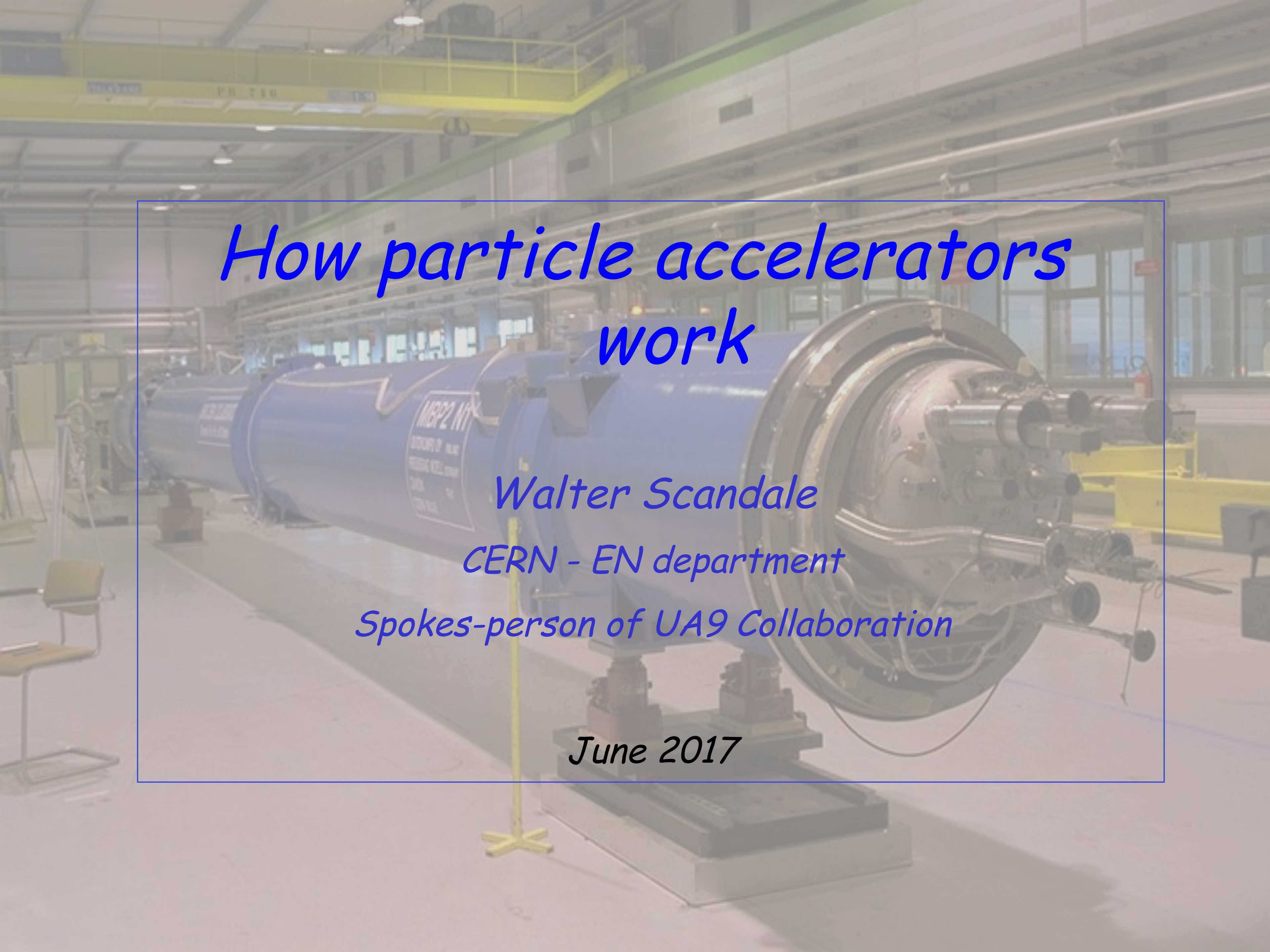


How particle accelerators work

Walter Scandale

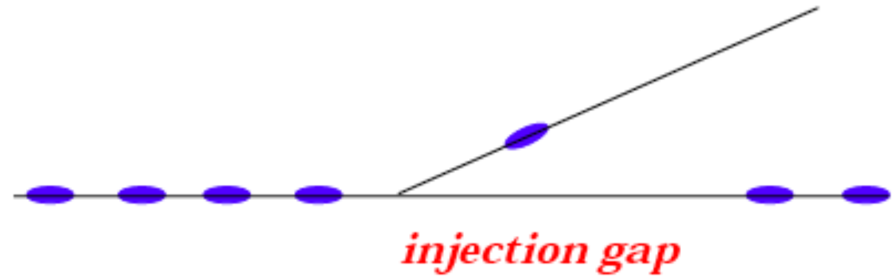
CERN - EN department

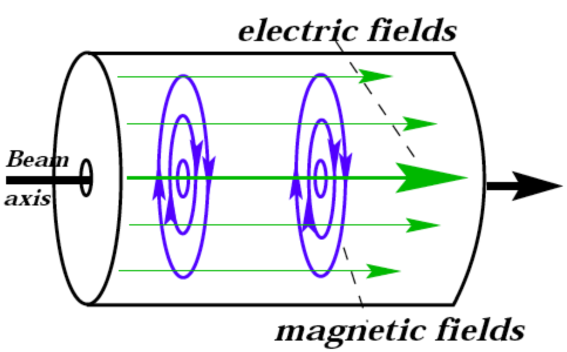
Spokes-person of UA9 Collaboration

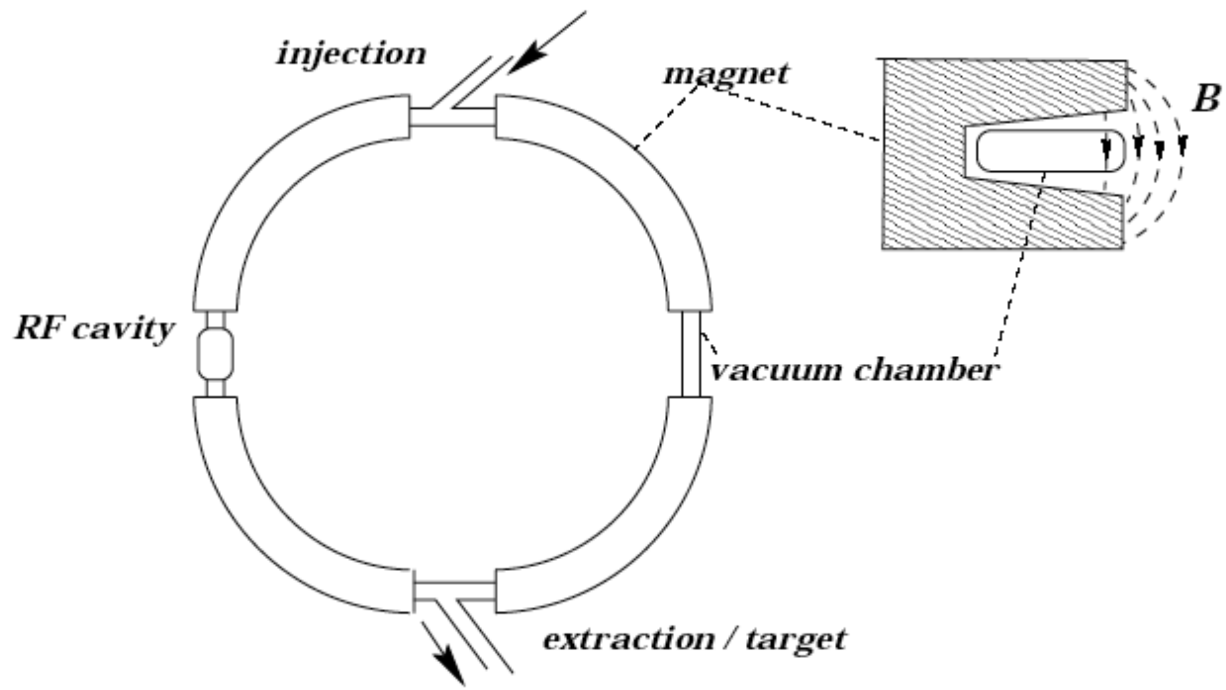
June 2017

Part I

Inventory of synchrotron components



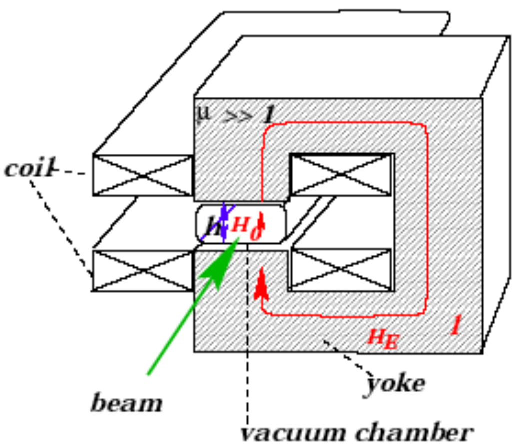




Bending magnet

$$\oint \mathbf{H} = \mathbf{I} \cdot \mathbf{N}$$

$$\mathbf{B} = \mu_0 \cdot \mu \cdot \mathbf{H}$$



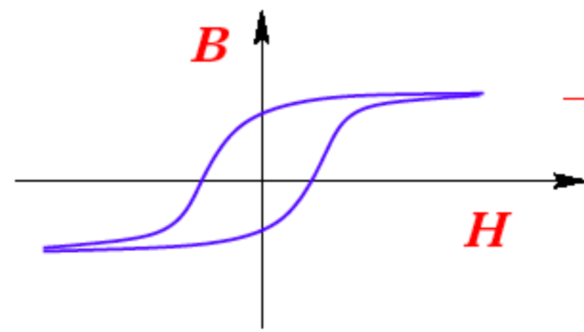
$\mu < 1$: *Dia*

$\mu > 1$: *Para*

$\mu >> 1$: *Ferro*

