a profit of $\Pi_1^* = \Pi_2^* = (a - c)^2/9$, but if they cooperate and act as a cartel, then they will have each a profit of $\Pi_C = (a - c)^2/8$. Comparing the profits in the two different situations, we can easily notice that $\Pi_C > \Pi_1^* = \Pi_2^*$ if the situation is treated as a one shot game. Nevertheless, a one shot game is not a practical approach to a real life situation. This is the reason why in the third part of presenting the Cournot duopoly we have compared collusion and deviation in an infinite Cournot game. We have concluded that if the two firms value the future $1/2$ or more than the present, then cooperation is a better choice.

In order to support our theoretical framework it is necessary to present a real life case that will sustain the previous presented scenario. We will take the case of OPEC and analyze the options of best-response (non- cooperation), collusion and as well take a look into what will happen if the firms collude and after deviate. The difference with the previously discussed model is that in the OPEC case the firms incur different costs, which is a more realistic scenario.

8.4. Real Case 2 – OPEC

8.4.1. The framework

OPEC, the Organization of the Petroleum Exporting Countries is a permanent intergovernmental consortium of major oil-producing countries. It was established in Baghdad, Iraq, 10-14 September 1960. The Organization, the way OPEC names itself in its Statute has at the moment 12 member countries: Algeria, Angola, Ecuador, Islamic Republic of Iran, Iraq, Kuwait, Socialist People's Libyan Arab Jamahiriya, Nigeria, Qatar, Saudi Arabia, United Arab Emirates and Venezuela. At the moment, the headquarters of OPEC is in Vienna, Austria.

OPEC's mission, as defined in its Statute is to '*to coordinate and unify the petroleum policies of its Member Countries and ensure the stabilization of oil markets in order to secure an efficient, economic and regular supply of petroleum to consumers, a steady income to producers and a fair return on capital for those investing in the petroleum industry*'. (www.opec.org)

On the 14th of October 2010, in Vienna, OPEC adopted a new Long Term Strategy (LTS). The LTS developed and incorporates extensive research and analysis for the future of the Organization. It has developed three potential scenarios: the Dynamics-as-Usual (DAU) scenario, the Prolonged Soft Market (PSM) scenario and the Protracted Market Tightness (PMT) scenario. In all of the scenarios, the main competitors, the non-OPEC oil-providers are also taken in consideration.

In the DAU scenario, the long-term assumption for the world economy sees a robust expansion in gross domestic product over the next two decades, with Asian economic growth still continuing to dominate.

In the PSM scenario, the downside risk to the world economy is taken in consideration, it is also assumed that the demand will be affected by new policies in developed and major developing countries.

The PMT scenario reflects optimism over the impact of the response of the governments to the global financial crisis. It sees geopolitical and economic conditions as conducive to improved economic growth prospects.

In the DAU scenario, global oil demand reaches over 105 million barrel a day (mb/d) by 2030, in PSM scenario it reaches 94 mb/d by 2030, whereas in the most optimistic scenario, the PMT scenario, it reaches 112 mb/d.

Treated in a separate section of their report, the non-OPEC supply is given special attention. It is recognized that the amount that OPEC has to supply depends on the future volumes of non-OPEC oil.

Making predictions about the supply of non-OPEC countries for all three different scenarios, it is anticipated that the supply of the non-OPEC countries to be larger in 2030 than in 2010, but the growth is expected to eventually slow down in all of the three scenarios. The factors that will influence the supply include the development of oil prices, legal and fiscal conditions or technological progress.

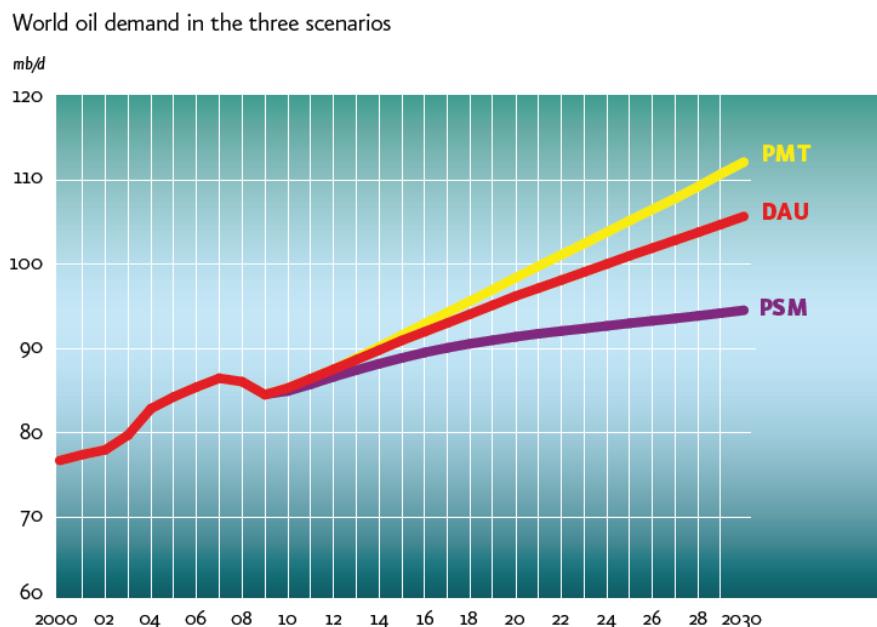


Figure 3

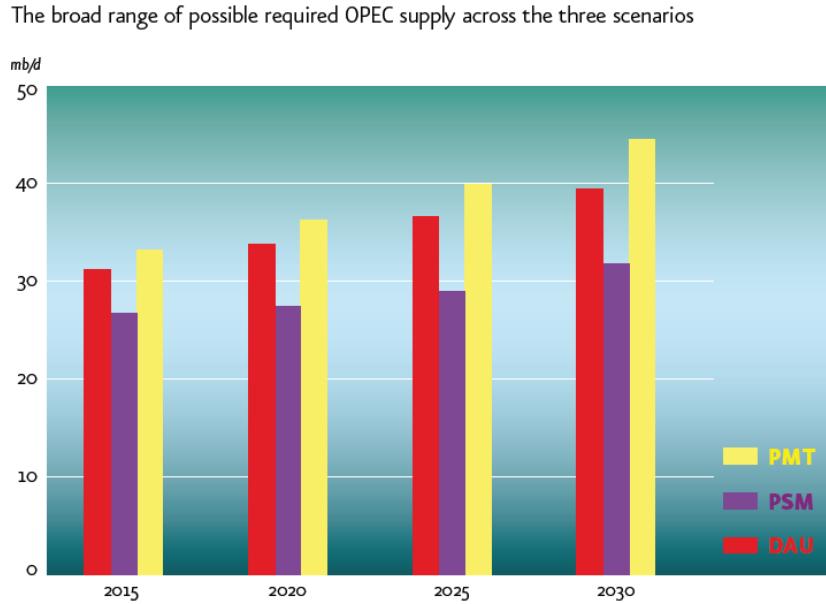


Figure 4

Figure 3 presents the increase in the world demand of oil during 2000 and 2030 in the three possible scenarios, whereas Figure 4 presents the possible required OPEC supply levels in 2015, 2020, 2025 and 2030 in all the three possible scenarios. We can easily observe that OPEC will not be required to fulfill all demand in any of the three situations, the rest of the demand being expected to be supplied by non-OPEC countries.

After we have presented the frame of the scenario, now we will put the real life case in a Cournot model and we will analyze that, by keeping prices high, OPEC quotas have made it worthwhile for non-OPEC producers to invest in new oil fields and increase production levels, which have put pressure on OPEC itself. Consequences include some members leaving, such as Ecuador in 1992, or others being known as cheating on their quotas (Dutta 82).

8.4.2. The model

In the model analyzed, we reduce the number of players to two, OPEC and non-OPEC producing countries. We will assume that the cost of production of a barrel of oil differs for the two players, $c_O = \$5$ for OPEC countries and $c_N = \$10$ for non-OPEC countries. We will as well assume that the demand curve for the oil market is $P = 65 - (Q_o + Q_N)/3$, where Q_o is the production of OPEC countries and Q_N is the production of non-OPEC

countries, both are measured in millions of barrels per day, mb/d. This case is *option 1*, the case when the two players are not cooperating (Dutta 81).

