4. Viscous Roll Damping

We are now at the point at which we can set up and solve the equations of motion for a floating body under the influence of small-amplitude waves in an inviscid fluid. However, the results will turn out to be quite unsatisfactory for one mode of motion in particular: rolling. The reason for this is that viscous effects contribute significantly to roll damping. Neglecting these effects will result in predictions which are considerably higher than measured values, particularly near resonance.

The viscous contribution is significant in the case of roll damping simply because the wavemaking contribution is so small for slender (ship-like) bodies. Forced rolling motion generally does not produce much of a disturbance to the fluid; in fact for a semicircular cylinder rolling about its axis, no waves are radiated and so the wavemaking damping is zero. However, we know that some effort is required to produce the motion; the resistance comes from frictional drag on the hull surface. In addition, if the hull has sharp corners (like a rectangular barge) there will be additional damping due to flow separation and eddy formation. Also, appendages contribute significantly to roll damping, particularly at speed. Thus in order to make reasonable predictions of rolling motion, we need to be able to account for these effects.

Unfortunately, the theoretical prediction of viscous roll damping is beyond the current state-of-the-art for ship-like bodies. The available tools of computational fluid dynamics (CFD) are being applied to the problem, but results are just becoming available for very simple geometries under somewhat idealized conditions (see Salui et.al. [2000], Korpus et.al. [1997]), for example). Thus we must use semi-empirical methods or experiments to determine the damping moments.

Note that, as was the case for added mass, the possibility exists for double-accounting if viscous roll damping is included in the "wave-induced roll moment" term; it could also justifiably be included in the "steady" roll moment (e.g., the coefficients $\tilde{d}_2$ and $\tilde{d}_{27}$ in Eq. (3.389d)). Care must be taken to avoid this duplication when incorporating wave-induced moments in the equations of motion.

4.1 Experimental determination

The best way to find the roll damping moment for a ship would be to impart a roll velocity to the hull and measure the hydrodynamic roll moment; this would have to be done for a range of angular velocities as well as forward speeds. Unfortunately this is seldom (if ever) possible in practice, even in model tests. Thus one is usually forced to resort to a so-called "roll decay test", in which rolling motion is induced by releasing the hull after somehow applying a roll inclination (for example), and the subsequent rolling motion is measured; the roll damping is computed from these measurements. To see how this is done, we will start with the single degree-of-freedom equation for small-amplitude "unforced" roll motion:

$$(I_{xx} + A_{44})\ddot{\phi} + B_{44T}\dot{\phi} + C_{44}\phi = 0 \tag{5.139}$$

where B_{44T} denotes "total" roll damping, including viscous effects, and ϕ is the roll angle. This equation can be written in a more standard form by dividing by the coefficient of the acceleration:

$$\ddot{\phi} + 2\nu\dot{\phi} + \omega_0{}^2\phi = 0 \tag{5.139a}$$

where

$$\nu \equiv \frac{B_{44T}}{2(I_{xx} + A_{44})}; \quad \omega_0 \equiv \sqrt{\frac{C_{44}}{I_{xx} + A_{44}}} \tag{5.139b}$$

You should recall that the solution of Eq. (5.139a) is of the form

$$\phi(t) = \phi_0 e^{\sigma t} \tag{5.140}$$

where ϕ_0 is the initial roll amplitude. Substituting Eq. (5.140) in Eq. (5.139) yields the auxiliary equation

$$\sigma^2 + 2\nu\sigma + \omega_0{}^2 = 0 \tag{5.141}$$

Solving for σ we obtain

$$\sigma = -\nu \pm \sqrt{\nu^2 - \omega_0{}^2} \tag{5.142}$$

Note that ν and ω_0 are always positive quantities (C_{44} is positive for a *stable* vessel as we showed back in Chapter 2). If $\nu > \omega_0$, Eq. (5.142) yields two real solutions and we can write (5.140) in the form

$$\phi(t) = e^{-\nu t}\left(A_1 e^{bt} + A_2 e^{-bt}\right) \tag{5.143}$$

where

$$b \equiv \sqrt{\nu^2 - \omega_0{}^2} \tag{5.144}$$

and A_1 and A_2 are determined from initial conditions. In this case the roll angle exponentially approaches the equilibrium value (zero) without oscillating.