Maxwell Equations:

$$\oint \mathbf{H} = \mathbf{h} \cdot \mathbf{H_0} + \mathbf{l} \cdot \mathbf{H_E}$$



$$B_{0\perp} = B_{E\perp}$$

$$H_0 = \mu \cdot H_E$$

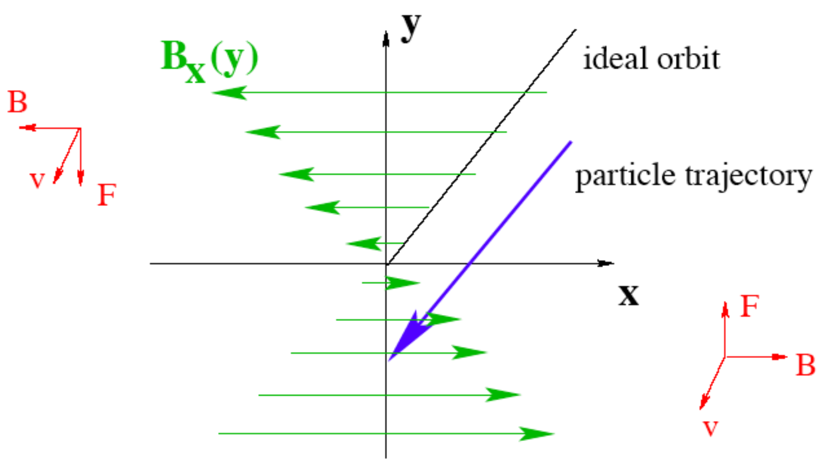
$$B_0 = \mu_0 \frac{N I}{h}$$

$$B[T] = 1.26 \frac{NI[kA - turns]}{h[mm]}$$

$$\frac{1}{\rho} [m^{-1}] = \frac{e \cdot B}{p} = 0.3 \cdot \frac{B\,[T]}{p\,[GeV]}$$

Efficient use of the current -> small gap height
Field quality -> determined by the pole shape
Field saturation -> 2 Tesla $B_{Earth} = 3\ 10^{-5}$ *Tesla*
B > 2 Tesla -> use superconducting magnets $B_{LHC} = 8.4$ *Tesla*

