EQUAZIONE DELLA MASSA

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \overline{V}\right) = 0$$

$$\frac{\partial \rho}{\partial t} + \overline{V} \cdot \nabla \rho + \rho \nabla \cdot \overline{V} = 0$$

EQUAZIONE DEL MOTO

$$\frac{\partial \overline{V}}{\partial t} + \overline{V} \cdot \nabla \overline{V} = -\frac{1}{\rho} \nabla p - \nabla (gz) + \bar{f}$$

$$\frac{\partial \overline{V}}{\partial t} + \nabla \left(\frac{V^2}{2}\right) - \overline{V} \times \left(\nabla \times \overline{V}\right) = -\frac{1}{\rho} \nabla p - \nabla (gz) + \bar{f}$$

$$\frac{\partial \overline{V}}{\partial t} - \overline{V} \times \left(\nabla \times \overline{V}\right) = \begin{cases} -\frac{1}{\rho} \nabla p_0 + \bar{f} & \left(p_0 = p + \rho \frac{V^2}{2} + \rho g z\right) \\ -\nabla H + T \nabla s + \bar{f} & \left(H = h + \frac{V^2}{2} + gz\right) \end{cases}$$

EQUAZIONE DELL'ENERGIA

$$\frac{\partial}{\partial t}\left(\frac{V^2}{2}\right) = -\frac{d^* p_0}{\rho} + d^* W_f = -d^* H + d^* q$$

Divergenza della Velocità

$$\nabla \cdot \overline{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = \frac{1}{Vol} \frac{dVol}{dt}$$

Gradiente della Velocità	Tensore degli sforzi
$\nabla \overline{V} = \begin{vmatrix} \frac{\partial V_x}{\partial x} & \frac{\partial V_y}{\partial x} & \frac{\partial V_z}{\partial x} \\ \frac{\partial V_x}{\partial y} & \frac{\partial V_y}{\partial y} & \frac{\partial V_z}{\partial y} \\ \frac{\partial V_x}{\partial z} & \frac{\partial V_y}{\partial z} & \frac{\partial V_z}{\partial z} \end{vmatrix}$	$\overline{\overline{\Pi}} = \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zy} & \tau_{zy} & \sigma_z \end{vmatrix}$

Accelerazione Spaziale	Forza per unità di volume
$\left(\overline{V} \cdot \nabla \overline{V}\right)_x = $ $= V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} + V_z \frac{\partial V_x}{\partial z}$	$\left(\nabla \cdot \overline{\overline{\Pi}}\right)_x = $ $= \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial y}$

Rotore della Velocità $\nabla \times \overline{V} = \overline{\Omega}$	
Coordinate Cartesiane	Coordinate Cilindriche
$\overline{\Omega} = \begin{vmatrix} \bar{i}_x & \bar{i}_y & \bar{i}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$	$\overline{\Omega} = \begin{vmatrix} \bar{i}_x / r & \bar{i}_r / r & \bar{i}_\vartheta \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial r} & \frac{\partial}{\partial \vartheta} \\ V_x & V_r & V_\vartheta \end{vmatrix}$

ESEMPIO DI SVILUPPO IN COORDINATE CILINDRICHE

$$rot(\overline{V}) = \nabla \times \overline{V} = \overline{\Omega}$$

$$\overline{\Omega} = \begin{vmatrix} \bar{i}_x / r & \bar{i}_r / r & \bar{i}_\vartheta \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \vartheta} \\ V_x & V_r & rV_\vartheta \end{vmatrix}$$

$$\Omega_x = \frac{1}{r}\left(\frac{\partial (rV_\vartheta)}{\partial r} - \frac{\partial V_r}{\partial \vartheta} \right)$$

$$\Omega_r = -\frac{1}{r}\left(\frac{\partial (rV_\vartheta)}{\partial x} - \frac{\partial Vx}{\partial \vartheta} \right)$$

$$\Omega_\vartheta = \left(\frac{\partial V_r}{\partial x} - \frac{\partial V_x}{\partial r} \right)$$

$$\overline{V} \times \overline{\Omega} = \begin{vmatrix} \bar{i}_x & \bar{i}_r & \bar{i}_\vartheta \\ V_x & V_r & V_\vartheta \\ \Omega_x & \Omega_r & \Omega_\vartheta \end{vmatrix}$$