However, for rolling motion it is more usual to have $\nu < \omega_0$, so that the solutions given by Eq. (5.142) are complex; in this case the solution is of the general form

$$\phi(t) = e^{-\nu t}\left(A_1 \cos bt + A_2 \sin bt\right),$$

or, equivalently,

$$\phi(t) = \phi_0 e^{-\nu t} \cos(bt + \delta) \tag{5.145}$$

where $b \equiv ib$ and δ is a phase angle. The motion in this case exhibits decaying oscillations. Thus we see that the quantity

$$b \equiv \sqrt{\omega_0{}^2 - \nu^2} \tag{5.146}$$

is the *damped natural rolling frequency*, and since $\nu = 0$ when $B_{44T} = 0$, ω_0 is the *undamped natural rolling frequency* (as we have already seen in connection with heave in Section 1.2 above).[w]

The intermediate case, when $\nu = \omega_0$, is known as "critically damped"; in this case Eq. (5.139b) gives

$$B_{44T} = B_{44CR} = 2\sqrt{(I_{xx} + A_{44})C_{44}} \tag{5.147}$$

where B_{44CR} is the "critical damping" coefficient. Thus the coefficient ν can be expressed in terms of the *fraction of critical damping* κ (see Section 1.2 above):

$$\nu = \frac{B_{44T}}{B_{44CR}}\omega_0 \equiv \kappa\omega_0 \tag{5.148}$$

---

[w] As alluded to in Section 1.2, the quantities b, ω_0 and ν as defined in Eqs. (5.139b) and (5.146) are actually functions of the wave frequency ω. To determine the true undamped natural frequency (for example), one could compute ω_0 using Eq. (5.139b) at a range of frequencies and plot the resulting values against ω. The true natural frequency is given by the point on this curve at which $\omega = \omega_0$.

and, from Eq. (5.146),

$$b = \omega_0 \sqrt{1-\kappa^2} \tag{5.149}$$

A typical time history of unforced rolling motion is shown on Figure 5.10. Successive extrema are labeled ϕ_j on the figure. In this case there is a maximum at t=0 and so the phase $\delta = 0$. According to Eq. (5.145), the extrema should be given by

$$\phi_j = (-1)^j \phi_0 e^{-j\pi \frac{\kappa}{\sqrt{1-\kappa^2}}} \tag{5.150}$$

where we have made use of Eqs. (5.148) and (5.1490). The ratio of successive maxima or minima is then given by

$$\frac{\phi_j}{\phi_{j+2}} = e^{\frac{2\pi\kappa}{\sqrt{1-\kappa^2}}}$$

or

$$\ln\left(\frac{\phi_j}{\phi_{j+2}}\right) = \frac{2\pi\kappa}{\sqrt{1-\kappa^2}} \tag{5.151}$$

which provides a means of determining κ from the data, if the damping coefficient (or fraction of critical damping) is independent of the roll angle (in which case the ratio of extrema is constant) and frequency (since in this type of test we can only look at the natural frequency[x]).

In practice, however, the ratio of extrema usually varies as a function of the roll angle due to nonlinear effects. To attempt to account for these nonlinearities we can re-write Eqs. (5.139) and (5.139a) as follows[y]:

$$(I_{xx} + A_{44})\ddot{\phi} + B_{44,1}\dot{\phi} + B_{44,2}\dot{\phi}|\dot{\phi}| + B_{44,3}\dot{\phi}^3 + C_{44}\phi = 0 \tag{5.152}$$

---

[x] The wavemaking contribution (which we know to be a function of frequency) would in this case be computed at $\omega = \omega_0$ and subtracted from B_{44T}, determined from κ using Eq. (5.148), to obtain the linear viscous contribution.