Option 1 – non-cooperative case

Having this information, we can now proceed and calculate the quantities, Q_o and Q_N , the price P of one barrel and the profits for each of the players. We name Π_o the profit for the OPEC countries and Π_N the profit for non-OPEC countries. We know that each of the profits is calculated by the general formula $\Pi_i = Q_i (P - c_i)$, where c_i is the cost of producing one barrel of oil and Q_i is the quantity related to the producer. Therefore, the profit of OPEC countries will be calculated with the formula $\Pi_o = Q_o (P - c_o)$ and the profit for non-OPEC countries will be calculated with the formula $\Pi_N = Q_N (P - c_N)$.

By solving the maximization problems:

$$\max_{Q_o} \Pi_o = Q_o (P - c_o)$$

and

$$\max_{Q_N} \Pi_N = Q_N (P - c_N)$$

we will find that $Q_o = 65$ mb/d and $Q_N = 50$ mb/d. The price $P = \$80/3$ and the profits are $\Pi_o = \$4225/3$ million and $\Pi_N = \$2500/3$ million. This is the best response function in the non-cooperative scenario, *option 1*.

Option 2- collusion

The next question that will be answered is what will happen if the non-OPEC countries would join the OPEC and together would form a cartel, which we will consider as *option 2*. If this would be the case, the cost of production would be lower for the ex-non-OPEC countries; it will drop from \$10 to \$5. Thus, the previous scenario would transform into one where the cost of production is $c = \$5$, the world demand is $P' = 65 - Q/3$ and the profit is defined by $\Pi' = Q(P' - c_o)$. By solving the maximization problem $\max_Q \Pi' = Q(P' - c_o)$, we obtain a total quantity of the market of $Q = 90$ mb/d sold at the price of $P' = \$35$ a barrel, which yields a cartel profit of $\Pi' = \$ 2700$ million.

While comparing the two options, we can see that even though $Q_N + Q_O > Q$, the cartel solution gives an overall more profitable outcome. By setting the price higher, from $P = \$80/3 = \26.6 per barrel to $P' = \$35$ per barrel, the profits for the cartel increase from $\Pi_O + \Pi_N = \$6725/3$ million = \$2241,66 million to $\Pi' = \$2700$ million.

We observe that due to lower production levels, but increased prices, the total profits are higher than in the non-cooperative situation. Even if the OPEC countries would pay the non-OPEC countries the profit that the latter would get in Nash equilibrium, namely \$2500/3 million, OPEC would still have \$5600/3 million left over, which is \$1375/3 million more than in the first option. In reality, the worse is that ex-members, such as Ecuador, have found ways to benefit from the OPEC quotas by being outside OPEC, they are no longer subject to the quotas, but still they benefit from the consequent higher prices (Dutta 82).

The question that is raised now is how the output and profit will be split between the OPEC and non-OPEC players in the scenario where they have formed a cartel. We have assumed that when forming the cartel, the marginal cost will drop to $c = \$5$. This is an unrealistic assumption, it is almost impossible for the cost to drop to half just because non-OPEC has joined the cartel. The explanation behind this is that in reality, only OPEC is producing the market quantity and then they are paying non-OPEC a specific amount of money in order not to produce any oil. The question remained is what are the terms that prevent non-OPEC from deviating from the cartel agreement, namely how large the amount paid to non-OPEC should be and what is the discount factor for non-OPEC, but as well as what is the discount factor for OPEC in order not to deviate from the cartel.

Until now we have analyzed the game as a one shot game. Next we will consider a repeated number of times game and calculate the discount factor in order to see how much the two players, OPEC and non-OPEC, have to value the present more than the future in order not to have an incentive to deviate from the cartel agreement.

Collusion vs. deviation for an indefinitely repeated number of times game

As mentioned before, the collusion profit is $\Pi' = \$2700$ million, but this profit is calculated for all the cartel members together. We have assumed that OPEC is producing the entire quantity on the market, namely $Q = 90$ mb/d and paying non-

OPEC a specific amount, which we will name X , in order not to produce any quantity of oil. The minimum payment X that non-OPEC will accept will be $X_{min} = \prod_N = \$2500/3 million = \$833,33 million$, since this is the amount which they will receive in the case of non-cooperation. The maximum OPEC will be willing to pay non-OPEC will be $X_{max} = \prod' - \prod_o = \$ 3875/3 million = \$1291,66 million$, saving for themselves at least the profit they would have obtained in the case of non-cooperation.

Since we are facing a game when the market is shared unequally, namely where the two players, when not cooperating, are producing different quantities, as well as when they are cooperating, we will have to calculate the discount factor related to each of the players. Therefore, δ_1 will be the discount factor for the OPEC countries and δ_2 for the non-OPEC countries.

We will first take a look at the OPEC deviation case. When OPEC has the incentive to deviate, it will deviate by not paying the agreed amount X to non-OPEC and keep the entire collusion profit $\prod_D^O = \prod' = \$ 2700 million$. Therefore, in this case the accumulated profit for the repeated game will be:

$$\prod_D^O + \delta_1 \prod_o + \delta_1^2 \prod_o + \delta_1^3 \prod_o + \dots = \prod' + \delta_1 \prod_o / (1 - \delta_1) = \$2700 + \$4225 \delta_1 / 3 (1 - \delta_1)$$

Whereas, in the collusion scenario, the accumulated profit for OPEC is $\prod_C^O = (\prod' - X) / (1 - \delta_1) = (\$2700 - X) / (1 - \delta_1)$. In order to calculate the payment X the following equation has to stand

$$\prod_C^O \geq \prod_D^O + \delta_1 \prod_o + \delta_1^2 \prod_o + \delta_1^3 \prod_o + \dots$$

Which is equivalent to $(\prod' - X) / (1 - \delta_1) \geq \prod' + \delta_1 \prod_o / (1 - \delta_1)$, therefore,

$$X \leq \delta_1 (\prod' - \prod_o)$$

Next we will analyze the case when non-OPEC deviates. In this case, non-OPEC, will deviate by producing as well an amount \prod_D^N and as well receiving the payment X in the first period. After deviating, OPEC will punish by returning to the non-cooperative solution which will force non-OPEC to do the same. Thus, after deviating, there will be again the case when OPEC will produce a quantity of $Q_N = 50mb/d$ which will give a profit of $\prod_N = \$833,33 million$.

We cannot be sure on the exact amount X which the two players will agree on, since this amount will be the result of a series of negotiations which will depend on other factors such as: the number of encounters between the two players, who has to make the first offer or how patient the players are. We will not make assumptions in order to calculate the payment since it is not the purpose of this paper.

In the case when non-OPEC deviates, it will produce a new quantity Q_N^* which will maximize the profit $\prod_D^N = Q_N^*(P^* - c)$, where $P^* = 65 - (Q + Q_N^*)/3$ and c is the cost that the cartel has, namely $c = \$5$. After solving the maximization function $\max_{Q_N} \prod_D^N = Q_N^*(65 - (Q + Q_N^*)/3 - c)$, we find that $Q_N^* = 45 \text{ mb/d}$ and that the extra profit, beside the payment X , that non-OPEC will have in the first period when deviating is $\prod_D^N = \$675 \text{ million}$. Hence, the accumulated profit in the deviation case is:

$$X + \prod_D^N + \delta_2 \prod_N + \delta_2^2 \prod_N + \delta_2^3 \prod_N + \dots = X + \prod_D^N + \delta_2 \prod_N / (1 - \delta_2) = X + \$675 + \$2500 \delta / (1 - \delta_2).$$

In the collusion case for non-OPEC, the accumulated profit is $\prod_C^N = X / (1 - \delta_2)$. In order to calculate the payment X , the following has to stand

$$\prod_C^N \geq X + \prod_D^N + \delta_2 \prod_N + \delta_2^2 \prod_N + \delta_2^3 \prod_N + \dots$$

This is equivalent to $X / (1 - \delta_2) \geq X + \prod_D^N + \delta_2 \prod_N / (1 - \delta_2)$, therefore

$$X \geq (\prod_N - \prod_D^N) + \prod_D^N / \delta_2.$$

Our purpose is to calculate the payment X that OPEC has to pay non-OPEC in order to cooperate by taking in consideration the way they value the future compared to their present, namely by taking in consideration the values of δ_1 and δ_2 . Therefore, if we put together the two equations, the payment X has to fulfill the following condition:

$$(\prod_N - \prod_D^N) + \prod_D^N / \delta_2 \leq X \leq \delta_1 (\prod' - \prod_O).$$

At the moment we have to consider two cases, one when OPEC and non-OPEC put the same value on the future compared to the present ($\delta_1 = \delta_2 = \delta$), *case A*, and *case B* when the two players, OPEC and non-OPEC have different values for the future when is compared to the present.