EQUAZIONE DI NAVIER-STOKES

$$\frac{D\overline{V}}{dt} = \frac{\partial \overline{V}}{\partial t} + \overline{V} \cdot \nabla \overline{V} = \frac{1}{\rho} \nabla \cdot \overline{\overline{\Pi}}$$

In coordinate cartesiane ortogonali :

$$\nabla \overline{V} = \begin{array}{c|ccc} & \bar{i}_x & \bar{i}_y & \bar{i}_z \\ \hline \bar{i}_x & \frac{\partial V_x}{\partial x} & \frac{\partial V_y}{\partial x} & \frac{\partial V_z}{\partial x} \\ \bar{i}_y & \frac{\partial V_x}{\partial y} & \frac{\partial V_y}{\partial y} & \frac{\partial V_z}{\partial y} \\ \bar{i}_z & \frac{\partial V_x}{\partial z} & \frac{\partial V_y}{\partial z} & \frac{\partial V_z}{\partial z} \end{array}$$

$$\overline{V} \cdot \nabla \overline{V} = \bar{i}_x \left(V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} + V_z \frac{\partial V_x}{\partial z} \right) + \bar{i}_y ($$

$$\overline{\overline{\Pi}} = \begin{array}{c|ccc} & \bar{i}_x & \bar{i}_y & \bar{i}_z \\ \hline \bar{i}_x & \sigma_x & \tau_{xy} & \tau_{xz} \\ \bar{i}_y & \tau_{yx} & \sigma_y & \tau_{yz} \\ \bar{i}_z & \tau_{zx} & \tau_{zy} & \sigma_z \end{array}$$

$$\nabla \cdot \overline{\overline{\Pi}} = \bar{i}_x \left(\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \right) + \bar{i}_y (......$$

Equazioni scalari :

$$\frac{\partial V_x}{\partial t} + V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} + V_z \frac{\partial V_x}{\partial z} = \frac{1}{\rho} \left(\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \right)$$

....

Pressione idrostatica:

$$p = -\frac{1}{3}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

Tensioni normali e tangenziali :

$$\sigma_x = -p + 2\mu\left(\frac{\partial V_x}{\partial x} - \frac{2}{3}\nabla\cdot\overline{V}\right)$$

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x}\right)$$

....

$$\frac{\partial\sigma_x}{\partial x} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left[2\mu\left(\frac{\partial V_x}{\partial x} - \frac{2}{3}\nabla\cdot\overline{V}\right)\right]$$

$$\frac{\partial\tau_{xy}}{\partial x} = \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x}\right)\right]$$

Equazione vettoriale di Navier - Stokes :

$$\rho\frac{D\overline{V}}{dt} = -\nabla p + \nabla\left(\frac{4}{3}\mu\nabla\cdot\overline{V}\right) - \nabla\times\left(\mu\nabla\times\overline{V}\right)$$

FORMA CONSERVATIVA

$$\frac{\partial \bar{u}}{\partial t} + \nabla \cdot \bar{f} = \bar{h}$$

DIVERGENZA

$$\bar{u} = \left\{ \begin{array}{ll} \rho A & \text{MASSA} \\ \rho \bar{V} A & \text{Q. d. M.} \\ \rho E A & \text{ENERGIA } \left(E = e + \frac{V^2}{2}\right) \end{array} \right\}$$

$$\bar{f} = \left\{ \begin{array}{ll} \rho \bar{V} A & \text{FLUSSO DI MASSA} \\ \rho \bar{V} \bar{V} A & \text{FLUSSO DI Q. d. M.} \\ (p + \rho E) \bar{V} \cdot A & \text{FLUSSO DI ENERGIA} \end{array} \right\}$$

$\bar{h}$ = TERMINI "SORGENTE"

ES: $p \frac{dA}{dx}$, $-2 C_f \rho V^2 \frac{A}{D}$ in Q. d. M.