[y] The restoring moment is also nonlinear; we will ignore this for the moment.

$$\ddot{\phi} + 2\nu\dot{\phi} + \alpha\dot{\phi}|\dot{\phi}| + \beta\dot{\phi}^3 + \omega_0{}^2\phi = 0 \tag{5.152a}$$

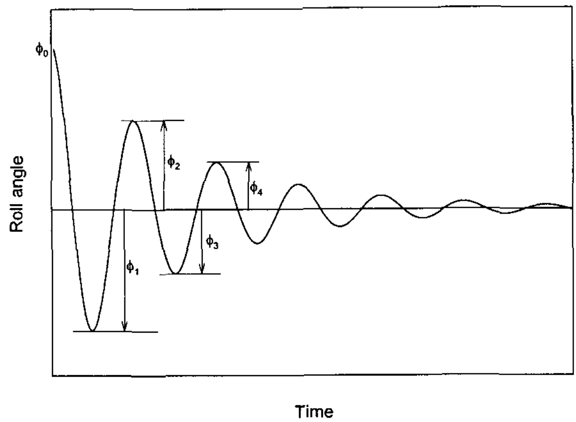

FIGURE 5.10 Typical roll decay time history

Returning now to the roll decay data, we will define the mean roll angle ϕ_m and roll decrement $\Delta\phi$ as

$$\phi_m = \frac{|\phi_j| + |\phi_{j+1}|}{2}; \quad \Delta\phi = |\phi_j| - |\phi_{j+1}|$$

By integrating Eq. (5.152a) over a half-period and equating the energy dissipated by damping to the work done by the restoring moment, we eventually obtain the following expression for the roll decrement as a function of the mean roll amplitude (Himeno[1981]):

$$\Delta\phi = \pi\kappa\phi_m + \frac{4}{3}\alpha\phi_m{}^2 + \frac{3\pi}{8}\beta\omega_0\phi_m{}^3 \tag{5.153}$$

Thus, one could find the coefficients κ, α and β (and so $B_{44,1}$, $B_{44,2}$ and $B_{44,3}$) by plotting the roll decrement against the mean roll angle (the "roll extinction curve"), and fitting a cubic polynomial to the data, at a range of vessel speeds. Unfortunately we are once again unable to identify any frequency dependence of the coefficients by this method, and in addition the coefficients are tacitly assumed to be independent of the amplitude of the roll motion. However it is certainly an improvement over neglecting the viscous contribution and should be fairly accurate at the frequency at which the rolling motion is largest, provided that the amplitude range of the extinction data brackets that experienced by the ship.

To illustrate the procedure, the first 8 extrema of a roll decay time history (such as that in Figure 5.9) are given in the second column of Table 5.1 below. The third and fourth columns contain the mean roll angle and the roll decrement, computed as indicated above. These are plotted on Figure 5.11. The results of the regression with a cubic polynomial (with constant term set equal to zero) are also shown.

By comparing the coefficients of the regression equation to those of Eq. (5.153) we can obtain the following values of the quantities κ, α and β:

$$\kappa = 0.18776/\pi = 0.060; \; \alpha = \frac{B_{44,2}}{I_{xx} + A_{44}} = 0.00456; \; \beta = \frac{B_{44,3}}{I_{xx} + A_{44}} = \frac{0.00056}{\omega_0}$$

Care must be taken in carrying out the curve fit, since it is possible to get nonsensical results if there is an insufficient number of data points available or if, for whatever reason, the data does not define a smooth curve (it seldom does). In these cases it is quite possible to obtain negative values for the regression coefficients; however, it is difficult to justify a negative fraction of critical damping physically. Similarly, the coefficient $B_{44,2}$, which is in essence a "crossflow drag coefficient" multiplied by a lever arm, is expected to be positive. The coefficient $B_{44,3}$, on the other hand, does not have a simple physical justification and so could be viewed as an empirical "adjustment" to the second-order term, which may be positive or negative, provided that the total nonlinear contribution is positive (we note that the values of $B_{44,3}$ presented by Himeno [1981] for four different ships, at a range of speeds, are all positive).

