

ESERCIZI SU DISEQUAZIONI ED INSIEMI DI DEFINIZIONE

1. Risolvere le disequazioni e i sistemi di disequazioni che seguono:

$$(a) \begin{cases} \frac{x}{2} + 2 \geq \frac{2x-1}{3} \\ x+1 \leq \frac{2}{3}x - \frac{5}{2} \end{cases}$$

$$(b) \frac{x-1}{3-2x} + 1 \leq 0$$

$$(c) \frac{x}{2} + 3x^2 > 0$$

$$(d) \begin{cases} x^2 - x + 2 > 0 \\ x^2 + 5x + 6 \leq 0 \\ x^2 - 5x > 0 \end{cases}$$

$$(e) \frac{x-1}{x^2+2x+1} \leq 0$$

$$(f) \frac{4-3x}{x^2-3x} > 1$$

$$(g) \left(\frac{x-1}{x+2}\right) \left(\frac{x-2}{x-3}\right) \geq 0$$

$$(h) \begin{cases} -1 - \frac{1}{x^2+1} \geq 0 \\ 2x^3 - 3x > 0 \end{cases}$$

$$(i) \frac{x^4 - 5x^2 + 4}{x^2 + 1} \leq 0$$

$$(j) \frac{2}{x^2+4} - \frac{3}{x^2-9} > 0$$

$$(k) \frac{x^4 - 16}{x^4 + x^3 - x - 1} \geq 0$$

$$(l) \frac{x^3 + 6x^2 + 11x + 6}{x+2} > 0$$

$$(m) \begin{cases} \frac{x^2 - 7x + 6}{2x - 4} \geq 0 \\ x^2 - 2 \leq 0 \end{cases}$$

2. Risolvere le seguenti disequazioni:

$$(a) \sqrt[4]{x} - 2 \geq 0$$

$$(b) \sqrt{x} + 1 < 0$$

$$(c) \sqrt[4]{x} \leq 0$$

$$(d) \sqrt[4]{x} + 3 \geq 0$$

$$(e) \sqrt[8]{x} - 1 \leq 0$$

$$(f) \sqrt[3]{2+x} < \sqrt[3]{x^2-4x-4}$$

$$(g) \sqrt[3]{2x+1-x^3} < 1-x$$

$$(h) \sqrt{x^2-3x} < \sqrt{3x-x^2-2}$$

$$(i) \sqrt{4-x^2} \leq 2-x$$

$$(j) \sqrt[3]{x^2} - 3\sqrt[3]{x} + 2 > 0$$

$$(k) 2x\sqrt{x} + \sqrt{x} > 2x + 1$$

$$(l) \sqrt{|x|-1} > |x|-2$$

$$(m) 3\sqrt{x+1} + \sqrt[4]{x+1} - 4 > 0$$

$$(n) \frac{\sqrt{x^2-x-2}-x+1}{x^2-4x} > 0$$

$$(o) \frac{\sqrt{x^2-1}-3x+4}{x^2+2x+1} < 0$$

$$(p) \frac{3-|x|}{\sqrt{x+2}} > \sqrt{x}$$

$$(q) |x|\sqrt{1-2x^2} > 2x^2 - 1$$

3. Determinare il campo di esistenza delle seguenti funzioni:

$$(a) f(x) = \sqrt{1 + \log_{\frac{1}{2}}(e^x - 2e^{-x} + 1)}$$

$$(b) f(x) = \log \left(\left(\frac{1}{3}\right)^{\sqrt{x^2+2x}} - \left(\frac{1}{3}\right)^{x+1} \right)$$

$$(c) f(x) = \left(2 \left(\log^2 x - \log x - 2 \right) - \log x - 1 \right)^\pi$$

$$(d) f(x) = \log \left(e^{\sqrt{x+1}} - e^{\frac{x}{\sqrt{1-x}}} \right)$$

$$(e) f(x) = \sqrt{\frac{|x^2+2|+1}{x^2-1}}$$

$$(f) f(x) = \left(|\sqrt{x^2+2x}-1| - x \right)^x$$

$$(g) f(x) = \log \left(\frac{|x^2+2-3x|+x-5}{2^{2x}-5 \cdot 2^x+6} \right)$$

$$(h) f(x) = \sqrt{\sqrt{2+\cos x} + \sqrt{2}\sin x}$$

$$(i) f(x) = \sqrt{2 \log_{\frac{1}{2}} |\cos x| - \log_{\frac{1}{2}} \frac{|\cos x|}{2}}$$