$\dot{Q}$ in ENERGIA

Flusso a Potenziale

$$\nabla \times \bar{V} = 0 \Rightarrow \bar{V} = \nabla\varphi \Rightarrow \begin{cases} V_x = \dfrac{\partial\varphi}{\partial x} \\ V_y = \dfrac{\partial\varphi}{\partial y} \end{cases}$$

Flusso a Potenziale Incomprimibile:

$$\nabla^2\varphi = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = 0$$

Flusso a Potenziale Comprimibile:

$$(1 - M_x^2)\frac{\partial^2\varphi}{\partial x^2} + 2M_x M_y \frac{\partial^2\varphi}{\partial x\partial y} + (1 - M_y^2)\frac{\partial^2\varphi}{\partial y^2} = 0$$

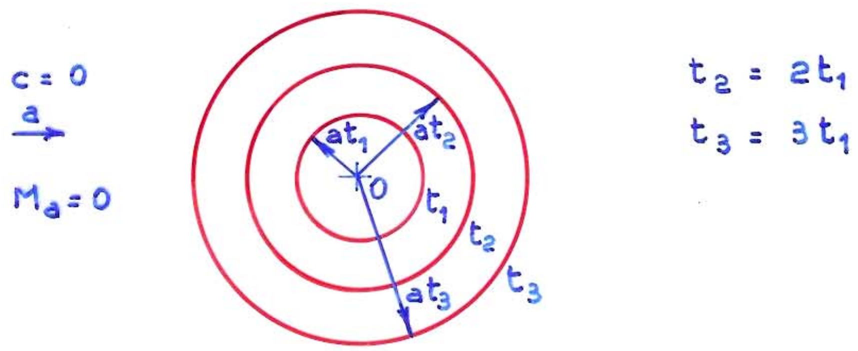
$$A\lambda^2 \qquad + \quad B\lambda \qquad\qquad C \qquad\qquad = 0$$

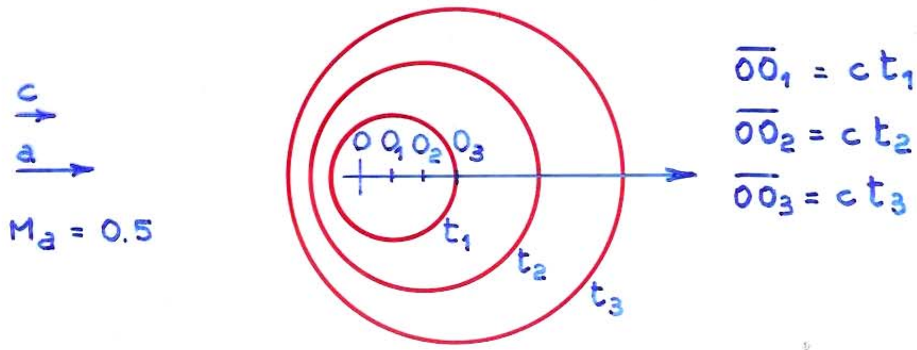
$$B^2 - 4AC = M^2 - 1$$

$$M \begin{cases} > 1 \Rightarrow \text{Iperbol.} \Rightarrow \{\lambda_\pm = tg(\vartheta \pm \alpha) \\ = 1 \Rightarrow \text{Parabol.} \\ < 1 \Rightarrow \text{Ellittico} \end{cases}$$

$$tg\vartheta = \frac{M_y}{M_x} = \lambda_o \qquad ; \qquad \sin(\alpha) = \frac{1}{M}$$