Case A: $\delta_1 = \delta_2 = \delta^*$

In the case when OPEC and non-OPEC have the same discount factors, the *payment X* has to fulfill the following equation: $(\prod_N - \prod_D^N) + \prod_D^N / \delta^* \leq X \leq \delta^* (\prod' - \prod_O)$, where δ takes values from 0 to 1.

Calculating the critical value of the discount factor from the equation $(\prod_N - \prod_D^N) + \prod_D^N / \delta^* = \delta^* (\prod' - \prod_O)$, we find that $\delta^* = 0,786$; therefore, after introducing the critical value of the discount factor in the equation, we reach the conclusion that the payment X^* will be *\$1016,25 million*, which is fulfilling the condition of existing between the minimum and maximum limits $X_{min} = \$2500/3 million = \$833,33 million$ and $X_{max} = \$3875/3 million = \$1291,66 million$.

In the case when the two players, OPEC and non-OPEC, put the same emphasis on the future compared to the present, by having the discount factor $\delta^* = 0,786$, the case when the payment will be $X^* = \$1016,25 million$, will also be the case when both of the players will be better off. In real life the payment is subject to negotiations which can also be analyzed by using game theory while making assumptions on the number of maximum moves that can be made, the discount factor and which one of the players will make the first proposal. After making the necessary assumptions, a best response will be found. We will not calculate this scenario since it is not the purpose of this paper.

Case B: $\delta_1 \neq \delta_2$

For the case when the two players, OPEC and non-OPEC, do not value the future compared to the present in the same way, in the sense that the two players have different discount factors ($\delta_1 \neq \delta_2$), then $(\prod_N - \prod_D^N) + \prod_D^N / \delta_2 \leq X \leq \delta_1 (\prod' - \prod_O)$. In order to make a graph which will help us to lower the area of existence of the payment X , we will have to find the critical values, therefore $(\prod_N - \prod_D^N) + \prod_D^N / \delta_2 = X$ and $\delta_1 (\prod' - \prod_O) = X$, where δ_1 and δ_2 can take values from 0 to 1.

By plugging in the values for \prod_N , \prod_D^N , \prod_D^N , \prod' and \prod_O we will have the following two functions:

$$675 / \delta_2 + 158,33 = X \text{ and}$$

$$1291,67 * \delta_1 = X.$$

After plugging in the data in a graph, the following results:

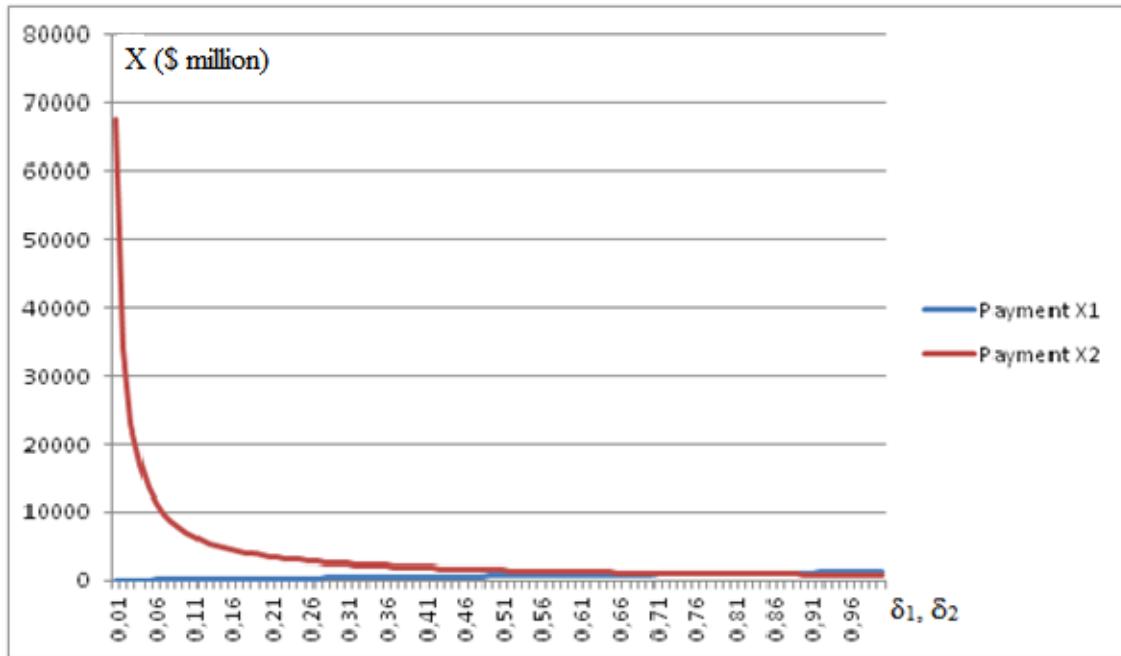


Figure 5

Figure 5 shows how the value of the payment that OPEC makes to non-OPEC in order not to produce oil varies as a function of the discount factor of each of the players. On the horizontal axis the two discount factors are represented taking values from 0 to 1, whereas on the vertical axis the value of the *payment X* is measured in *\$ million*. There are two graphs that intersect in the point previously discussed when $\delta_1 = \delta_2 = \delta^*$.

Payment X₁ presents the payment *X* as a function of the discount factor of OPEC, δ_1 and the profits related to it, $X = \delta_1 (\Pi' - \Pi_o)$ which after plugging in the numbers transforms into the function $1291,67 * \delta_1 = X$. *Payment X₁* is the payment that OPEC considers making, while *payment X₂* is the payment non-OPEC considers receiving. *Payment X₂* presents the payment *X* as a function of the discount factor related to non-OPEC, δ_2 and as well the profits related to it, $X = (\Pi_N - \Pi_D^N) + \Pi_D^N / \delta_2$, which, after plugging in the numbers transforms in $675 / \delta_2 + 158,33 = X$. Both of the discount factors can take values from 0 to 1 and the minimum and maximum values that the payment can take are $X_{min} = \Pi_N = \$2500/3 million = \$833,33 million$ and $X_{max} = \Pi' - \Pi_o = \$3875/3 million = \$1291,66 million$.

A limitation when making the graph was that the constraints of the payment, $X_{min} = \Pi_N = \$833,33 million$ and $X_{max} = \Pi' - \Pi_o = \$1291,66 million$ were not taken in

consideration in order to have a wider picture of the situation, though, this step will be fulfilled while making the analyzing the graph. The data used for drawing the graph are showed in Appendix.

From *Figure 5* we can see that the two graphs intersect, but since the scale on the horizontal axis is too wide, the point where the two representations intersect cannot be easily observed. Therefore, in *Figure 6* the same graph is represented in a closer scale, starting with $\delta = 0,2$. The reason behind this step is simply for a better observation of the findings.