Quadrupole magnet



Vertical focusing
Horizontal defocusing

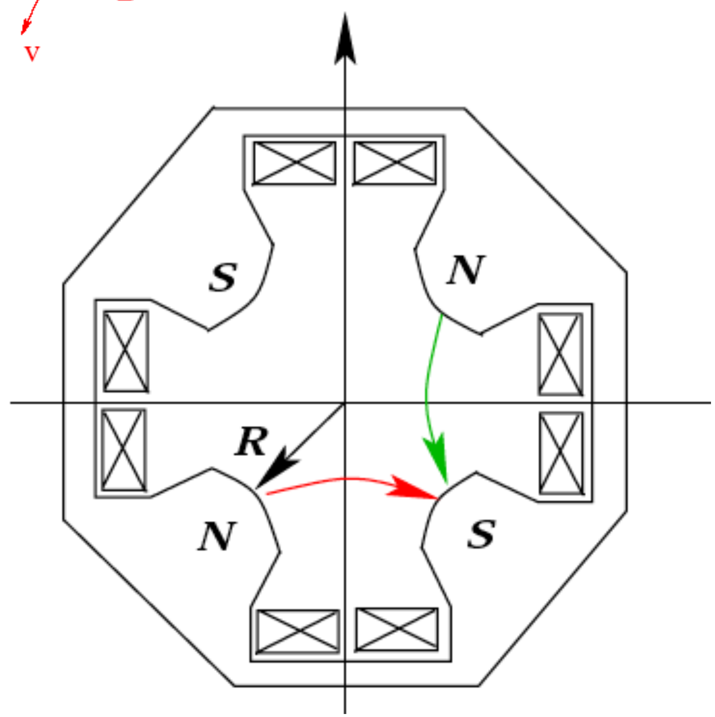
$$B_x = -g \cdot y$$

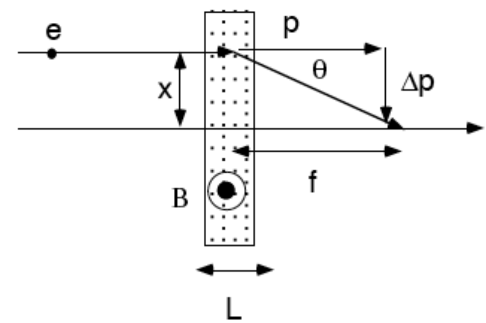
$$B_y = -g \cdot x$$

$$F_x = g \cdot x$$

$$F_y = -g \cdot y$$

g=gradient [T/m]

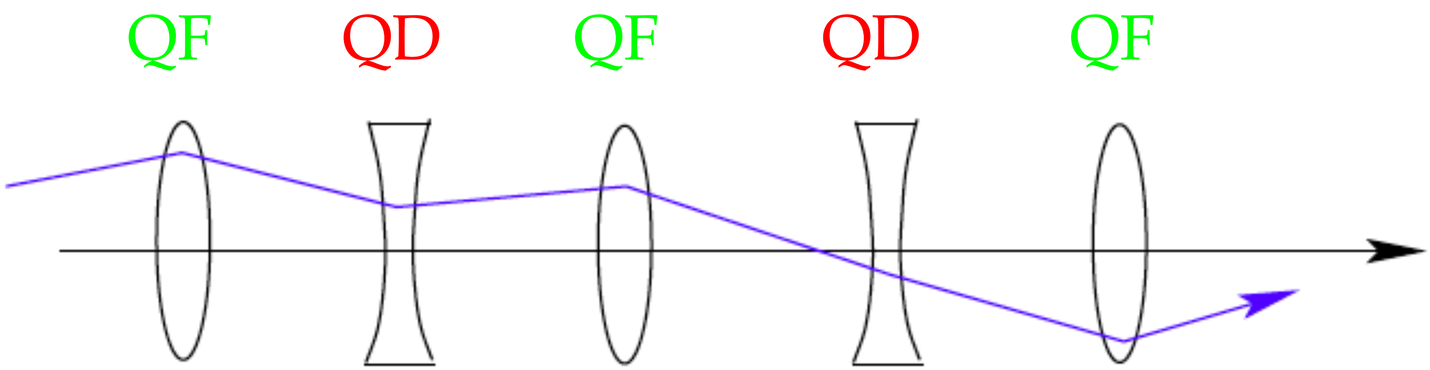




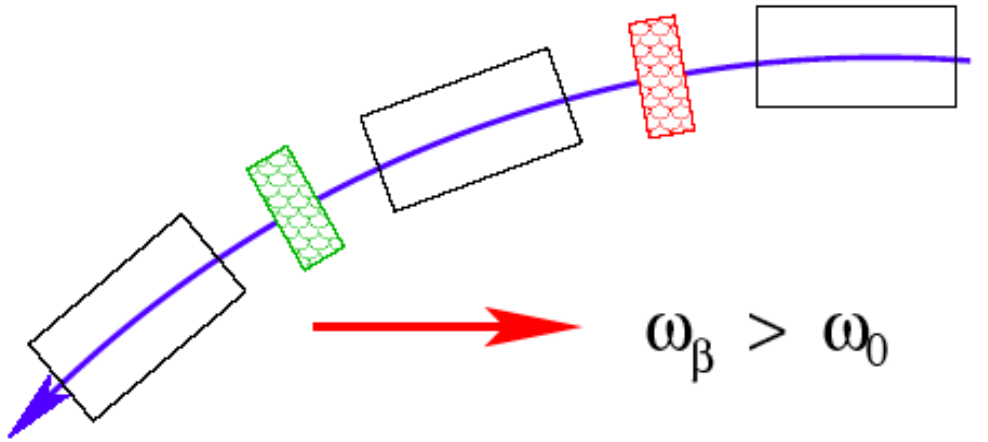
$$\delta p = F\delta t = evB_y \frac{L}{v} = eB_y L$$