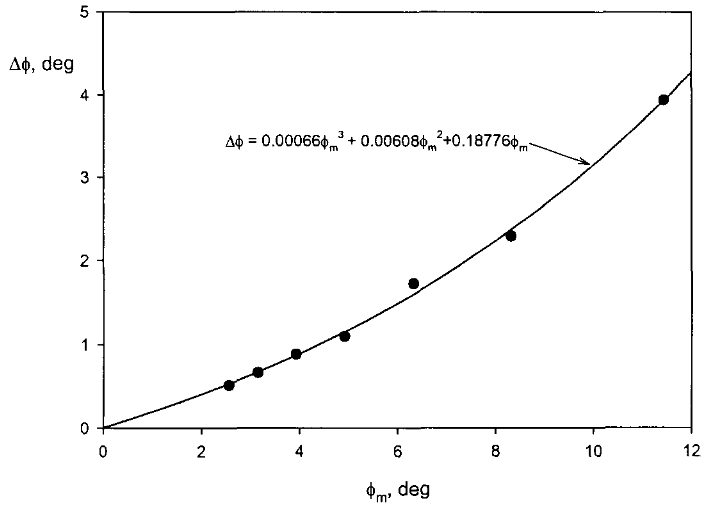

FIGURE 5.11 Roll extinction curve and regression results

TABLE 5.1 Roll decay data

j	ϕ_j deg	ϕ_m deg	$\Delta\phi$ deg
0	13.40		
		11.43	3.93
1	9.46		
		8.32	2.28
2	7.18		
		6.32	1.72
3	5.46		
		4.91	1.09
4	4.37		
		3.92	0.88
5	3.48		
		3.15	0.66
6	2.82		
		2.56	0.51
7	2.31		

4.1.1 General single degree-of-freedom response

At this point it is useful to digress somewhat to write the *general* expression for the single degree-of-freedom response[z] for a floating body in the frequency domain (Eq. (5.18) was the solution for heave motion), in terms of the dimensionless quantities defined above:

$$\frac{|x_j|}{A} = \frac{|X_j|/(M_{jj} + A_{jj})}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\kappa_j^2 \omega_0^2 \omega^2}} \tag{5.154a}$$

which can be re-written in the simple form

$$\frac{|x_j|}{A} = \frac{|X_j|/C_{jj}}{\sqrt{(1 - \Lambda^2)^2 + 4\kappa_j^2 \Lambda^2}} \tag{5.154b}$$

[z] Note that the effects of coupling among the modes of motion may be significant; the degree of coupling is a function of hull shape as well as the choice of the origin. Thus the single DOF equations should in general be used with caution.

if $C_{jj} \neq 0$ (that is, if there is a natural frequency in mode j; thus Eq. (5.154b) holds for j = 3, 4 and 5), where

$$\Lambda \equiv \frac{\omega}{\omega_0} \tag{5.154c}$$

If the natural frequency is low enough to permit the wave exciting force to be approximated using the Froude-Krylov component, Eqs. (5.134) and (5.137), and if we transfer the moments to a coordinate system with origin at the CG[aa], it can be shown that

$$|X_3| \approx C_{33}; \quad |X_4| \approx kC_{44}; \quad |X_5| \approx kC_{55}$$

so that Eq. (5.154b) becomes

$$\frac{|x_3|}{A} \approx \frac{1}{\sqrt{\left(1-\Lambda^2\right)^2 + 4\kappa_3{}^2\Lambda^2}}; \quad \frac{|x_{4,5}|}{kA} \approx \frac{1}{\sqrt{\left(1-\Lambda^2\right)^2 + 4\kappa_{4,5}{}^2\Lambda^2}} \tag{5.155}$$

which can be regarded as "magnification factors" for the responses relative to the wave amplitude or wave slope. At the resonant frequencies ($\Lambda = 1$), Eq. (5.155) reduces to

$$\frac{|x_3(\omega_0)|}{A} \approx \frac{1}{2\kappa_3}; \quad \frac{|x_{4,5}(\omega_0)|}{kA} \approx \frac{1}{2\kappa_{4,5}} \tag{5.156}$$

The behavior of the magnification factor with the frequency ratio Λ for several different values of the damping ratio is illustrated on Figure 5.12 (note the logarithmic vertical scale).