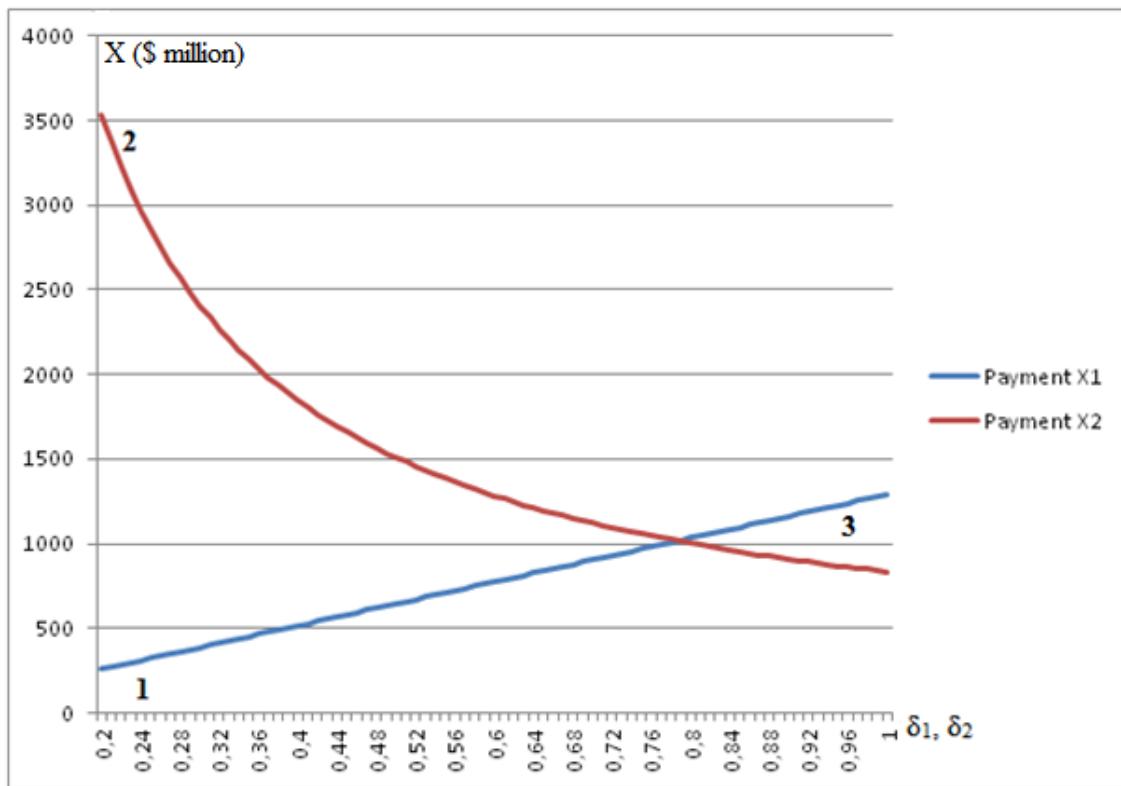


Figure 6

We will start by analyzing each player's case. When looking at the case of OPEC, we have the information that without collusion, its profit will be $\prod_O = \$1408,33$ million which further more gives us the information that the maximum payment it is willing to pay to non-OPEC is $X_{max} = \prod' - \prod_O = \$3875/3$ million = $\$1291,66$ million.

The function for *payment* X_1 is an upward sloping curve ($X = 1291,67 * \delta_1$). We observe that if the discount factor is low, then the *payment* X_1 that OPEC is willing to pay will be low, e.g. for $\delta_1 = 0,2$ then $X_1 = \$258, 334$ million (Appendix). The discount factor of $\delta_1 = 0,2$ shows that OPEC values the future 0,2 more than the present and it is not a high enough value in order to trigger collusion. OPEC is aware that non-OPEC will not settle for a *payment* X which is lower than the minimum $X_{min} = \prod_N = \$833,33$ million that they would earn in a non-cooperative situation. Since OPEC does not put a big emphasis on the future due to the low discount factor, it does not have the incentive of making a good offer of payment to non-OPEC.

We can observe that as the discount factor increases, which means that OPEC puts more emphasis on the future, it also starts to be more willing to pay a bigger amount to non-OPEC until reaching the moment when is willing to pay the minimum amount of $X_{min} = \$833,33$ million. When $833,33 = 1291,67 * \delta_1$, $\delta_1 = 0,645$. Hence, for a discount factor greater than $\delta_1 = 0,645$, OPEC is willing to pay the minimum amount expected by non-OPEC.

While the discount factor will be growing higher than 0,645, the payment will be growing as well, reaching the maximum value of $\$1291, 66$ million when the discount factor will be $\delta_1 = 1$. After reaching this point of valuing the future enough, OPEC starts acknowledging the important results of cooperation and is willing to accept a high enough *payment* X which will facilitate cooperation.

In *Figure 6*, the feasible area of the payment that OPEC is willing to pay to non-OPEC it is marked **1**. It is on the right of the function of *payment* X_1 .

Now we will analyze the case of non-OPEC. The function for *payment* X_2 is a downward sloping curve ($X = 675 / \delta_2 + 158,33$). The minimum payment that non-OPEC will accept is $X_{min} = \prod_N = \$833,33$ million since this is the profit it will make in a non-cooperative environment. If it does not receive the minimum payment, cooperation is not possible.

We observe that for a low discount factor of non-OPEC, non-OPEC is expecting a high payment, e.g. if $\delta_2 = 0,2$ then $X_2 = \$3533,33$ (Appendix). We can observe that if the discount factor related to non-OPEC has the value $\delta_2 = 0,2$, it is not high enough to facilitate collusion since non-OPEC will expect a too high payment of $X_2 = \$3533,33$,

which will not be delivered by OPEC due to the maximum amount it is willing to pay $X_{max} = \$1291,66$ million.

Therefore, since the function is downward sloping and taking in consideration the value of the maximum payment, X_{max} , cooperation will start being possible in the moment when the following equation is fulfilled $1291,66 = 675/\delta_2 + 158,33$, which gives a discount factor $\delta_2 = 0,595$. We interpret this as: if the discount factor is 0,595 or more, non-OPEC is willing to accept a payment lower or equal to $\$1291,66$ million, which will be part of the interval X_{min} and X_{max} .

While the discount factor will be growing at a higher value than 0,595, the expected payment will be lowering until the point when it will reach the minimum value of $\$833,33$ million in the moment when the discount factor $\delta_2 = 1$. The reason is that non-OPEC will put enough emphasis in valuing the future in order to realize the important effects of cooperation.

In *Figure 6*, the feasible region for the payment that non-OPEC is willing to accept is marked **2**. It is the region on the right of *Payment X₂*.

We observe that while in the case of OPEC, the greater the discount factor is the greater the payment will be, whereas in the case of non-OPEC, the greater the discount factor is the smaller the payment will be. There are two reasonable interpretations behind this. The first one is that by looking at the graph, we observe that function for *payment X₁* is upward sloping therefore the greater the discount factor δ_1 is, the greater *payment X₁* will be. While looking at the graph of *payment X₂*, which is downward sloping, the greater the discount factor δ_2 is the smaller the *payment X₂* will be. The second reason behind the result is that because OPEC is the one that is paying the *amount X* to non-OPEC, its desire is that the payment to be as low as possible, but the moment that it starts valuing the future more and knowing that accumulated profit in time is greater in the case of cooperation, OPEC will be willing to pay more in order to achieve collusion. In the case of non-OPEC is the other way around, in the sense that non-OPEC is the one receiving the *payment X* from OPEC, thus, its wish is that the payment to be as high as possible. In the moment non-OPEC starts to value the future more, it will be the case when non-OPEC will be willing to accept a lower payment in order to make

cooperation possible, knowing that in the future will yield higher accumulated profit than in a non-cooperative environment.

The area that is marked **3** in *Figure 6* is the feasible area that fulfills both players' requirements. The feasible area **3** starts at the intersection of the graph of *payment X₁* with the *payment X₂* when $\delta_1 = \delta_2 = 0,786$ and $X = \$1016,25 \text{ million}$. It has as an upper limit the graph function of *payment X₁* and as a lower limit the graph function of *payment X₂*. Therefore, in order for cooperation to be attainable, the two players, OPEC and non-OPEC have to value the future at least 0,786 times more than the present. Any values of the discount factor greater than 0,786 will yield feasible payments. Since the discount factor is $\delta = 0,786$, and it is a high value, we can state that cooperation will not be easy to achieve.

To summarize all the analysis, in order for OPEC and non-OPEC to cooperate, they have to value the future at least 0,786 more than the present, case when the payment will be higher than $X^* = \$1016,25 \text{ million}$. As mentioned before, it is impossible to calculate the real value of the payment since it is subject to negotiations between OPEC and non-OPEC.

8.4.3. Conclusion

To conclude the case, we can observe that when colluding, the joint profit is higher in the collusion case, $\Pi' = \$ 2700 \text{ million}$, compared to $\Pi_O + \Pi_N = \$ 6725/3 \text{ million} = \$ 2241,66 \text{ million}$, in the non-cooperative environment. Even though the profit is higher, in order to cooperate, it has to be advantageous for both of the players to maintain collusion. As proved before, this decision is in fact depending on how, in the first place, on the way OPEC and non-OPEC value the future compared to the present and secondly on how the negotiations between the two will go in order for both of them to be satisfied by the result achieved. After these constraints have been fulfilled, it is clear that by cooperating, both of the players will be better off.

After presenting the way cooperation can be achieved while competing in quantities, Cournot model, we will now turn to the case when firms will compete in prices, the Bertrand model.

