Master of Science course in Mathematical Engineering
a.a. 2016-2017

Numerical Methods
Prof. G.Toraldo

edited by Dott.ssa R. Campagna
15/05/17

`Matlab Spline Toolbox:` ***some functions***

1. We want to generate the B-form of a cubic spline (order $k = 4$) on the interval $[1, 3]$, with interior breaks $1.5, 1.8, 2.6$, and with two continuous derivatives in the internal knots. Remember that the knot multiplicities govern the smoothness of the spline across the knots.

   - Define the appropriate knot sequence by introducing repetitions in the set of the breakpoints and by imposing that the first and the last knots have multiplicity equal to 4, by using the Matlab function `augknt`.
   - Describe if the b-spline, defined at the previous point, is a continuous function in each knot; say how many derivatives are continuous in each knot, according to the order of the b-spline and to the multiplicity of the knot itself.
   - Modify the knot multiplicity so that the function has a jump in the first derivative at 2.6 and so that it is continuous with its derivative at 1.8.

2. The matlab function `bspline(t)` plots a B-spline and its polynomial pieces defined on the t set of knots.

```
bspline(t) plots the B-spline with knot sequence t,  as
well as the polynomial pieces of which it is composed.

bspline(t,window) does the plotting in the subplot window
specified by window; see the MATLAB® command subplot
for details.

pp = bspline(t) plots nothing but returns the ppform
```

`of the B-spline.`

- Define a set of simple knots, plot the b-spline with this knot sequence; say:
  - the b-spline order;
  - the smoothness of the spline across the knots;
  - the degree of the polynomials defining the b-spline in each subset;
- Change the number and multiplicity of knots as desired. Observe how the regularity of the function varies.
- look at the "pp-form" of the b-spline by `pp = bspline(t)`.

3. `bspligui` starts a graphical user interface (GUI) for exploring how a B-spline depends on its knots. As you add, move, or delete knots, you see the B-spline and its first three derivatives change accordingly *dinamically*. Answer to the following questions about the B-spline with knot sequence $t_0 \leq \ldots \leq t_k$:

   (a) What is the sign of the b-spline on the open interval (t_0, t_k)?

   (b) At what knots is it zero? Consider both simple external knots and multiple external knots with multiplicity k.

   (c) How is the value of the b-spline outside $[t_0, t_k]$? Does the GUI `bspligui` plot the b-spline outside the close set $[t_0, t_k]$?

   (d) The B-spline is piecewise polynomial of order k; what are the degrees of its polynomial pieces between two breakpoints? Answer also by looking at the first three derivatives of the B-spline.

   (e) How change the degree of the piecewise polynomial of the b-spline every time you add/delete a knot?

   (f) Each knot t_j is a break for the B-spline, but it is permissible for several knots to coincide. What is maximally the number of nontrivial polynomial pieces? How is the multiplicity of each knot in this case? What is minimally the number of nontrivial polynomial pieces? How is the multiplicity of each knot in this case?

   (g) The smoothness of the B-spline across a break depends on the multiplicity of the corresponding knot. If the break occurs in the

knot sequence m times, then the $(k-m)th$ derivative of the B-spline has a jump across that break, while all derivatives of order lower than $(k-m)$ are continuous across that break. Thus, by varying the multiplicity of a knot, you can control the smoothness of the B-spline across that knot. Make some example to describe how the smoothness of the B-spline (and of its derivatives) change by adding or removing knots, i.e. by changing the multiplicity of the corresponding both internal ad external knots.