For heave and pitch motions, the damping is generally quite substantial, and magnification factors less than 2 are typical. However, for typical ship forms the total roll damping moment is small (generally less than 5% of critical for ships without bilge keels). Thus the roll magnification factor may be greater than 10 at resonance. This could result in large roll motions if the encountered wave system contains a significant amount of energy near the roll resonant period, particularly if the operator cannot alter course to reduce the excitation (due, for example, to loss of power). The (undamped) roll resonant frequency was given in Eq. (5.139b) above; the period is

[aa] For other choices of the origin, coupling among the various motions must be considered; the same conclusions will eventually be reached, however.

$$T_{04} = 2\pi\sqrt{\frac{I_{xx} + A_{44}}{C_{44}}}$$

which can be approximated for typical ship forms using (Beck[1989])

$$T_{04} \approx \frac{2.27B}{\sqrt{gGM_T}} \tag{5.157}$$

where B is the beam. Roll resonant periods for typical large ships are in the range of 10 to 16 seconds (possibly considerably longer for containerships, which generally have low values of GM_T). As shown in Table 4.1, wave modal periods in this range are most probable in Sea States 6 and higher, which are relatively uncommon events. However, *encounter* periods in this range are quite possible in lower sea states, in following seas. Thus there is motivation to increase the roll damping and various methods have been applied.

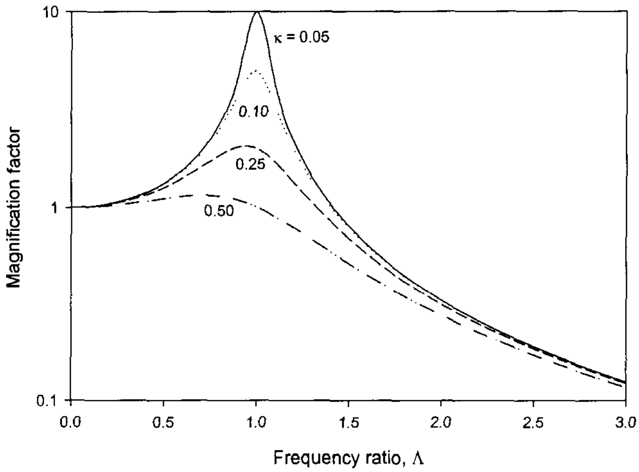

FIGURE 5.12 Magnification factor (Eq. (5.155)) vs. frequency ratio

The simplest and most common of these are *bilge keels*, which are usually nothing more than flat plates installed along the bilge. The bilge keels are generally aligned with the local longitudinal flow lines (determined from the results of flow visualization tests) to minimize resistance. They usually extend about one-third of the length of the ship, and spans of 0.5m - 1.0m are common on large ships (the bilge keels are typically sized so that they do not extend below the baseline or outboard of the maximum beam). The bilge keels can increase the roll damping by a factor of 2 or more, thus reducing the magnification factor at resonance by this factor. This reduction of rolling motion is generally well worth the small price in the form of increased resistance (the increase in power is typically on the order of 3%). Other anti-roll devices are discussed below.

4.2 Prediction of roll damping

As discussed above, the only techniques that are presently generally available for the prediction of viscous roll damping of realistic ship forms consist of semi-empirical formulas. The principal weakness of these methods is that their use is restricted to the hullforms comprising the database of each formulation. In addition, the fit of these expressions to the data that they represent can only be described as fair, particularly for the expressions representing the effects of forward speed.

The most popular semi-empirical method currently in use is the "component analysis" described by Himeno [1981]. In this method (actually an amalgamation of the methods of several researchers) the total roll damping is broken down into its components, consisting of:

- Friction on the hull surface (B_F);
- "Eddy damping" caused by flow separation at the bilge or near the stem and stern (B_E);
- Damping induced by lift forces on the hull (B_L); and
- Bilge keel damping, due to:
    - Normal force on the bilge keels (B_{BKN});
    - Pressure on the hull induced by the flow around the bilge keels (B_{BKH}); and
    - Wavemaking damping of the bilge keels (B_{BKW}).

