2.4 Interconnection of the Structure with Aerodynamics

Structural and aerodynamic grids are connected by interpolation. This allows the independent selection of grid points of the structure and aerodynamic elements of the lifting surfaces/bodies in a manner best suited to the particular theory. The structural model for a wing may involve a one-, two- or three-dimensional array of grid points. The aerodynamic theory may be a lifting surface theory or a strip theory. A general interpolation method is available that will interconnect the various combinations. Any aerodynamic panel or body can be subdivided into subregions for interpolation, using a separate function for each.

The interpolation method is called splining. The theory involves the mathematical analysis of beams and plates (see Figure 2-2). Three methods are available:

- linear splines, which are a generalization of an infinite beam and allow torsional as well as bending degrees of freedom
- surface splines, which are solutions for infinite uniform plates
- an explicit user-defined interpolation

Several splines, including combinations of the three types, can be used in one model. For example, a model may use one spline for the horizontal tail and three splines for the wing (a surface spline for the inboard section, and a linear spline for the outboard wing section, and the explicit interpolation for the aileron). Separation into subregions allows discontinuous slopes (e.g., at the wing-aileron hinge), separate functions (for wing and tail), and smaller regions. Smaller regions reduce the computing time and may increase the accuracy.

The structural degrees of freedom have been chosen in MSC/NASTRAN as the independent degrees of freedom; the aerodynamic degrees of freedom are dependent. A matrix is derived that relates the dependent degrees of freedom to the independent ones. The structural degrees of freedom may include any grid components. Two transformations are required: the interpolation from the structural deflections to the aerodynamic deflections and the relationship between the aerodynamic forces and the structurally equivalent forces acting on the structural grid points.

The splining methods lead to an interpolation matrix $\left[G_{kg}\right]$ that relates the components of structural grid point deflections $\{u_g\}$ to the deflections of the aerodynamic grid points $\{u_k\}$,

$$\{u_k\} = \left[G_{kg}\right]\{u_g\} \qquad (2\text{-}22)$$

The derivation of the elements of $\left[G_{kg}\right]$ is discussed in the following sections for the surface spline and linear spline interpolation methods. Attention is first given to the transformation between the aerodynamic and structural force systems. This transformation is found from the requirement that the two force systems be "structurally equivalent" rather than statically equivalent. Structural equivalence means that the two force systems deflect the structure equally. Statically equivalent force systems, as used on a whiffletree in a static structural strength test, do not result in equal deflections. It is the deflections rather than resultant loads that are of primary interest in aeroelasticity. The concept of structural equivalence is discussed by Schmitt (1956) and Rodden (1959b).