$$\theta \approx \frac{\delta p}{p} = \frac{eB_y L}{p}$$

Alternate gradient focusing

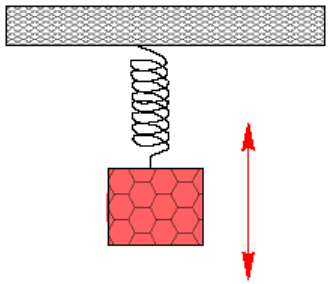


Idea: ***cut the arc sections in focusing and defocusing elements***



Mechanical analogy for alternate gradient

- *oscillator (spring):*



$F = -g \cdot y$

→ $\Omega^2 \propto g$, $A \propto \frac{1}{g}$ for a fixed energy

- *strong focusing:*

→ **small amplitudes**

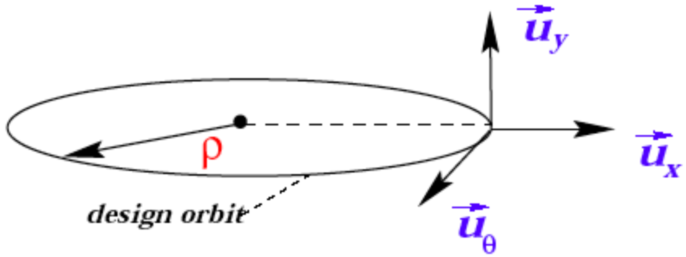
→ **small vacuum chamber**

→ **efficient magnets**

→ **high oscillation frequency**

Basic 2-D equation of motion in a dipolar field

$$x(t) = a \cdot \sin(\omega \cdot t + \phi_0)$$



$$\omega = \omega_{rev}$$

$$\omega_{rev} = 2 \cdot \pi \cdot \frac{V}{L}$$

$$\omega_{rev} = \frac{V}{\rho}$$

$$\frac{d^2x}{dt^2} = -v^2 \cdot \frac{1}{\rho^2} \cdot x$$

$$\frac{d}{dt} = \frac{ds}{dt} \cdot \frac{d}{ds}$$

$\frac{ds}{dt} \rightarrow v$

$$\frac{dx}{ds} = \frac{p_x}{p_0}$$

$$\rightarrow \quad \frac{d^2x}{ds^2} = -\frac{1}{\rho^2} \cdot x$$

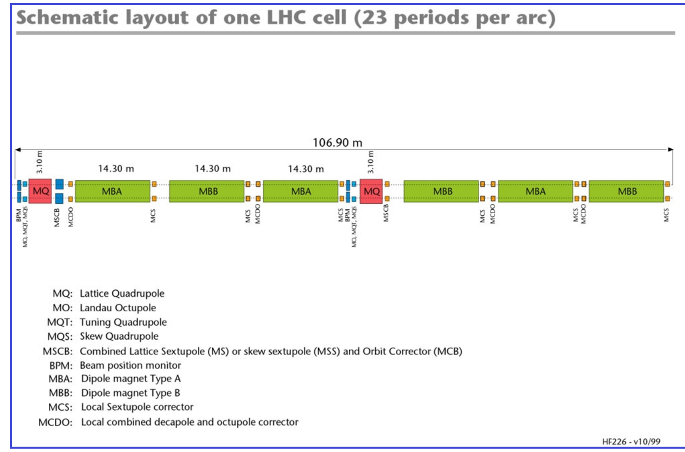
Basic 2D equation of motion

Hills Equation:

$$\frac{d^2x}{ds^2} + K(s)\cdot x = 0; \qquad K(s) = \begin{cases} 0 & \text{drift} \\ 1/\rho^2 & \text{dipole} \\ \dfrac{q\cdot g}{p} & \text{quadrupole} \end{cases}$$

$$K(s) = K(s+L)$$

[general: $K(s)\cdot x = F/(p\cdot v)$]



$$K(s) = \text{const.} \longrightarrow x = A\cdot \sin(\sqrt{K}\cdot s + \phi_0)$$

$$x'' - \left(k - \frac{1}{\rho^2}\right)x = \frac{1}{\rho}\frac{\Delta p}{p_0}$$

$$z'' + kz = 0$$

Floquet Theorem:

$$x = \sqrt{A\cdot \beta(s)}\cdot \sin(\phi(s) + \phi_0)$$

$$\beta(s) = \beta(s+L); \qquad \phi(s) = \int \frac{1}{\beta}\, ds$$

- β and ϕ are determined by the arrangement of the magnets in the tunnel
- individual trajectories are determined by A and ϕ_0

$\longrightarrow$ differential equation for β !