9. Bertrand Duopoly

9.1. Traditional Bertrand model

Before starting to show what will happen when firms will compete in pricing, we have to set the scene. As before, there are two firms competing in the market, producing homogeneous goods, which are substitutes in the consumer's utility function. As a result, the only difference between the two goods will be the price charged by the firms. Therefore, the consumers will buy the good with the lowest price, assuming that they have fast access to any of the two products. If the firms will charge the same price, we make the assumption that the market is divided equally between them. Another assumption that we will make is that each of the firms will be able to supply the demand it faces. The demand function is $q = D(p) = a - p$. There will be no fixed costs for any of the two firms, but a marginal cost of c .

In this case, Bertrand model, the strategies of the firms consist of the prices they choose. Therefore, each firm's strategy space is $S_i = [0, \infty)$, which can be a nonnegative real numbers, with a typical strategy being s_i , a price choice (Gibbons 22).

The profit of firm i will be $\Pi^i(p_i, p_j) = (p_i - c) D_i(p_i, p_j)$ where the demand $D_i(p_i, p_j)$ is $D(p_i) = a - p_i$ if $p_i < p_j$; $(1/2)D(p_i) = (a - p_i)/2$ if $p_i = p_j$ and 0 if $p_i > p_j$. The aggregate profit, $\min(p_i - c)D(p_i)$, cannot exceed the monopoly profit $\Pi^m = \max(p - c)D(p)$. Since the marginal cost is c , any of the two firms will want to have $p_i \geq 0$ and $p_i \geq c$ in order to have a payoff $\Pi_i \geq 0$. As well, it has to be taken in consideration that the predictions that are made will yield the result that the sum of the profits of both firms are greater than 0 and smaller than the monopoly profit, $0 < \Pi^1 + \Pi^2 < \Pi^m$ (Tirole 210).

The firms will choose their prices simultaneously. We will find the Nash equilibrium, in this case called Bertrand equilibrium, by assuming that the two firms will choose their prices following $p_1^* > p_2^* > c$. In this manner, firm 1's profit will be 0 and firm 2's profit will have to supply the entire demand. If firm 1 decreases its price to $p_1 = p_2^* - \varepsilon$ (where $\varepsilon > 0$, ε is very small), it will obtain the entire market demand $D(p_2^* - \varepsilon)$, and it will have a positive profit margin of $p_2^* - \varepsilon - c$. Thus, firm 1 will not choose $p_1^* > p_2^* > c$ since it will not be in his best interest (Tirole 210).

Now we assume that the firms will choose their prices according to $p_1^* = p_2^* > c$, which brings each of the firms a profit of $D(p_1^*)(p_1^* - c)/2$. If firm 1, for example, decreases its price slightly to $p_1^* - \varepsilon$, the effect will be an increase in its profits to $D(p_1^* - \varepsilon)(p_1^* - \varepsilon - c)$, the market share of the firm increasing in a discontinuous manner. None of the two firms will charge a price lower than the unit cost c because this will yield negative profits, therefore the situation on the market will be that one or both of the firms will be charging the marginal cost and making no profit. In order to demonstrate this statement, the assumption that the firms will charge $p_1^* > p_2^* = c$ will be made. Thus, firm 2, which makes no profit, could increase its prices slightly, but still supply all demand and make a positive profit – which is a contradiction (Tirole 210).

We have been proving that the firms will reach the point when the price charged will be the marginal cost, thus not yielding any profit. The conclusion we have reached is the definition of *Bertrand paradox* and it is hard to believe that firms in industries with a minimum number of firms will not manage to manipulate the market in such a way to make profits (Tirole 211). It is also a not a realistic model since in real life many differences can occur. We will take the differences into consideration and analyze them. We will look into more detail at a duopoly case where the firms have different marginal costs and a duopoly where there are some differences in the products offered.

Before going further with analyzing different scenarios of Bertrand competition, we will discuss what will be happening if we analyze the *real case 2* previously described, under Bertrand competition instead of Cournot competition.

9.2. *Real case 2 under Bertrand competition*

OPEC and non-OPEC are the two players on the oil market producing homogenous goods, oil. We have assumed that they are the only producers on the market and that they can produce an unlimited amount of oil. OPEC incurs the marginal cost of $c_O = \$5$ for a barrel of oil and non-OPEC countries has a marginal cost of $c_N = \$10$ per barrel of oil.

If there is the case of competing in prices, the OPEC countries will see the opportunity for being able to benefit from lower marginal cost which is in fact a critical advantage. OPEC will charge a price of $P_B = \$ 9,99$ a barrel, this way managing to obtain the entire market since it is not profitable for non-OPEC countries to charge a price lower than their marginal cost of $c_N = \$10$. Due to the fact that the firms are competing in prices, OPEC will become the monopolist on the market, hence the demand on the market will be $P = 65 - Q/3$, Q is the quantity on the market, in this case, the quantity produced by the monopolist OPEC. After solving the maximization problem of $\Pi_B = Q_B (P_B - c_O)$ we find that $Q_B = 90 \text{ mb/d}$ will be the quantity demanded by the market at a price of $P_B = \$ 9,99$ which yields a profit of $\Pi_B = \$449,1 \text{ million}$. We can recall that under Cournot competition, in the collusion case, the same quantity $Q = 90 \text{ mb/d}$ was on the market. The reason for this result is that in both of the cases, Bertrand competition with different costs and collusion under Cournot competition, the entire quantity demanded by the market is produced by OPEC. The difference between the two cases is the profits that result. In the case of Cournot competition the joint profit will be $\Pi' = \$ 2700 \text{ million}$ under a market price of $P' = \$35$ a barrel. Even though OPEC has to pay the amount X which cannot be calculated exactly since it is subject to negotiation, OPEC will still have a minimum profit of $\Pi_O = \$4225/3 \text{ million}$ (because OPEC will not agree, in the Cournot competition to have a lower profit under collusion when compared to the non-cooperative scenario). Whereas under Bertrand competition the profit will be only $\Pi_B = \$449,1 \text{ million}$, profit reached by selling a barrel of oil at the price of $P_B = \$ 9,99$.

It is easy to observe the tremendous difference in profits, as well as in prices and the main reason behind this is that price competition is tougher than quantity competition. For example, in the case of OPEC and non-OPEC, in a price war, OPEC has the advantage of a lower cost and therefore it will use it and there is no room for even considering a collusion in the case of Bertrand competition since OPEC has the advantage and it can easily obtain the entire market demand.

Even though competing in prices will assure OPEC the entire market, the profits obtained are much lower than the profits under quantity competition. Therefore, it is much more profitable to compete in quantities rather than in prices in this case.

Since the conclusion defined in the Bertrand paradox is not that a realistic result with a wide applicability in real life, researchers have tried to find solutions to this paradox. The paradox can be resolved by relaxing on any one of the assumptions made initially (Tirole 211).

9.3. The Edgeworth Solution

Edgeworth (1987) tried to solve the Bertrand paradox by implementing capacity constraints, which is a more realistic approach. We will prove this by looking at a potential real-life example. Assume that in a small town there are only two hotels, hotel A and hotel B. Each hotel has, of course, a limited number of rooms that cannot be increased in the short-run. The question that arises now is if the Bertrand equilibrium $(p_A^*, p_B^*) = (c, c)$ still stands. If this is the case, the hotels will make zero profit. Now suppose that hotel A increases its prices slightly. Hotel B will face a demand of $D(c)$ that it cannot satisfy, thus some of the customers will pay the higher price in order to stay at hotel A, helping hotel A to achieve positive profits. Therefore, the Bertrand solution is not the equilibrium anymore. In conclusion, in a case like this (a duopoly with constraints in capacity) there is no point of having a strong competition in prices. Here, cooperation is possible, cooperation on an agreed price which is high enough so that the firms will have a positive profit. (Tirole 212)

After introducing capacity constraints and showing how the Bertrand paradox can be avoided, we will not take the case when the game is played more than one time, thus relaxing the temporal dimension.