This method "can be safely applied to the case of [an] ordinary ship hull form with single screw and rudder if the ship is in its normally loaded condition" (Himeno[1981]). The resulting damping moment is a function of frequency, amplitude, and forward speed. The relative contributions of the various components are shown on Figure 5.13.

The Himeno method is, unfortunately, a little complicated. The expressions for the eddy damping and bilge keel-induced hull pressure components are presented as 2-D sectional values, which must be integrated over the length of the hull. No expression is available for the bilge keel wavemaking component; Himeno states that "for bilge keels with ordinary breadth of B/60 to B/80 [B = maximum beam], we can safely neglect the wave effect of bilge keels".

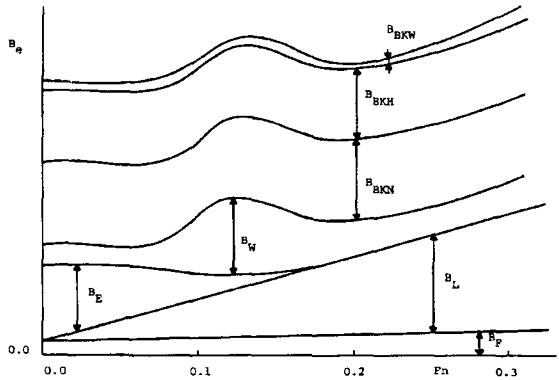

FIGURE 5.13 Contributions of components of roll damping (from Himeno [1981])

An alternative, less complicated (but probably also less accurate) approach, originally formulated by Watanabe and Inoue [1964], is also presented by Himeno[1981]; a simplified version was developed by de Kat (see Beck et.al. [1989]). This formulation is based on "an extensive series of model tests and some theoretical considerations on the pressure distribution on the hull caused by ship roll motion" at zero speed (a forward-speed "multiplier" was later proposed by another investigator). Himeno likens the approach to a "drag coefficient" for the hull and bilge keels; thus the formulation would appear to be applicable only to the quadratic coefficient $B_{44,2}$; this is further supported by the fact that the method is presented in terms of the so-called "N-coefficient", where

$$\Delta\phi = N\phi_m^{\ 2}.$$

So, despite the fact that Himeno and Beck et. al. present formulas for both $B_{44,1}$ and $B_{44,2}$, there is little justification for a linear term in this approach.

Using the expressions given in Himeno's report, we obtain the following expression for the quadratic damping coefficient:

$$B_{44,2} = h\left[1.42\frac{C_B T}{L} + 2\frac{A_{BK}\sigma_0}{L^2} + 0.01\right] f(Fn, \Lambda) \tag{5.158}$$

where

$$h = \left[\left(\frac{KG - T/2}{B}\right)^3 + \left(\frac{T}{B}\right)^2 \frac{L}{4B} + \frac{cB}{64T}\right] \frac{mB^2}{4\pi^2 C_B} \frac{180}{\pi} ,$$

$$c \approx 1.1994 C_{WP}^{\ 2} - 0.1926 C_{WP}$$

and KG is the height of the CG from the keel, C_{WP} is the waterplane area coefficient, m is the mass of the ship, and σ_0 is a "bilge keel efficiency" factor, shown as a function of the aspect ratio of the bilge keel on Figure 5.14. The expression for the factor c is the result of fitting a curve to the results of the more complicated equations given by Himeno; the results match to within 1% for $0.55 \leq C_{WP} \leq 1.0$. The factor $f(Fn,\Lambda)$ represents the effect of forward speed:

$$f(Fn, \Lambda) = 1 + 0.8 \frac{1 - e^{-10Fn}}{\Lambda^2} \tag{5.158a}$$

where Fn is the Froude number and Λ is the frequency ratio (eq. (5.154c)). These formulas are based on "detailed analysis of many typical commercial ships, including some very high block coefficient tankers". An additional caveat added by Himeno [1981] stems from the fact that roll decay data was used, limiting the applicability of the formulas to the natural roll frequency: "...these formulas should be applied to the case of a normally loaded ship, and then only near the natural frequency...".