PROPAGAZIONE DELLE PERTURBAZIONI IN VARI REGIMI DI MOTO

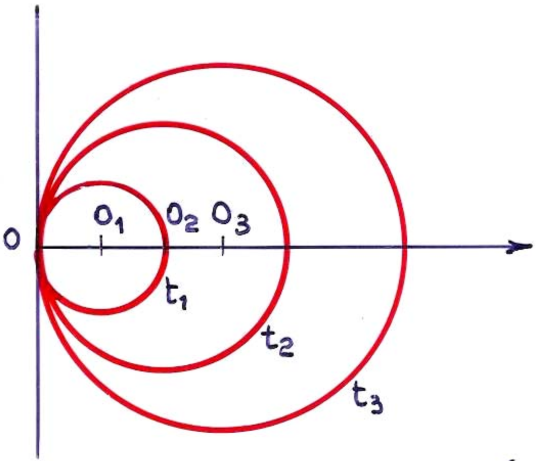




$\vec{c}$

$\vec{a}$

$M_a = 1$

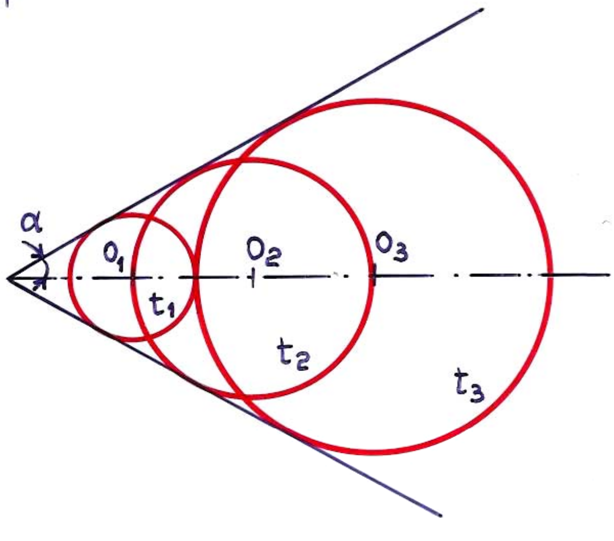


$\vec{c}$

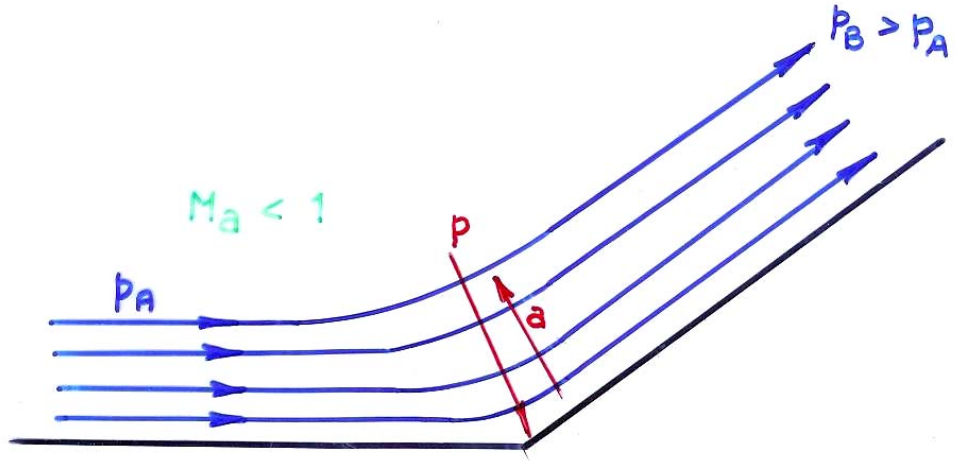
$\vec{a}$

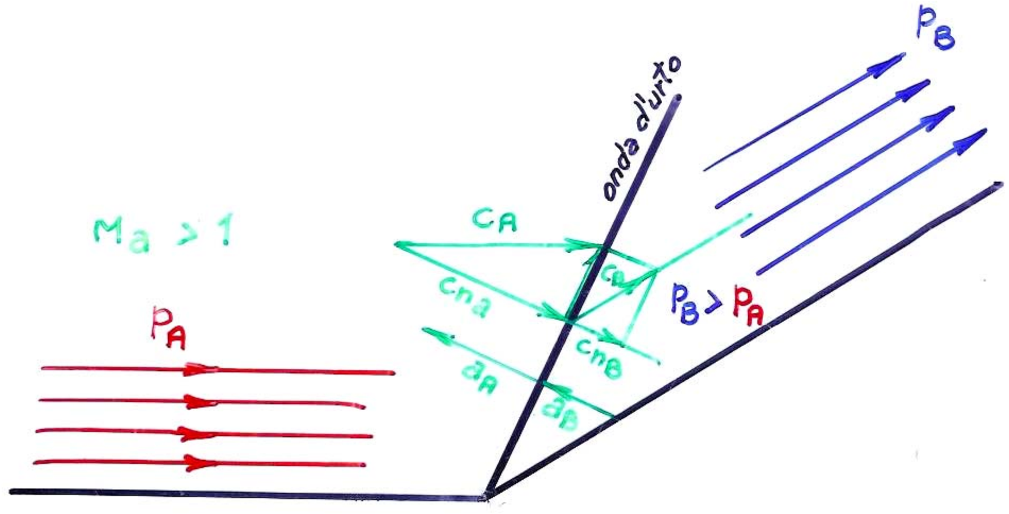
$M_a = 2$

$\text{sen}\ \alpha = \frac{1}{M_a}$



COMPORTAMENTO DI UNA CORRENTE IN PRESENZA DI UNA PERTURBAZIONE





MOTO 1D NON STAZIONARIO

(Adiabatico e senza attrito) $\Rightarrow \partial p = a^2 \partial \rho$

Massa - $$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x} = 0$$

Moto - $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\begin{cases} Massa \quad \rho a^2 \dfrac{\partial u}{\partial x} \quad --- \quad + u\dfrac{\partial p}{\partial x} \quad + \dfrac{\partial p}{\partial t} \quad = 0 \\ Moto \quad \rho u \dfrac{\partial u}{\partial x} \quad + \rho\dfrac{\partial u}{\partial t} \quad + \dfrac{\partial p}{\partial x} \quad --- \quad = 0 \end{cases}$$

<u>Linea caratteristica</u>: $\lambda = \dfrac{dt}{dx} \Rightarrow \dfrac{d}{dx} = \left(\dfrac{\partial}{\partial x} + \lambda\dfrac{\partial}{\partial t}\right)$

$$\lambda_{\pm} = \left(\frac{dt}{dx}\right)_{\pm} = \frac{1}{u \pm a} \qquad \Rightarrow \qquad dp \pm \rho a du = 0$$

Equaz. Linee di Mach — Equaz. di Compatibilità

MOTO A POTENZIALE

(Esempio di equazione ellittica del 2° Ordine nello spazio 2D)