$$(j) f(x) = \sqrt{x-5}$$

- (k) $f(x) = \frac{x+25}{x^2-3x+2}$
- (l) $f(x) = \sqrt{\frac{2x-1}{x+2}}$
- (m) $f(x) = x \log(5-x^2)$
- (n) $f(x) = \sqrt[3]{\frac{x-4}{x-6}}$
- (o) $f(x) = \sqrt{2 \cos x + 1}$
- (p) $f(x) = \sqrt{\log \frac{x-2}{x}}$
- (q) $f(x) = \sqrt[8]{\log(3x^2+2x)}$
- (r) $f(x) = \frac{1}{\sqrt{\sin x}} + \frac{1}{\sqrt{\cos x}}$
- (s) $f(x) = \sqrt{x|x|+3}$
- (t) $f(x) = \sqrt{|x-3|-|x+4|}$
- (u) $f(x) = \sin\left(\frac{x}{x+1}\right)$
- (v) $f(x) = \frac{x^2-1}{e^{1/x}-e}$
- (w) $f(x) = \frac{x^{\sin x}+1}{e^{\sin x}}$
- (x) $f(x) = \log(x^{1/3}-x^{1/2})$
- (y) $f(x) = \sqrt{4-x^2}$
- (z) $f(x) = (x-x^2)^{\sqrt{5}}$
- (α) $f(x) = \sqrt{\frac{x+1}{x-1}} + \sqrt{\frac{x-3}{x}}$
- (β) $f(x) = \sqrt{5-|x|}$
- (γ) $f(x) = \sqrt{\frac{2|x|+3}{x-1}}$
- (δ) $f(x) = \sqrt{9-3^x} + \sqrt{2^x-1}$
- (ε) $f(x) = \sqrt{\sqrt{x-2}-\sqrt{2x-5}}$
- (ζ) $f(x) = (\sin x - 1)^\pi$
- (η) $f(x) = (x-1)^{\sqrt{3-2x}}$
- (θ) $f(x) = \log \frac{x-2}{x}$
- (ι) $f(x) = \log(\sin x - \cos x)$
- (κ) $f(x) = \log(e^x - 1)$
- (λ) $f(x) = \sqrt{x + \sqrt{x^2+6}}$
- (μ) $f(x) = \frac{\log(1-x^2)}{x}$
- (ν) $f(x) = \log(1-3^{2 \cos x - \sqrt{3}})$
- (ξ) $f(x) = \frac{\arccos(x^2-3)}{\log_{10}(x-5)}$
- (ο) $f(x) = \log_3 \left(\log_4 \left(\frac{|x+2|}{|x-1|} \right) \right)$
- (ρ) $f(x) = \log(2 \cos^2 x + 5 \cos x - 3)$
- (π) $f(x) = \arcsin(2^{2x} - 3 \cdot 2^x + 1)$
- (σ) $f(x) = \log \left[\left(\frac{1}{3} \right)^{\sqrt{x^2+2x}} - \left(\frac{1}{3} \right)^{x+1} \right]$
- (τ) $f(x) = \sqrt{\sin x - \frac{1}{2}}$
- (υ) $f(x) = \log(\log(\tan x))$
- (φ) $f(x) = \log \left(\frac{2^x - 1}{1 - 3^{x+1}} \right)$
- (χ) $f(x) = \sqrt{1 - \tan^2 x}$
- (ψ) $f(x) = \sqrt{8^{x+1} - 2^{x^2}}$
- (ω) $f(x) = \sqrt{\log_{1.5}(x^2+4x)-1}$
- (A) $f(x) = \log(x+1-\sqrt{x^2-x-2})$
- (B) $f(x) = \arcsin(x - \log_3(3^{2x}-3^{x+1}+2))$
- (C) $f(x) = \sqrt{\arctan x}$
- (D) $f(x) = \sqrt{e^{\frac{1}{\log x}} - x}$
- (E) $f(x) = \log \left(4 \sin x - \frac{1}{\cos x} \right)$
- (F) $f(x) = \sqrt{\log^2 x + \log x - 2}$
- (G) $f(x) = \log(e^x - 2e^{-x} + 1)$
- (H) $f(x) = \arctan \left(\frac{x^2+1}{\sqrt{\log x}} \right)$
- (I) $f(x) = \arcsin \left(\frac{x-1}{x+4} \right)$
- (J) $f(x) = \sqrt{\frac{\pi}{2} - \arccos(\log x)}$
- (K) $f(x) = \arcsin \left(\sqrt{4+2x-x^2} + 1-x \right)$
- (M) $f(x) = \sqrt{\arctan \left[\frac{1}{\sqrt{3}} - \left(\frac{1}{3} \right)^{\sin^2 x} \right]}$
- (N) $f(x) = \sqrt{|x^2-x|-|x+3| + \log_{\frac{1}{2}}(2^{2x}-2^{x+1}-3)}$
- (O) $f(x) = \arcsin(x^2-3|x|+3)$
- (P) $f(x) = \log \left(\frac{\sqrt{x^2-x-2}-x+1}{x^2-4x} \right)$
- (Q) $f(x) = (4 \log^2 x - 1)^\pi$
- (R) $f(x) = \log_{\frac{1}{2}} \left(\frac{3 \arcsin x - \pi}{\pi - 4 \arctan x} \right)$
- (S) $f(x) = \tan(3^x + 1)$