$$\frac{1}{2}\beta\beta'' - \frac{1}{4}\beta'^2 + K\beta^2 = 1$$

Basic 2D equation of motion

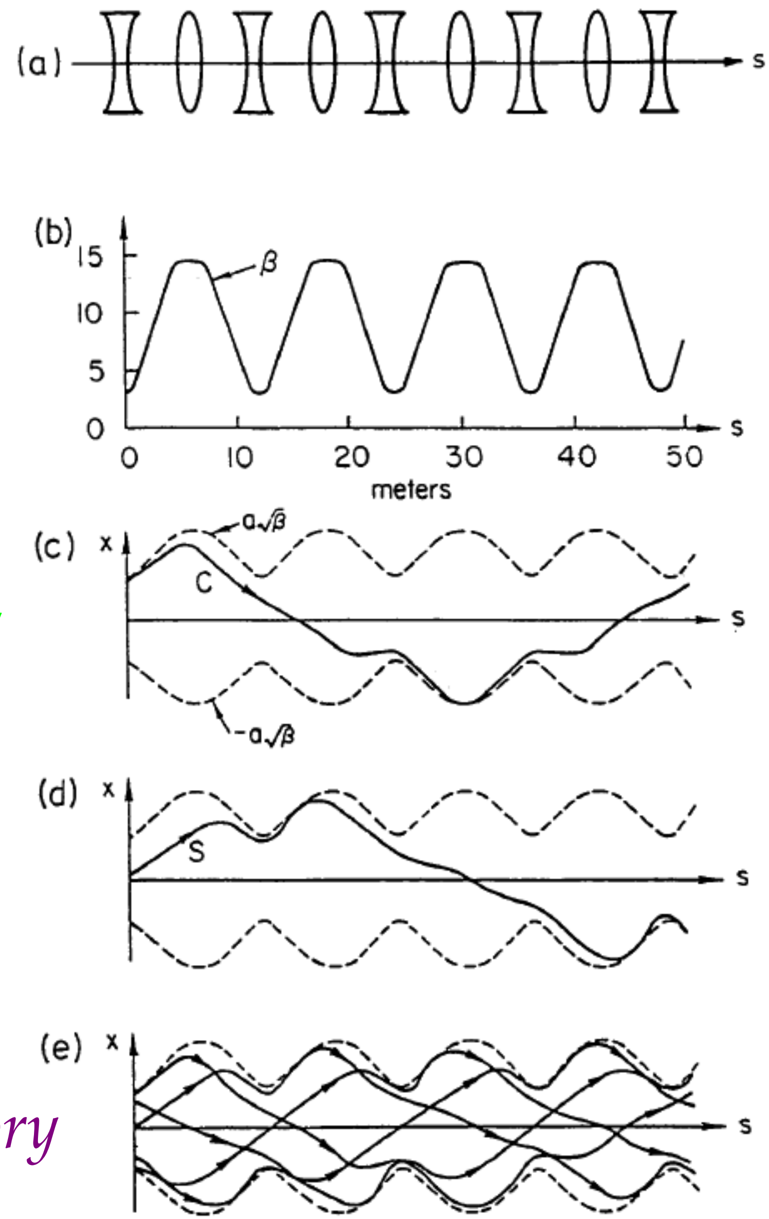
FODO structure

Periodic envelop

Cos-like trajectory

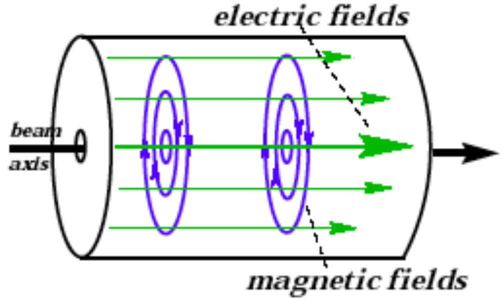
Sin-like trajectory

Multi-turn trajectory



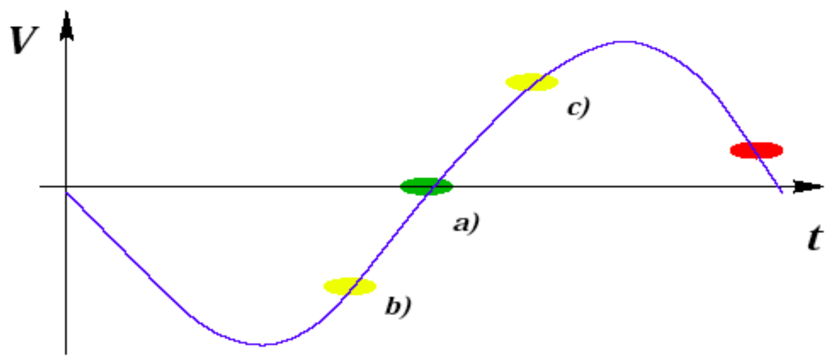
Longitudinal stability

RF Cavity



- **assume:** $p = p_0 + \Delta p \longrightarrow \omega = \omega_0 + \Delta\omega$

- **voltage in cavity:**



$\longrightarrow$ **longitudinal stability**

$$\frac{\Delta T}{T} = \frac{\Delta L}{L} - \frac{\Delta v}{v} = \left(\alpha - \frac{1}{\gamma^2}\right) \cdot \delta \qquad \eta \equiv \left(\frac{1}{\gamma_{tr}^2} - \frac{1}{\gamma^2}\right) \quad \text{where} \quad \alpha \equiv \frac{1}{\gamma_{tr}^2}$$

Momentum compaction

- *increase particle energy*

$\longrightarrow$ *velocity increase*

shorter revolution time

$\longrightarrow$ *momentum increase*

longer revolution time

- *transition energy*

$$\frac{\Delta R}{R} = \alpha \cdot \frac{\Delta p}{p}$$

$$\alpha = \frac{1}{\gamma_t^2} \qquad \alpha \approx \frac{1}{Q^2}$$

$\longrightarrow$ *E error depends on transition energy!*

Beam Distribution:

$$\langle p \rangle = p_0$$

$$p = p_0 + \Delta p \cdot \cos(\omega_s \cdot s)$$

Chromaticity and sextupole magnet

Problem:

$$K\ (quadrupole) = \frac{e \cdot g}{p} \rightarrow Q = Q_0 + \xi \cdot \frac{\Delta p}{p_0}$$

- **Large Machine (LEP / LHC):** → **requires correction!**

$$\xi \approx 100 \div 500$$

$$\frac{\Delta p}{p_0} \approx 10^{-3}$$

Dispersion orbit

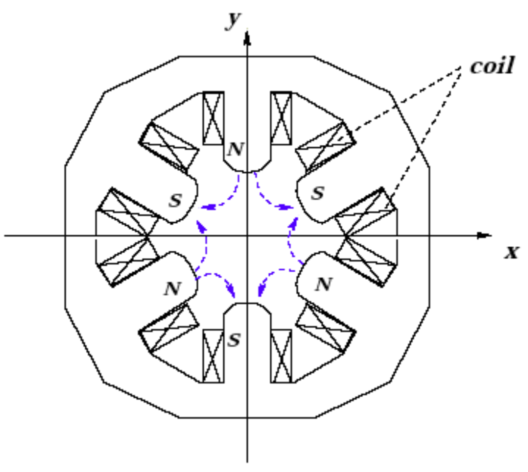
- **Dipole:** $\frac{1}{\rho} = \frac{q \cdot B}{p}$

→ **energy error leads to orbit error**

- **Equation of motion:**

$$x'' - K(s) \cdot x = \frac{1}{\rho} \cdot \frac{\Delta p}{p_0}$$

$$\rightarrow x(s) = x_0(s) + D(s) \cdot \frac{\Delta p}{p_0}$$

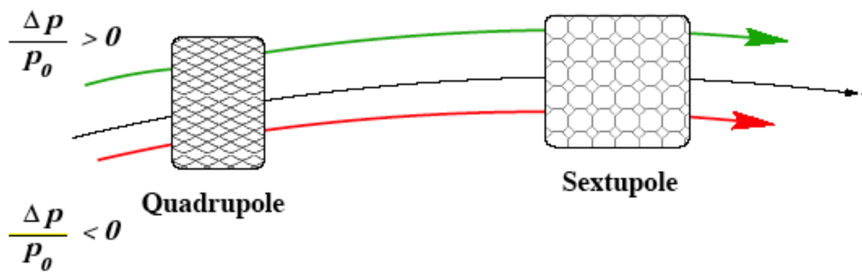


$$B_x = \tilde{g} \cdot x \cdot y$$

$$B_y = \frac{1}{2} \cdot \tilde{g} \cdot (x^2 - y^2)$$

$$[\tilde{g}] = T / m^2$$

Chromaticity correction and non-linear resonance



$$x(s) = x_0(s) + D(s) \bullet \frac{\Delta p}{p_0}$$

→ *offset in sextupole*

→ $Q = Q_0 + \Delta Q_Q\left(\frac{\Delta p}{p_0}\right) + \Delta Q_S\left(\frac{\Delta p}{p_0}\right)$