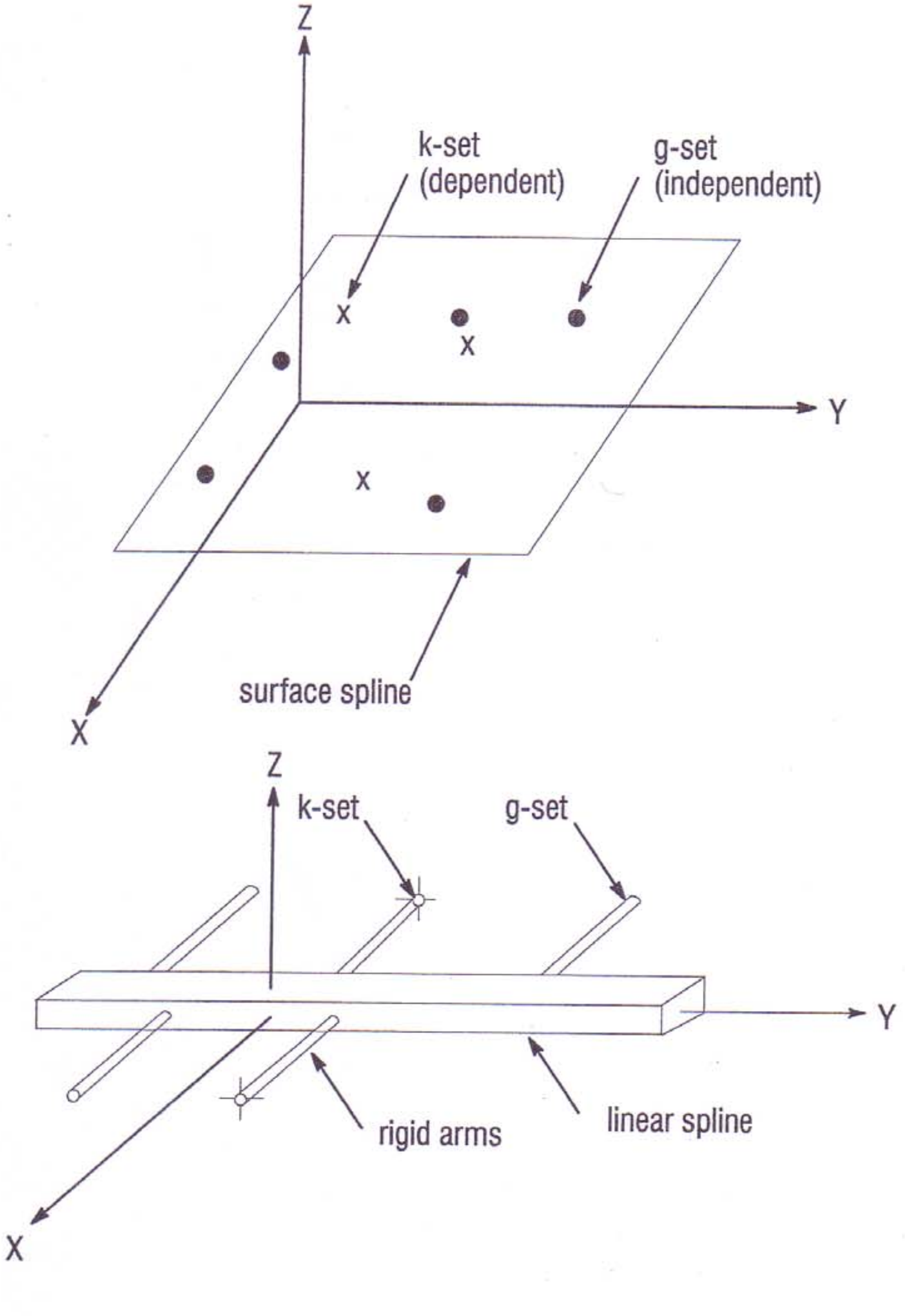

Figure 2-2. Splines and Their Coordinate Systems.

The aerodynamic forces $\{F_k\}$ and their structurally equivalent values $\{F_g\}$ acting on the structural grid points therefore do the same virtual work in their respective deflection modes,

$$\{\delta u_k\}^T \{F_k\} = \{\delta u_g\}^T \{F_g\} \tag{2-23}$$

where δu_k and δu_g are virtual deflections. Substituting Eq. (2-22) into the left-hand side of Eq. (2-23) and rearranging yields

$$\{\delta u_g\}^T \left(\left[G_{kg}\right]^T \{F_k\} - \{F_g\} \right) = 0 \tag{2-24}$$

from which the required force transformation is obtained because of the arbitrariness of the virtual deflections.

$$\{F_g\} = \left[G_{kg}\right]^T \{F_k\} \tag{2-25}$$

Equations (2-22) and (2-25) are both required to complete the formulation of aeroelastic problems in which the aerodynamic and structural grids do not coincide, i.e., to interconnect the aerodynamic and structural grid points. However, since the transpose of the deflection interpolation matrix is all that is required to connect the aerodynamic forces to the structure, it is appropriate in further discussions to refer to the interconnection problem as simply the problem of interpolating from the structural to the aerodynamic grid points.

Theory of Linear Splines

A linear spline is a "beam" function, $w(x)$, which passes through the known deflections, $w_i = w(x_i)$ with twists $\phi_i = \phi(x_i)$. Bending deflections are easily solved by the three-moment method, which is appropriate for simple beams. However, an extension of the method is required for splines with torsion, rigid arms, and attachment springs. The following derivations outlined are based on analogy with the surface spline derivation.

Bending Bars

Equation for deflection

$$EI\frac{d^4w}{dx^4} = q - \frac{dM}{dx} \tag{2-44}$$

where q is a distributed transverse load and M is a distributed moment.

A symmetric fundamental solution for $x \neq 0$ is used for lateral loads $q = P\delta(x)$ where $\delta(x)$ is the Dirac δ-function, and an antisymmetric fundamental solution is used for moments. The solution for the general case is found by superimposing the fundamental solutions,

$$w(x) = a_0 + a_1x + \sum_{i=1}^{N}\left[-\frac{M_i(x - x_i)|x - x_i|}{4EI} + \frac{P_i|x - x_i|^3}{12EI}\right] \tag{2-45}$$

$$\theta(x) = \frac{dw}{dx} = a_1 + \sum_{i=1}^{N} \left[-\frac{M_i |x - x_i|}{2EI} + \frac{P_i (x - x_i) |x - x_i|}{4EI} \right] \tag{2-46}$$