$$\nabla^2 \varphi = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0$$

$\varphi(x,y)$ **= potenziale della velocità**

EQUAZIONE DI CONDUZIONE DEL CALORE

(Esempio di equazione parabolica nello spazio-tempo)

$$\frac{\partial(\rho c T)}{\partial t} = \frac{\partial}{\partial x} \lambda \frac{\partial T}{\partial x}$$

con [ρ = ρ (T) ; c = c(T) ; λ = λ (T)] ; T = T(x,t)

se (ρ, c, λ = cost) :

$$\rho c \frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial x^2}$$

MOTI RELATIVI

Accelerazione Assoluta

$$\bar{a} = \frac{D\overline{V}}{dt} = \frac{\partial \overline{V}}{\partial t} + \overline{V} \cdot \nabla \overline{V}$$

Accelerazione Relativa (*r* = riferimento relativo)

$$\bar{a}_r = \frac{D_r W}{dt} = \frac{\partial_r \overline{W}}{\partial t} + \overline{W} \cdot \nabla_r \overline{W}$$

Relazione tra velocità assoluta e velocità relativa

$$\overline{V} = \overline{W} + \overline{U} = \overline{W} + \overline{\omega} \times \bar{r}_r$$

Relazione tra accelerazione assoluta e velocità relativa (per ω = *cost*)

$$\bar{a} = \bar{a}_r + \overline{\omega} \times (\overline{\omega} \times \bar{r}_r) + 2\overline{\omega} \times \overline{W}$$

acc. centrip. acc. Coriolis

Relazione tra instazionarietà nei riferimenti assoluto e relativo

$$\frac{\partial (..)}{\partial t} = \frac{\partial_r (..)}{\partial t} + \overline{U} \cdot \nabla (..)$$