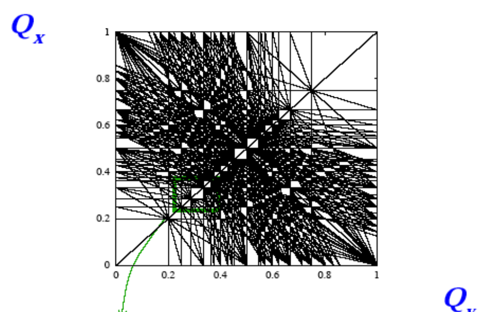
≈ 0

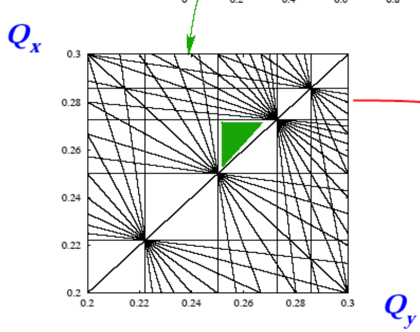
Tune Diagram

resonances: $n \cdot Q_x + m \cdot Q_y + r \cdot Q_s = p$

strength: $h \propto A^{n+m+s}$

→ *avoid low order resonances!*





limits for b_n and bunch intensity

- **Lorentz Force:**

$$x' = \frac{F}{v \cdot p} \longrightarrow x' = q \cdot \frac{B_y}{p}$$

- **Sextupole Magnet:** $B_y = \frac{1}{2} \cdot \tilde{g} \cdot x^2$

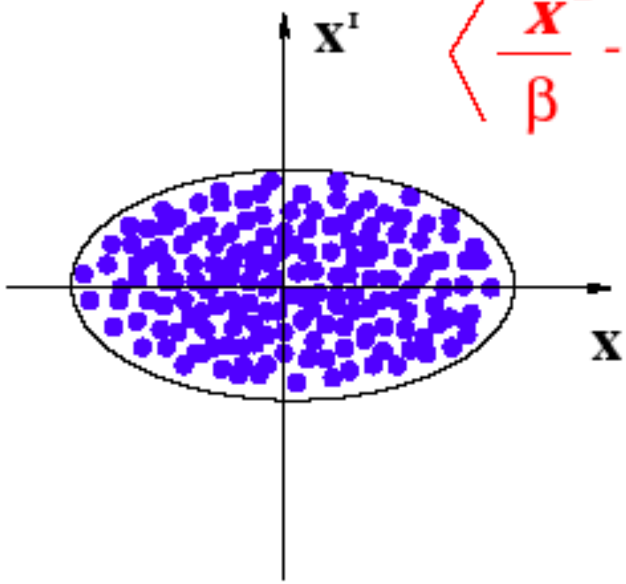
$$\longrightarrow \Delta x' = \int \frac{F}{v \cdot p}\, ds$$

$$= \frac{1}{2} \cdot \frac{1}{v} \cdot \frac{q}{m} \cdot \tilde{g} \cdot x^2$$

- **Non-linear Perturbation:**
  - *amplitude growth*
  - *detuning with amplitude*
  - *coupling*

Emittance

- ***beam ensemble:***



$$\left\langle \frac{x^2}{\beta} - \beta' \bullet x \bullet x' + \beta \bullet x'^2 \right\rangle = \varepsilon$$

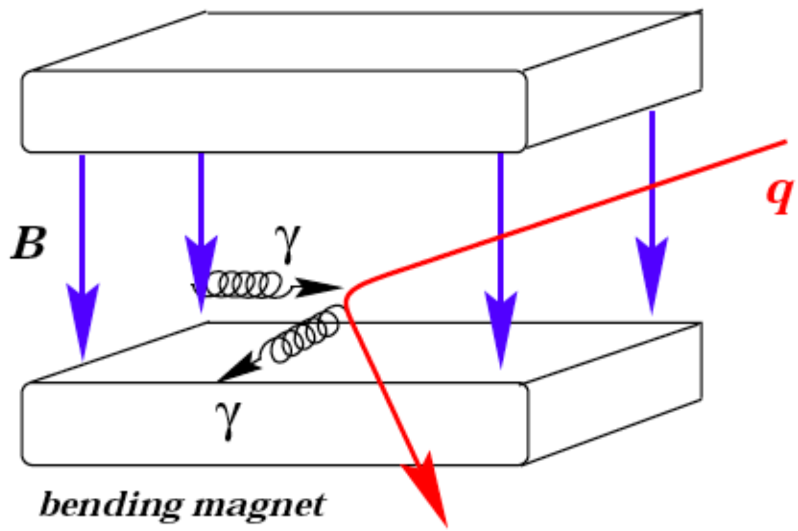
$$\varepsilon = \frac{\textit{area}}{\pi}$$

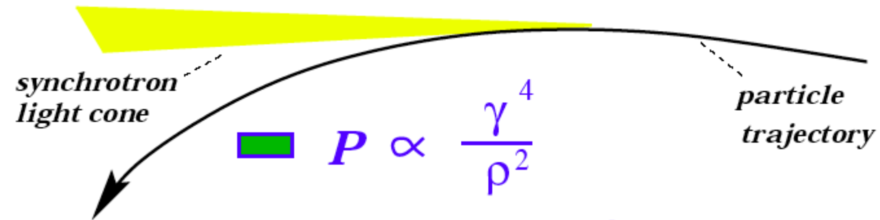
→ ε ***describes the beam quality***

→ $\sigma = \sqrt{\varepsilon \bullet \beta}$ ***describes the beam size***

Synchrotron radiation

Quantum Picture:





- $P \propto \frac{\gamma^4}{\rho^2}$
- $\langle E_\gamma \rangle \propto \frac{\gamma^3}{\rho}$
- polarised
- radiation fan in bending plane
- opening angle $\propto \frac{1}{\gamma}$

	E [GeV]	ρ [km]	N [10^{12}]	U [MeV]	P [MW]	u_c [keV]
LEP 1	45	3.1	4.7	260	2.1	90
LEP 2	100	3.1	4.7	2800	23	715
LHC	7000	3.1	312	0.007	0.005	0.04