9.4. The Temporal Dimension

Another critical assumption in the Bertrand solution is that the game is played only once. When having two firms, firm 1 and firm 2, this is the reason why $p_1 = p_2 > c$ is not a solution. In this case, if firm 2, for example, would lower the price to $p_2 - \varepsilon$ then it will gain all the market demand and there will be no consequences in the future since the

players will not interact anymore. This is rarely the case in real life, therefore, as it will be shown next, the players have to take in consideration the effects that a deviation will have on their relationship with the other player (Tirole 212)

We have seen that when competing in prices there is a high probability that the firm will end up charging the marginal cost as a price, which has as a result a profit $\Pi = 0$ for both firms. Therefore, we will analyze what will happen if the firms collude and act as a monopoly. The two firms that will cooperate are the same as in the ones in the model when the Bertrand paradox was presented. There are two firms producing homogenous goods with a marginal cost of c . The demand follows the same pattern: for firm i , $D(p_i) = a - p_i$ if $p_i < p_j$, $D(p_i) = 0$ if $p_i > p_j$ and $D(p_i) = (a - p_i)/2$ if $p_i = p_j$. The profit in this case will be $\Pi_M = (p_M - c)(a - p_M)$. By solving the maximization problem with respect to p_M , we find the market price of $p_M = p_1 = p_2 = (a + c)/2$, which will sell a total quantity on the market of $q_M = (a - c)/2$, produced in equal quantities by the two firms on the market, $q_1 = q_2 = (a - c)/4$. This brings a profit of $\Pi_C = \Pi_I = \Pi_D = (a - c)^2/8$.

If firm 2 decides to deviate from the cooperation agreement and set the price at $p_2 = p_D = p_M - \varepsilon$, where ε is very small, then firm 2 will have to satisfy the entire demand (when calculating we ignore ε since it has a very low value) $q_2 = q_M = (a - c)/2$, which will give firm 2 a profit of $\Pi_D^D = (a - c)^2/4$ and firm 1 a profit $\Pi_I = 0$. After deviating, the collusion breaks and the future profits will return to the non-cooperation level of $\Pi = 0$ due to the low price set $p = c$.

Therefore, the accumulated profit when cooperating will be:

$$\Pi_C + \delta\Pi_C + \delta^2\Pi_C + \dots = \Pi_C/(1 - \delta) = (a - c)^2/8(1 - \delta)$$

whereas when deviating, the accumulated profit is:

$$\Pi_D + 0 + 0 + \dots = \Pi_D = (a - c)/4.$$

Since the two firms are identical, the profits for the two cases, cooperation and deviations, are the same.

Thus, calculating the discount factor by solving the equation $\Pi_C/(1 - \delta) \geq \Pi_D$ we get $\delta \geq \frac{1}{2}$. Hence, if $\delta \geq \frac{1}{2}$, cooperation is better off and the firms value the future $\frac{1}{2}$ more than the present.

The reason why the two firms produce the same quantities and earn the same profit is because the two firms are identical, which is not that often found in real life. Another observation we can make is that the price on the market, the quantity produced by each of the two firms, the total quantity on the market, the profits of the two firms are the same when the two firms collude in a price competition or when they collude in a quantity competition. The reason behind this is that we have used the same simple basic model of two identical firms (the two firms are having the same marginal cost c) producing two homogenous goods. In reality it is different; there are plenty of differences between the competing firms and the firms are always trying to improve their position in the market and gaining the competitive advantage. These advantages can be a lower marginal costs, better advertising or product innovation, just to name a few.

Another solution to the Bertrand paradox is introducing difference between the products produced, case which will be analyzed in the next section.

9.5. Product differentiation

Another critical assumption in the Bertrand solution is that the products that the two firms are selling are identical which makes the consumer indifferent about the product and will buy the lowest-priced one. If we relax this assumption, the producers will not charge the marginal cost (Tirole 212).

We assume there are two firms on the market, firm 1 and firm 2. Firm 1 and firm 2 will simultaneously choose the prices for their products and the demand for firm i will be:

$q_i(p_i, p_j) = a - p_i + bp_j$, where b reflects the extent to which product of firm 1 is a substitute of product of firm 2. Beside the simultaneity of the actions of the two firms, we also assume that the two firms do not have fixed costs for production, only marginal cost which is constant at $c < a$.

As in the previous case of Bertrand competition, the two firms on the market are the two players of the game and the strategies available are the different prices that can be

chosen by the firms, s_i which is part of the strategy space $S_i = [0, \infty)$, and can be a nonnegative real numbers (Gibbons 22).

As before, the payoffs for each of the firms are considered its profits. Therefore, the general form of the profit of firm i when choosing the price p_i and firm j choosing price p_j is:

$$\Pi_i(p_i, p_j) = q_i(p_i, p_j^*) (p_i - c) = (a - p_i + bp_j^*) (p_i - c).$$

By solving firm's i optimization problem $\max_{p_i} \Pi_i$, the solution we reach will be:

$$p_i^* = (1/2)(a + bp_j^* + c).$$

Therefore, after satisfying and solving the equations $p_1^* = (1/2)(a + bp_2^* + c)$ and $p_2^* = (1/2)(a + bp_1^* + c)$ the result it will yield is $p_1^* = p_2^* = (a + c)/(2 - b)$. When this equations are satisfied, the price pair found, (p_1^*, p_2^*) will be a Nash – equilibrium (Gibbons 22). In this case, the profits of the two firms are $\Pi_1 = \Pi_2 = (a - c + bc)^2/(2 - b)^2$.

There are many ways for the products to be seen as different, one can be by measuring the level of substitution of the two products, as presented before, or another example can be differentiating the products by the level of accessibility for the customers.

Imagine a small town where there are only two bakeries, one of them situated in the center of the city and the other one in the periphery of the city. They both sell the same homemade bread, having the same recipe. The difference is in price and the location of the store. The bakery 1, the one that is located downtown, charges the price $p_1 = c$, while bakery 2 charges a slightly higher price $p_2 = c + \varepsilon$. It is true that the customers would have the incentive to buy from bakery 1 since it has a lower price for the same good, but if their household is closer to bakery 2 or if bakery 2 is easier to reach than bakery 1, they would prefer paying the small difference ε rather than loose time and money on transportation to bakery 1. The fact that bakery 2 is more accessible to some customers helps it to reach a higher profit than bakery 1 for the simple reason that it charges a higher price. It is true that when comparing the quantities sold, bakery 1 will have an advantage, but in the end, a business is profit oriented. Since we are framing a potential real life situation, the result of the first stage of this game is that bakery 1 will

also increase its prices. This way, the Bertrand solution, that the two firms will charge the marginal cost, is not a solution for this case.

It has been shown that through traditional Bertrand competition the firms will reach the situation of charging the marginal cost c . This situation, the one of Bertrand paradox can be solved by relaxing on some of the assumptions made, as presented previously.

After presenting both of the classical methods of competition, in quantities and in prices, it has been chosen to analyze the case of competing in R&D due to the importance of the field in the current business environment. R&D is the one that helps firms develop new products or new ways of production and introducing innovation through higher standards in the market.

10. Research and development

Research and development (R&D) is crucial not only from an individual business perspective, but as well from an economy-wide viewpoint. Solow (1957) concluded his discoveries with the idea that special attention has to be given to the firms' incentives to innovate and implement new technologies (Tirole 389).

John Seely Brown, Director Emeritus at Xerox Palo Alto Research Center believes that innovation is invention implemented and taken over the market (Chesbrough ix).

Innovation can be divided into two categories: product innovation and process innovation. Product innovation refers to creating new goods or services, while process innovation refers to creating new ways of producing existing products or new ways of lowering the cost of production of new products. Even though the innovation can be divided into two categories, there cannot be drawn a clear line between the two types. An example can be the case when one firm's product innovation can lead to another firm's process innovation (Tirole 389). The reason can be that the latter firm could already have had a version of the 'new' product of the first firm, but now with the new innovative product, it can improve its production cost.

Regarding process and product innovation, it has been proven in different researches that there exists a connection between the two types of innovation and the size of the firm. It has been found that large firms devote a higher proportion of their R&D expenditure on process innovation than smaller firms. When looking at the different expenditures incurred by different sized firms in product R&D, the difference is in no way dramatic (Fritsch & Meschede 2001).

As mentioned, R&D has an extremely important role especially in today's globalized business world, competition is growing at an increasing rate. Companies have to find ways in order to remain on the competitive edge; this can for example be done through cooperation in R&D, case which will be discussed in the next section.