MOTI RELATIVI

Equazione della Massa

$$\frac{\partial_r \rho}{\partial t} + \nabla_r \cdot \left(\rho \overline{W}\right) = 0$$

$$\frac{\partial_r \rho}{\partial t} + \overline{W} \cdot \nabla_r \rho + \rho \nabla_r \cdot \overline{W} = 0$$

Equazione del Moto

$$\frac{D\overline{V}}{dt} = \frac{D\overline{W}}{dt} - \omega^2 \overline{r}_r + 2\overline{\omega} \times \overline{W} = -\frac{1}{\rho}\nabla p - \nabla\left(gz\right) + \bar{f}$$

$$\frac{\partial \overline{W}}{\partial t} + \nabla\left(\frac{W^2}{2}\right) - \overline{W} \times \left(\nabla \times \overline{W}\right) + 2\overline{\omega} \times \overline{W} = -\frac{1}{\rho}\nabla p - \nabla\left(gz\right) + \nabla(\omega^2 r_r^2) + \bar{f}$$

$$\frac{\partial \overline{W}}{\partial t} - \overline{W} \times \left(\nabla \times \overline{W} + 2\overline{\omega}\right) = \begin{cases} -\frac{1}{\rho}\nabla p_{0r} + \bar{f} & ; p_{0r} = p + \rho\left(\frac{W^2}{2} - \frac{\omega^2 r_r^2}{2}\right) + gz \\ -\nabla H_r - T\nabla s + \bar{f} & ;\ H = h + \left(\frac{W^2}{2} - \frac{\omega^2 r_r^2}{2}\right) + gz \end{cases}$$

Equazione dell’ Energia

$$\frac{\partial_r}{\partial t}\left(\frac{W^2}{2}\right) = -\frac{d_r^* p_{0r}}{\rho} + d_r^* W_f = -d_r^* H_r + d_r^* q$$

MOTI RELATIVI

Rotore della Velocità Relativa

$$\left(\nabla \times \overline{W}\right) = \overline{\Omega}_w = \begin{vmatrix} \bar{i}_x/r & \bar{i}_r/r & \bar{i}_\vartheta \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \vartheta} \\ W_x & W_r & W_\vartheta \end{vmatrix} = \begin{matrix} \bar{i}_x\left(\dfrac{\partial W_\vartheta}{\partial r} - \dfrac{1}{r}\dfrac{\partial W_r}{\partial \vartheta}\right) + \\ \bar{i}_r\left(\dfrac{1}{r}\dfrac{\partial W_x}{\partial \vartheta} - \dfrac{\partial W_\vartheta}{\partial x}\right) + \\ \bar{i}_\vartheta\left(\dfrac{\partial W_r}{\partial x} - \dfrac{\partial W_x}{\partial r}\right) \end{matrix}$$

Condizione di Moto a Potenziale

$$\left(\nabla \times \overline{W}\right) = -2\overline{\omega} \Rightarrow \begin{cases} \left(\dfrac{\partial W_\vartheta}{\partial r} - \dfrac{1}{r}\dfrac{\partial W_r}{\partial \vartheta}\right) = -2\omega \\ \left(\dfrac{1}{r}\dfrac{\partial W_x}{\partial \vartheta} - \dfrac{\partial W_\vartheta}{\partial x}\right) = 0 \\ \left(\dfrac{\partial W_r}{\partial x} - \dfrac{\partial W_x}{\partial r}\right) = 0 \end{cases}$$

Blade-to-Blade Macchina Radiale
Blade-to-Blade Macchina Assiale
Piano Meridiano (Through Flow)

$$\left(\frac{\partial W_\vartheta}{\partial r} - \frac{1}{r}\frac{\partial W_r}{\partial \vartheta}\right) = -2\omega \quad \Rightarrow \quad \frac{\partial W_r}{\partial \vartheta} = 2r\omega + r\frac{\partial W_\vartheta}{\partial r}$$