Promemoria Formule

Formule di somma/differenza per seno e coseno:

$$\cos(\alpha \pm \beta) = \cos\alpha\,\cos\beta \mp \sin\alpha\,\sin\beta \qquad \sin(\alpha \pm \beta) = \sin\alpha\,\cos\beta \pm \cos\alpha\,\sin\beta$$

Formule di prostaferesi:

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\,\cos\frac{\alpha-\beta}{2} \qquad \sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\,\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\,\cos\frac{\alpha-\beta}{2} \qquad \cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\,\sin\frac{\alpha-\beta}{2}$$

Limiti notevoli:

$$\lim_{x\to 0}\frac{\sin x}{x} = 1 \qquad \lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}$$

$$\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^x = e \qquad \lim_{x\to 0}\frac{\ln(1+x)}{x} = 1 \qquad \lim_{x\to 0}\frac{e^x-1}{x} = 1$$

Formule di Taylor centrate in $x_0 = 0$:

$$\begin{aligned}
e^x &= 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + o(x^n)\\
\ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots + (-1)^{n+1}\,\frac{x^n}{n} + o(x^n)\\
\sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + (-1)^n\frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})\\
\cos x &= 1 - \frac{x^2}{2} + \frac{x^4}{4!} + \ldots + (-1)^n\frac{x^{2n}}{(2n)!} + o(x^{2n+1})\\
\tan x &= x + \frac{x^3}{3} + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \frac{62}{2835}x^9 + o(x^{10})\\
\arctan x &= x - \frac{x^3}{3} + \frac{x^5}{5} + \ldots + (-1)^n\frac{x^{2n+1}}{(2n+1)} + o(x^{2n+2})\\
\frac{1}{1-x} &= 1 + x + x^2 + x^3 + \ldots + x^n + o(x^n)\\
(1+x)^\alpha &= 1 + \alpha\,x + \frac{\alpha(\alpha-1)}{2}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \ldots\\
&\quad + \frac{\alpha(\alpha-1)(\alpha-2)\ldots(\alpha-n+1)}{n!}x^n + o(x^n)
\end{aligned}$$