These are written in matrix notation as

$$\begin{Bmatrix} w(x) \\ \theta(x) \end{Bmatrix} = \left[\begin{array}{cc|cc|cc} 1 & x & \frac{|x - x_1|^3}{12EI} & \cdots & -\frac{(x - x_1)|x - x_1|}{4EI} & \cdots \\ 0 & 1 & \frac{(x - x_1)|x - x_1|}{4EI} & \cdots & -\frac{|x - x_1|}{2EI} & \cdots \end{array} \right] \begin{Bmatrix} a_0 \\ a_1 \\ \hline P_1 \\ \vdots \\ P_N \\ \hline M_1 \\ \vdots \\ M_N \end{Bmatrix} \tag{2-47}$$

To satisfy the boundary condition at infinity, $w(x)$ must approach a linear function. This requires

$$\sum P_i = 0 \tag{2-48}$$

$$\sum (x_i P_i + M_i) = 0 \tag{2-49}$$

These are recognized as the equations of equilibrium. The unknowns a_i, P_i, and M_i are found from

$$\begin{Bmatrix} 0 \\ 0 \\ \hline w_1 \\ \vdots \\ w_N \\ \hline \theta_1 \\ \vdots \\ \theta_N \end{Bmatrix} = \begin{bmatrix} 0 & R_1^T & R_2^T \\ R_1 & A_{11} & A_{21} \\ R_2 & A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \\ \hline P_1 \\ \vdots \\ P_N \\ \hline M_1 \\ \vdots \\ M_N \end{Bmatrix} \tag{2-50}$$

where it has been assumed that $x_1 < x_2 \cdots < x_N$, and

$$R_1^T = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_N \end{bmatrix}$$

$$R_2^T = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \end{bmatrix}$$

$$A_{11} = \begin{bmatrix} 0 & \dfrac{(x_2 - x_1)^3}{12EI} & \cdots & \dfrac{(x_N - x_1)^3}{12EI} \\ \dfrac{(x_2 - x_1)^3}{12EI} & 0 & \cdots & \dfrac{(x_N - x_2)^3}{12EI} \\ \vdots & \vdots & \cdots & \vdots \\ \dfrac{(x_N - x_1)^3}{12EI} & \dfrac{(x_N - x_2)^3}{12EI} & \cdots & 0 \end{bmatrix}$$

$$A_{21} = \begin{bmatrix} 0 & -\dfrac{(x_2 - x_1)^2}{4EI} & \cdots & -\dfrac{(x_N - x_1)^2}{4EI} \\ \dfrac{(x_2 - x_1)^2}{4EI} & 0 & \cdots & -\dfrac{(x_N - x_2)^2}{4EI} \\ \vdots & \vdots & \cdots & \vdots \\ \dfrac{(x_N - x_1)^2}{4EI} & \dfrac{(x_N - x_2)^2}{4EI} & \cdots & 0 \end{bmatrix}$$

$$A_{22} = \begin{bmatrix} 0 & -\dfrac{(x_2 - x_1)}{2EI} & \cdots & -\dfrac{(x_N - x_1)}{2EI} \\ -\dfrac{(x_2 - x_1)}{2EI} & 0 & \cdots & -\dfrac{(x_N - x_2)}{2EI} \\ \vdots & \vdots & \cdots & \vdots \\ -\dfrac{(x_N - x_1)}{2EI} & -\dfrac{(x_N - x_2)}{2EI} & \cdots & 0 \end{bmatrix}$$

Torsion Bars

Equation for twist:

$$GJ\left(\frac{d^2\phi}{dx^2}\right) = -t \tag{2-51}$$

where t is a distributed torque. The solution is

$$\phi(x) = \left[\begin{array}{c|cccc} 1 & -\frac{|x_1 - x|}{2GJ} & -\frac{|x_2 - x|}{2GJ} & \cdots & -\frac{|x_N - x|}{2GJ} \end{array}\right] \left\{\begin{array}{c} a_0 \\ \hline T_1 \\ T_2 \\ \vdots \\ T_N \end{array}\right\} \tag{2-52}$$

To satisfy the condition that ϕ = constant for large x requires the equilibrium condition

$$\sum T_i = 0 \tag{2-53}$$

Then the unknowns a_0 and T_i are found by solving

$$\left\{\begin{array}{c} 0 \\ \hline \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{array}\right\} = \left[\begin{array}{c|cccc} 0 & 1 & 1 & \cdots & 1 \\ \hline 1 & 0 & -\frac{|x_2 - x_1|}{2GJ} & \cdots & -\frac{|x_N - x_1|}{2GJ} \\ 1 & -\frac{|x_2 - x_1|}{2GJ} & 0 & \cdots & -\frac{|x_N - x_2|}{2GJ} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & -\frac{|x_N - x_1|}{2GJ} & -\frac{|x_N - x_2|}{2GJ} & \cdots & 0 \end{array}\right] \left\{\begin{array}{c} a_0 \\ \hline T_1 \\ T_2 \\ \vdots \\ T_N \end{array}\right\} \tag{2-54}$$