- γ-rays: Co_{60} → 1.3 MeV
- X-rays: → keV
- Visible Light: → eV

LEP 1 → X-rays

LEP 2 → γ -rays

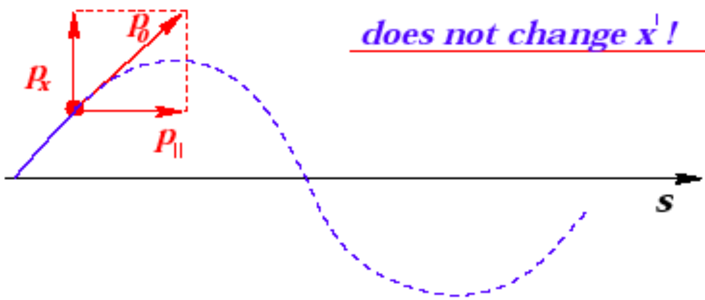
LHC → UV light

Synchrotron radiation and beam size

Adiabatic damping

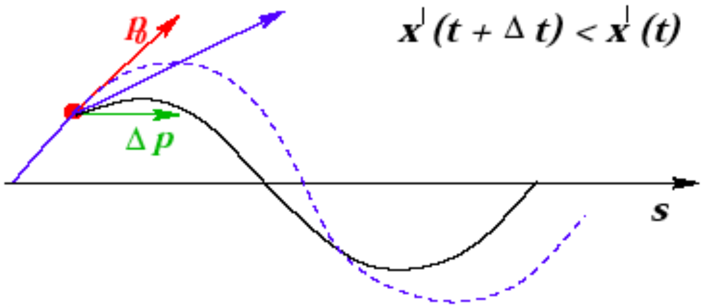
- $x' = \frac{p_x}{p_{\parallel}}$

→ synchrotron radiation does not change x' !



- Acceleration:

$x'(t + \Delta t) < x'(t)$



→ the beam shrinks as it gets accelerated!

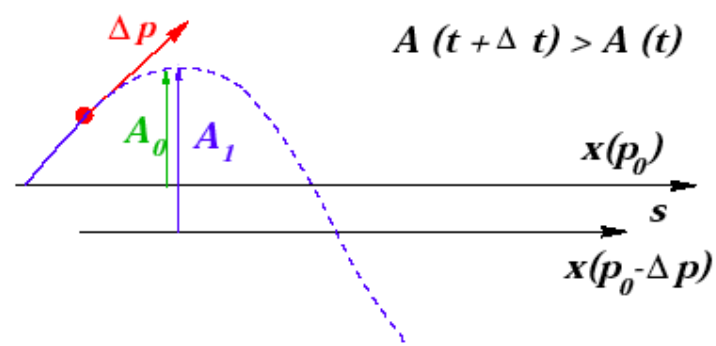
damping $\propto \frac{1}{\gamma}$

Synchrotron light emission

Synchrotron radiation + acceleration → continuous damping

- Limits:
  - quantum excitations → small but finite beam size
  - dispersion
  - orbit curvature → synchrotron radiation increases beam size

$A(t + \Delta t) > A(t)$



synchrotron radiation + dispersion → excitation

Effect of synchrotron light

- ***Pro:***
  - *synchrotron radiation*
    → *dedicated light sources*
  - *vertical damping*
    → *flat beams*
  - *damped motion*
    → *not sensitive to errors*

- ***Contra:***
  - *power loss*
    → *large storage ring energy limit*
  - *radiation excitation*
    → *the horizontal beam size increases with γ !*

Collective effects

Communication of individual particles via:

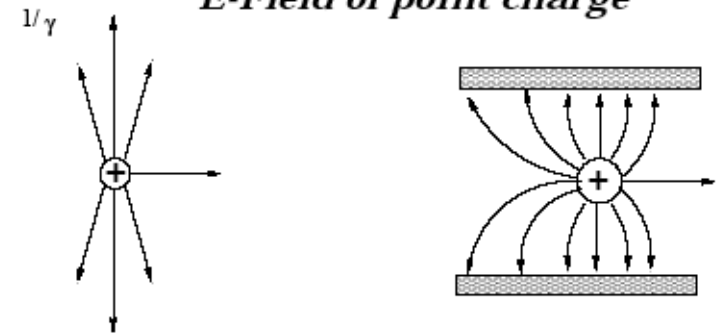
- *direct coulomb interaction*
- *residual gas ionisation*
- *synchrotron radiation*
- *image charges on the vacuum chamber*

→ *change of individual particle motion*

→ *collective motion*

→ *stability?* { *beam size* / *oscillation modes* }

E-Field of point charge



- *electro-magnetic field of the bunch*
- *image charge currents*

change in bunch distribution ⇄ *force changes*

→ *equilibrium?*

- *wakefields*

→ *particle-environment interaction*

Instabilities and feedback

→ *beam stability depends*

on the surface properties

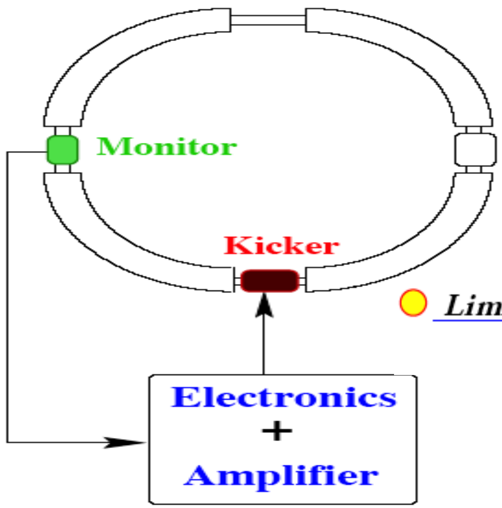
+

geometry of the vacuum chamber

→ *careful design of all vacuum equipement*

- *General Rules:*
  - *smooth transitions*
  - *shielded discontinuities*
- *Quantitative Analysis:*
  - *evaluate E and B for a given test distribution*
  - *study the beam stability by super-imposition*