4.3 Equivalent linear roll damping

Unfortunately, it is not easy to make use of the nonlinear roll damping terms in a frequency-domain analysis (they are by definition excluded from our standard linear "small amplitude" model). However, these effects can be extremely important, particularly near resonance where the roll motion is dominated by the damping moment. To incorporate the nonlinear effects in the linear model, it is common to define an "equivalent linear damping" coefficient, which is generally a function of the roll amplitude and frequency. One way to define the equivalent linear damping is to determine the linear damping coefficient that produces the same energy dissipation in a half-cycle of motion as the actual (nonlinear) process (similar to the analysis that led to Eq. (5.153)). This method leads to the following expression:

$$\begin{aligned} B_{44,1}\dot{\phi} + B_{44,2}\dot{\phi}|\dot{\phi}| + B_{44,3}\dot{\phi}^3 &= B_{44e}\dot{\phi} \\ B_{44e} = B_{44,1} + \frac{8\omega}{3\pi}\phi_0 B_{44,2} &+ \frac{3\omega^2}{4}\phi_0{}^2 B_{44,3} \end{aligned} \tag{5.159}$$

where ϕ_0 is the amplitude of the roll motion. Thus we need to know the roll motion before we can solve the roll equation! In practice this difficulty is usually surmounted by employing an iterative procedure, in which an initial amplitude is assumed for the first calculations and subsequently "tuned" based on the results.

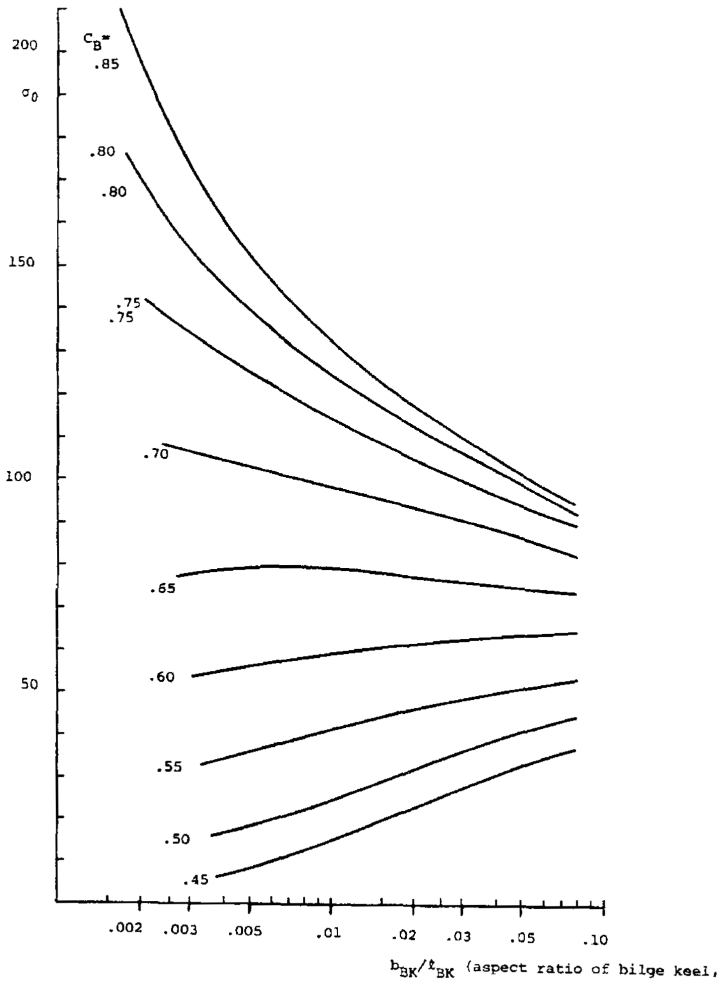

FIGURE 5.14 Bilge keel efficiency σ_0 (Eq. (5.158)), from Himeno [1981]