10.1. Cooperative and non-cooperative R&D in duopoly with spillovers

D'Aspremont and Jacquemin (1988) have developed a model in order to compare the effects of cooperation in R&D between fierce competitors. They have had considered the case of a duopoly where cooperation can be present in one or both of the two stages considered of the game. Therefore, they have considered two types of agreement. First, R&D cooperation can appear at the so-called 'precompetitive stage' when firms share basic information and effort in the R&D stage, but remain competitors on the market. The second type of agreement presents the case of an extensive collusion between firms, creating common policies at the product level as well. This type of cooperation helps reducing the difficulties in protecting intellectual property. The main reason is to allow the partners involved, who have achieved together innovation, to control together as well the process of production, in order to recuperate jointly their initial R&D investments (D'Aspremont & Jacquemin, 1988).

As a real-life example of the second type of agreement previously mentioned, where the firms collude in the entire process, during the R&D process and as well in the implementation phase, is EUREKA. EUREKA is a pan-European framework for R&D collaboration. The initiative of forming the consortia was based on the idea that knowledge can be improved if organizations with complementary knowledge had the

opportunity of working together through R&D activities in a cooperative manner. The main purpose of EUREKA is to conduct applied research with the intention of exploiting its commercial opportunities, whereas the aim of the consortia is to strengthen European competitiveness by helping organizations to work together through collaborative R&D projects in most fields of advanced civil technology, such as transportation industries, communication or energy, just to name a few (Monthe et all, 2000).

The example on which D'Aspremont and Jacquemyn (1988) have worked in their research in R&D cooperation and noncooperation consists of 'a simple yet elegant symmetric duopoly model of R&D and spillovers to compare several equilibrium concepts' (Henriques 1990).

The model implies that the two firms on the market are facing an inverse demand function

$$D^{-1}(Q) = a - bQ,$$

where $Q = q_1 + q_2$ is the total quantity on the market and $a, b > 0$.

Each of the firms incurs a cost of production

$$C_i(q_i, x_i, x_j) = [A - x_i - \beta x_j] q_i, \quad i = 1, 2, i \neq j$$

with $0 < A < a$, $0 < \beta < 1$; $x_i + \beta x_j \leq A$; $Q \leq a/b$. The term β indicates the spillover parameter.

As it can be seen, the cost of production is a function of its own production q_i , of the amount of research x_i it undertakes and the amount of research x_j that its rival undertakes. As shown, both of the function, D^{-1} and C are linear functions (D'Aspremont et all, 1988).

In their paper, D'Aspremont and Jacquemyn (1988), when analyzing the cases of cooperation and noncooperation, have also considered the existence of spillovers. The R&D spillovers or externalities imply that some benefits of each firm's R&D will flow to other firms without payment. In the model presented earlier, the external effect of *firm j* R&D is to lower *firm i*'s unit production cost. An interpretation of this situation

can be the case when the successful inventions of rivals can be replicated at a lower cost of firm i than if it were invented by firm i itself (D'Aspremont et all, 1988).

The model presented is a two stage game. As mentioned, there are two firms on the market. At the first stage, the firms choose their level of research x_1 and x_2 , while in the second stage; they choose the quantities q_1 and q_2 that they will produce. Taking this in consideration, we will analyze three cases: when the two firms act non-cooperatively in both of the stages of the game, while choosing R&D levels and as well when choosing quantities (*a*); in the second case we will introduce cooperation when choosing R&D levels, while the competition in quantities stay at a non-cooperative level (*b*); in the third case, the firms cooperate at both of the stages, R&D and quantities, forming a scenario that can be analyzed as a monopoly (*c*).

Case a – non-cooperation at both stages

By not cooperating at any of the stages, the profit of firm i , conditional on x_1 and x_2 will be:

$$\Pi_i = [a - bQ]q_i - [A - x_i - \beta x_j]q_i - \gamma x_i^2/2, j \neq i, i = 1, 2.$$

By solving the maximization functions $\max_{q_1} \Pi_1$ and $\max_{q_2} \Pi_2$, where $\Pi_1 = [a - bQ]q_1 - [A - x_1 - \beta x_2]q_1 - \gamma x_1^2/2$ and $\Pi_2 = [a - bQ]q_2 - [A - x_2 - \beta x_1]q_2 - \gamma x_2^2/2$, we find that the Nash- Cournot equilibrium can be computed to be:

$$q_i = [(a - A) + (2 - \beta)x_i + (2\beta - 1)x_j]/3b.$$

We can notice that $q_1 + q_2 \leq (1/3b)[2(a - A) + 2A] \leq a/b$.

Therefore, we have found the quantity that will be produced in a non-cooperative environment by using backward induction; we now have to compute the level of R&D that the two firms will choose in this situation of non-cooperation. Thus, at the first stage of the game, the profits can be written as:

$$\Pi_i^* = (1/9b)[(a - A) + (2 - \beta)x_i + (2\beta - 1)x_j]^2 - \gamma x_i^2/2, j \neq i, i = 1, 2.$$

There exists an unique and symmetric solution that satisfies the maximization function, taking in consideration that the second-order conditions require that $[2(2 - \beta)^2]/9b - \gamma < 0$. For the unique solution, the following are true:

$$x_i^* = [(a - A)(2 - \beta)]/[4,5b\gamma - (2 - \beta)(1 + \beta)] \text{ with } i = 1, 2.$$

$$Q^* = q_i^* + q_j^* = 2(a - A)/3b + 2(\beta + 1)x_i^*/3b = [2(a - A)/3b]\{4,5b\gamma/[4,5b\gamma - (2 - \beta)(1 + \beta)]\}.$$

Case b – cooperation through R&D in the first stage, while at the second stage, non-cooperation is maintained

At the first stage the firms will maximize their joint profits, as a function of x_1 and x_2 :

$$\Pi' = \Pi_1^* + \Pi_2^* = 1/9b \{[(a - A) + (2 - \beta)x_1 + (2\beta - 1)x_2]^2 - \gamma x_1^2/2 + [(a - A) + (2 - \beta)x_2 + (2\beta - 1)x_1]^2 - \gamma x_2^2/2\}, j \neq i.$$

We assume that the R&D level chosen by the firms will be the same, $x_1 = x_2 = x'$, therefore, while taking in consideration that $(2/9)(1 + \beta)^2 < b\gamma$, the unique solution for the equilibrium when the firms are cooperating through R&D will be:

$$x' = (1 + \beta)(a - A)/[4,5b\gamma - (1 + \beta)^2];$$

$$Q' = 2(a - A)/3b + 2(1 + \beta)x'/3b = [2(a - A)/3b]\{4,5b\gamma/[4,5b\gamma - (1 + \beta)^2]\}.$$

If we compare the two cases, *case a* and *case b*, we can observe that especially in the case of large spillovers, meaning $\beta > 0,5$, we observe that if we introduce cooperation through R&D, the level of R&D will be higher $x' > x^*$. If we compared the quantities that will result in the first two cases, we can state that cooperation through R&D increases the quantities produced, $Q' > Q^*$ especially in the case of large spillovers when $\beta > 0,5$. Even though we have found that $x' > x^*$ and $Q' > Q^*$, what is more important is that the profits of the two firms to be higher under cooperation. The latter condition is always sufficient to trigger cooperation.

Case c – monopoly

In this case, as mentioned before, firms cooperate in both of the stages of the game, they cooperate through R&D, as well as when choosing quantities. In this manner, the situation on the market will transform from a duopoly into a monopoly.