Feedback System

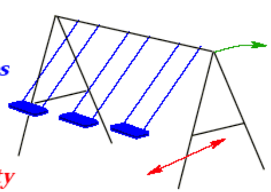


- *Limit:*

  power and bandwidth

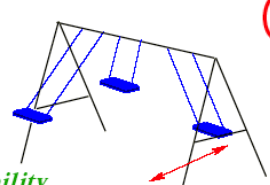
Landau Damping

- *Three Coupled Oscillators:*
  - *equal frequencies*



→ *instability*

- *Three Coupled Oscillators:*
  - *different frequencies*

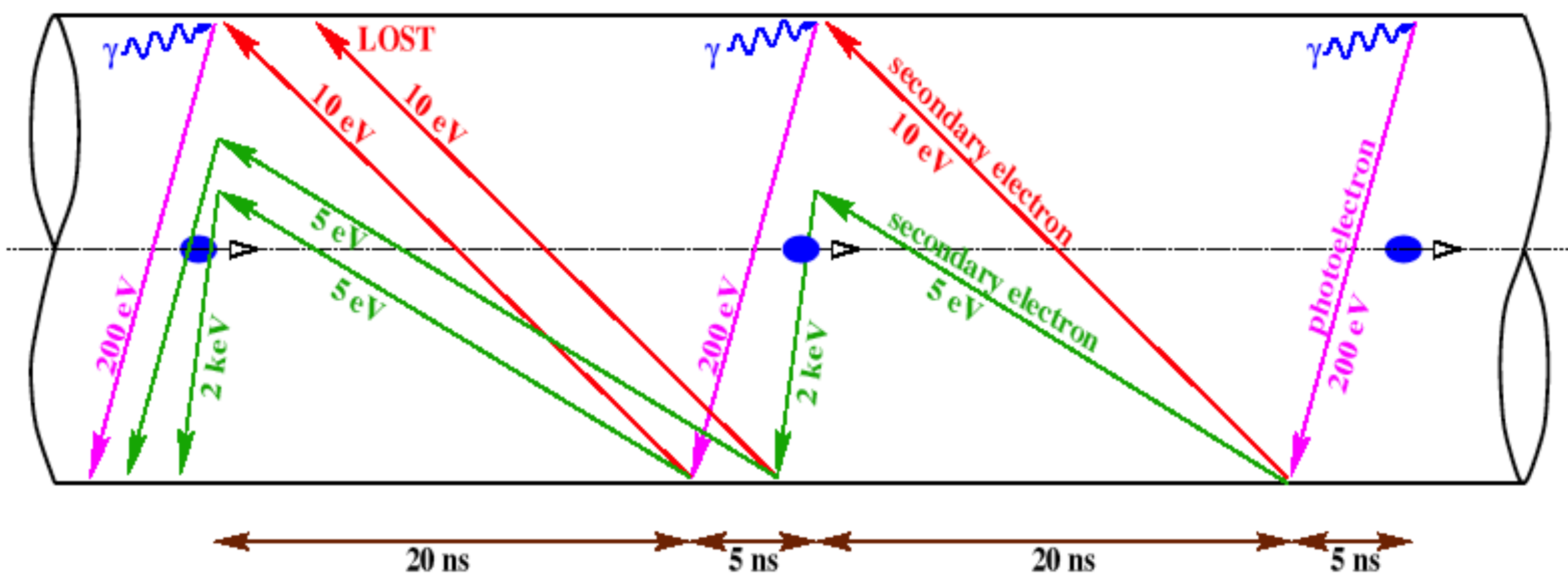


→ *no instability*

- *Limit:*
  - *frequency spread (tune spread)*
  - *single particle resonances*

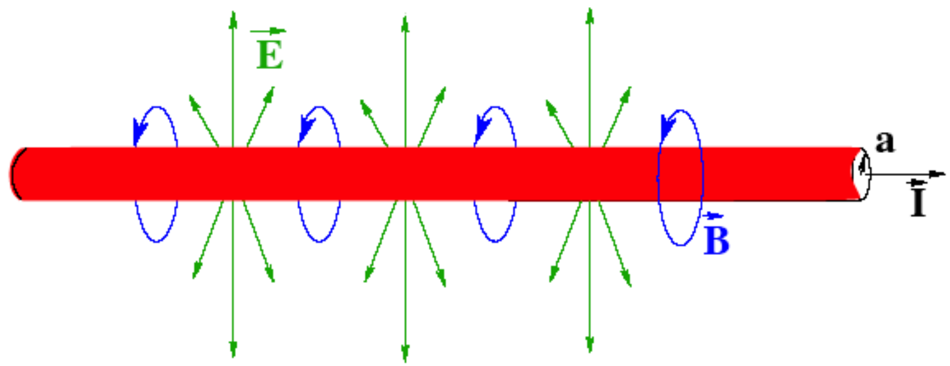
Electron Cloud Instability

- Synchrotron light removes electrons from chamber wall
- Electrons are accelerated by the beam
- Electrons hit vacuum chamber and generate more electrons
- Electron cloud ⟶ instability and heat losses!



Space charge

Line Current:



- $E_r = \frac{2\lambda e}{a^2} \cdot r; \quad r < a; \ \lambda = \frac{N}{2\pi R}$

- $B_\theta = \frac{v}{c^2} \cdot E$

$$x'' + \left(\frac{Q}{R}\right)^2 \cdot x = \left[q \cdot E_r - q \cdot v \cdot B_\theta\right] \cdot \frac{1}{m \cdot v^2}$$

$\propto x$

$$\rightarrow \Delta Q \propto \frac{I \cdot R}{\gamma^2}$$

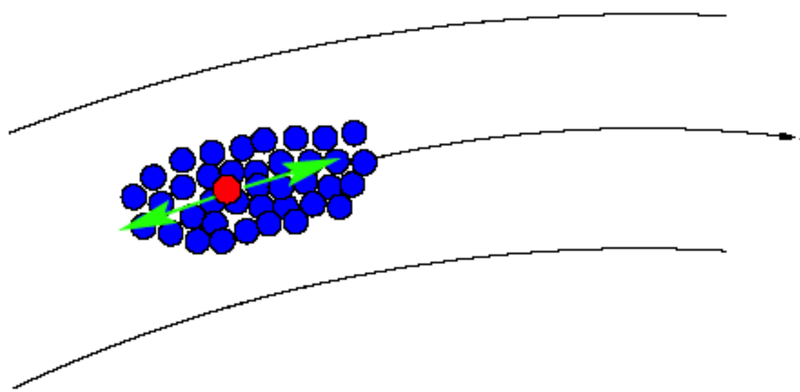
- resonances limit the total beam current

Beam size

Intra Beam Scattering:

- *each particle performs longitudinal*
  + transverse oscillations
  → *uncorrelated motion!*

Coulomb Scattering → Emittance Growth



- *Emittance blow-up:*

$$\varepsilon\,(t+\Delta t) = \left(1+\frac{\Delta t}{\tau}\right)\cdot\varepsilon\,(t)$$

- *Growth rate depends on beam size:*

$$1/\tau \propto \frac{N}{\varepsilon_t^2 \cdot \varepsilon_l}\cdot A\cdot Z$$