At the second stage, the joint profit is given by the equation and it is conditional of x_1 and x_2 :

$$\Pi = (a - bQ)Q - AQ + (x_1 + \beta x_2)q_1 + (x_2 + \beta x_1)q_2 - \gamma x_1^2/2 - \gamma x_2^2/2.$$

Assuming that the two firms choose the same level of R&D, $x_1 = x_2 = x$, the symmetric solution $q_1'' = q_2''$ will lead to:

$$Q = q_1 + q_2 = [(a - A) + (1 + \beta)x]/2b.$$

Therefore, at the preceding stage, the joint profit will become

$$\Pi'' = (1/b)\{[a - A + (1 + \beta)x]/2\}^2 - \gamma x^2.$$

By solving the maximization problem while taking in consideration that $(1 + \beta)^2/4 < b\gamma$, we will find the unique solution for the symmetric cooperative equilibrium in R&D and in production, which will be:

$$x'' = (a - A)(1 + \beta)/[4b\gamma - (1 + \beta)^2];$$

$$Q'' = (a - A)/2b + (1 + \beta)x''/2b = [(a - A)/2b]\{4b\gamma/[4b\gamma - (1 + \beta)^2]\}.$$

When comparing the results obtained in this case with the ones from the previous two cases, we observe that the collusive output, for a given level of R&D, is smaller than the non-cooperative one, $Q'' < Q^*$ iff $5\beta^2 + 4\beta - 1 < 3b\gamma$, but this will not be the case when the optimal amount of R&D will be incorporated. The collusive amount of R&D (*case c*) varies with β and is, for enough larger spillovers $\beta > 0,41$, higher than the fully non-cooperative case (*case a*). As expected, when compared to the case of cooperation through R&D stage (*case b*), the amount of R&D is higher in the case of complete collusion (*case c*), $x' < x''$. The reason behind this finding is that due to complete collusion, there is lower competition on the product market; therefore, the surplus of the research will lead to more R&D expenditure. Even though the amount of R&D is higher in *case c*, when we compare the quantities produced, we will observe that there is a

higher level of quantities produced under limited cooperation (*case b*), than in the case of fully cooperation (*case c*), $Q' > Q''$ iff $(1 + \beta)^2 < 3b\gamma$.

Conclusions of the model

We will summarize the conclusions by comparing the level of R&D implemented by the firms in each of the three cases, conclusion 1, and by comparing the quantities produced in each of the three cases, conclusion 2. In their study, in the last section of their article, D'Aspremont and Jacquemyn (1988) have also calculated the social optimum and comparing the previous results with it.

Conclusion 1

The conclusions that can derivate from the model analyzed with respect to the levels of R&D are that cooperation only in R&D (*case b*) increases both the expenditures in R&D, when compared to the non-cooperative case (*case a*) when the spillovers levels are high enough, $\beta > 0,5$, whereas in the second comparison, *case b* and *case c*, the level of R&D that the model implies it is always higher under cooperation at both of the stages. Therefore, the following classification results:

$$x^* < x' < x''.$$

This classification stands especially in the case when the spillover parameter is high, $\beta > 0,5$. In this case, the social optimum is closer to being achieved when the firms are cooperating at both of the stages, while when through noncooperation, the social optimum is more distant.

Conclusion 2

When analyzing the results about the quantities that will be produced under the three cases discussed, the results are not that easily comparable due to further conditions at different stages of the comparison.

While comparing *case a* and *case b*, for large level of spillovers ($\beta > 0,5$), $Q^* < Q'$. While comparing *case b* and *case c*, the following condition has to be fulfilled $(1 + \beta)^2$

$< 3b\gamma$, in order to be able to make the following classification: $Q' > Q''$. In the third comparison, *case a* compared to *case c*, the condition of $5\beta^2 + 4\beta - 1 < 3b\gamma$ has to be achieved in order to have the case of $Q'' < Q^*$.

If all the conditions mentioned above are fulfilled, the following overall classification of the quantities produced in the three scenarios taken in consideration can be made:

$$Q'' < Q^* < Q'.$$

Therefore, concerning the quantities produced, the social optimum is closest to being achieved while cooperating at the ‘precompetitive stage’, when the firms will cooperate in R&D.

The overall conclusion reached is that cooperation will play an important role when the market characteristic is that there are few firms competing and the firms are using R&D that generates spillovers. The limitation of the study is that the real-life model has been simplified by neglecting some crucial aspects of R&D activities (D’Aspremont et all, 1988) .

11. Further research

A first step that can be done as a future analysis is that of including in a future paper the limitations we have put. As well, it should also be included and studied more into detail the competition on different levels in R&D. It is a field which can show several interesting results.

12. Conclusion

Throughout the analysis we have reached the conclusion that if certain factors are fulfilled, cooperation will yield better off results than the competitive scenario. The analysis has been made on two types of competition, in prices and in quantities, as well as on a simple model of competition in R&D.

We have observed and proved that the profits thus, the desire of obtaining higher monetary rewards is enough to trigger cooperation, but as well it can also be the factor that will make the players having the incentive to deviate after agreeing on cooperating.

We observe a common factor on which the results of the cooperation depend on. The discount factor it is the most important trigger tool in cooperation. It has to be high enough in order for cooperation to exist. If this condition is not fulfilled, cooperation is not possible to appear between the firms.

The discount factor can be in the same time a trigger of cooperation and as well a factor that will influence the results of cooperation by influencing the profits.

It has been shown that cooperation can differ from case to case. In some of the cases we encounter a basic model which makes it easy to analyze, whereas in other cases, it is a more demanding analysis, as the case of OPEC under Cournot competition.

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14. Appendix

The calculations shown in the appendix refer to section 8.4.2 and represent the values of the payment X_1 and X_2 which are calculated while taken in consideration the value of the discount factor by using the following equations:

$$1291,67 * \delta_1 = X_1 \text{ and}$$

$$675 / \delta_2 + 158,33 = X_2.$$

It has to be emphasized that δ_1 and δ_2 take the values that δ takes in the following table:

δ	X1	X2
0	0	
0,01	12,9167	67658,33
0,02	25,8334	33908,33
0,03	38,7501	22658,33
0,04	51,6668	17033,33
0,05	64,5835	13658,33
0,06	77,5002	11408,33
0,07	90,4169	9801,187
0,08	103,3336	8595,83
0,09	116,2503	7658,33
0,1	129,167	6908,33
0,11	142,0837	6294,694
0,12	155,0004	5783,33
0,13	167,9171	5350,638
0,14	180,8338	4979,759
0,15	193,7505	4658,33
0,16	206,6672	4377,08
0,17	219,5839	4128,918
0,18	232,5006	3908,33
0,19	245,4173	3710,962
0,2	258,334	3533,33
0,21	271,2507	3372,616
0,22	284,1674	3226,512
0,23	297,0841	3093,113
0,24	310,0008	2970,83
0,25	322,9175	2858,33
0,26	335,8342	2754,484
0,27	348,7509	2658,33

0,28	361,6676	2569,044
0,29	374,5843	2485,916
0,3	387,501	2408,33
0,31	400,4177	2335,749
0,32	413,3344	2267,705
0,33	426,2511	2203,785
0,34	439,1678	2143,624
0,35	452,0845	2086,901
0,36	465,0012	2033,33
0,37	477,9179	1982,654
0,38	490,8346	1934,646
0,39	503,7513	1889,099
0,4	516,668	1845,83
0,41	529,5847	1804,671
0,42	542,5014	1765,473
0,43	555,4181	1728,097
0,44	568,3348	1692,421
0,45	581,2515	1658,33
0,46	594,1682	1625,721
0,47	607,0849	1594,5
0,48	620,0016	1564,58
0,49	632,9183	1535,881
0,5	645,835	1508,33
0,51	658,7517	1481,859
0,52	671,6684	1456,407
0,53	684,5851	1431,915
0,54	697,5018	1408,33
0,55	710,4185	1385,603
0,56	723,3352	1363,687
0,57	736,2519	1342,541
0,58	749,1686	1322,123
0,59	762,0853	1302,398
0,6	775,002	1283,33
0,61	787,9187	1264,887
0,62	800,8354	1247,04
0,63	813,7521	1229,759
0,64	826,6688	1213,018
0,65	839,5855	1196,792
0,66	852,5022	1181,057
0,67	865,4189	1165,793
0,68	878,3356	1150,977
0,69	891,2523	1136,591
0,7	904,169	1122,616
0,71	917,0857	1109,034

0,72	930,0024	1095,83
0,73	942,9191	1082,988
0,74	955,8358	1070,492
0,75	968,7525	1058,33
0,76	981,6692	1046,488
0,77	994,5859	1034,953
0,78	1007,503	1023,715
0,79	1020,419	1012,76
0,8	1033,336	1002,08
0,81	1046,253	991,6633
0,82	1059,169	981,5007
0,83	1072,086	971,583
0,84	1085,003	961,9014
0,85	1097,92	952,4476
0,86	1110,836	943,2137
0,87	1123,753	934,1921
0,88	1136,67	925,3755
0,89	1149,586	916,757
0,9	1162,503	908,33
0,91	1175,42	900,0882
0,92	1188,336	892,0257
0,93	1201,253	884,1365
0,94	1214,17	876,4151
0,95	1227,087	868,8563
0,96	1240,003	861,455
0,97	1252,92	854,2063
0,98	1265,837	847,1055
0,99	1278,753	840,1482
1	1291,67	833,33