Thinking Strategically

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(Based on notes by David Myatt)

Objective:

- What is Game Theory?
- Simultaneous moves games.
- The representation of games in strategic form.
- Dominant strategy equilibrium.
- The iterated deletion of strictly dominated strategies.
- Pure strategy Nash equilibrium.
- Reaction functions (best responses).
- Finding Nash equilibria with both discrete and continuous action spaces.
- Supermodular and submodular games.

What is Game Theory?

- Economics is a *social science*: it aims to investigate, understand and explain human behaviour.
- The social sciences are (partly) concerned with decision making:
  - Individual decision-making: e.g. consumer theory.
  - Interactive decision-making: the outcome depends on decisions of many people.
- Classical microeconomics analyzes market systems:
  - Individuals react in response to a set of prices.
  - Prices tie together the diverse decisions in an economy.
- But many situations are fundamentally *strategic*:
  - The outcome for an individual depends on the decisions made by others.
  - Her best decision depends on such anticipated decisions.
- This is the study of game theory, where we:

(i) Define what a game actually is.

(ii) Derive solution concepts, i.e. ways to play that meet certain criteria. We can then think about whether they are a good idea.

Fisher: “No business man assumes that his rival’s output or price will remain constant any more than a chess player assumes that his opponent will not interfere with his effort to capture a knight.”

Myerson (1999): “Nash Equilibrium in the History of Economic Theory” *Journal of Economic Literature* 37, 1067-1082.

In 1994, the Nobel Prize in Economics was awarded to three game theorists. These lectures will be closely related to their pioneering ideas.

- **John Nash Jr**, in a few years, made a number of outstanding contributions that changed the path of social science. His work helped frame social interactions in the strategic form, and provided a way to analyze them.
- **Reinhard Selten** paid closer to attention to the dynamic structure of games. When modelled in the extensive form, we are forced to consider the temporal relationship between moves. Some moves are made because of other hypothetical moves that will never be carried out. This is a world of threats and promises, and these threats must be credible. Selten introduced the notion of subgame perfection.
- **John Harsanyi** studied games of incomplete information. The simplest formulation of games supposes that we know all of the ingredients of games. Harsanyi worked out how to model situations where this is not the case, for instance, where we are unsure of the exact payoffs of the game.

Gul (1997): “Nobel Prize for Game Theorists: The Contributions of Harsanyi, Nash and Selten” *Journal of Economic Perspectives* 11, 159-174.

Strategic Scenarios

- **Entry in an Industry:** An incumbent firm is already in an industry, and is faced by a potential entrant. This entrant may choose to enter the industry or stay out. If entry occurs, the incumbent may or may not start a price war.
- **Prisoner's Dilemma**: Two suspects are arrested for a crime, and interviewed separately. If they both keep quiet, they go to prison for 1 year. If one supplies evidence she is freed, and her partner is imprisoned for 9 years. If both supply evidence, they are imprisoned for 6 years.
- **Coordination:** Two friends need to meet up to discuss their love for economics. One prefers to go to the cafe, the other prefers to meet in the pub. They would both rather meet than miss each other, but need to choose simultaneously where to go.
- **Electoral Competition**: Two political parties choose their advertising budgets. The electoral outcome depends on their relative advertising, but they dislike campaign spending.

In each example, the outcome for each player depends on the decisions of others. In all but one, the optimal decision also depends on the decisions of others. We can model all of these scenarios as games.

Ingredients of a Game

- A game consists of:

  Players: A list of players, possibly including "nature".

  Moves: What each player can do and when.

  Information: What each player knows when he moves.

  Payoffs: A list of payoffs for each player, as a function of the moves made by *all* players.

- The actions of a player form the strategies of the game:

  Strategy: A *complete plan of action* for a player — it specifies how a player will act in *every possible* circumstance in which she may be called upon to move.

  Strategy Profile: A collection of strategies for each player — a way to play the game.

- The representation of a game often takes one of two forms:

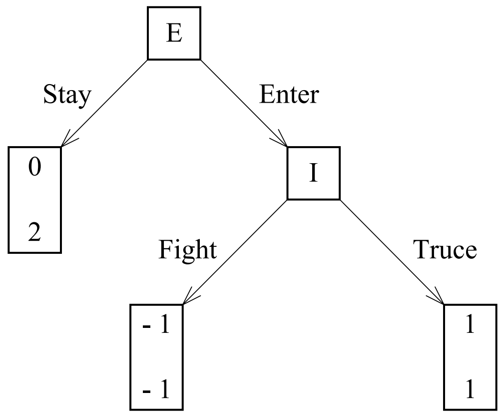
  Extensive Form: Represents the game as a tree, detailing the players, their moves, and their final payoffs.

  Strategic Form: Collects together the strategies for each player, typically as a matrix. Each cell represents a strategy profile.

Example: Entry in an Industry

An incumbent firm is already in an industry, and is faced by a potential entrant. This entrant may enter the industry or stay out. If entry occurs, the incumbent may start a price war (fight), or accommodate the entrant (truce).

- This Entry Deterrence Game may be represented using an *extensive form* tree:



- Alternatively, we can use a *strategic form* matrix:

	Fight	Truce
Stay	2 0	2 0
Enter	-1 -1	1 1

Strategic Form Games

- A *strategic form* game consists of:

  Players: A finite set $I = \{1, 2, \ldots, n\}$, with member i.

  Actions: For each player $i \in I$, a nonempty set A_i of available actions (the moves), with member a_i. We let $a \equiv (a_1, ..., a_n)$ be a list of actions chosen by each player. This is an *outcome* of the game.

  Payoffs: For each player $i \in I$, a preference relation on the set of outcomes $A = \times_{j \in I} A_j$. This will be represented by a payoff function $\pi_i : a \in A \to \mathbb{R}$ such that $\pi_i(a)$ is the payoff of player i if the outcome of the game is a.

- Payoffs are not necessarily von Neumann-Morgenstern utilities — we are restricting to *pure* strategies (definite actions). Extension to mixed strategies requires the use of vNM payoffs.

- A typical representation of a strategic form game is a matrix of payoffs — this representation is appropriate when actions are taken once and simultaneously.
- Alternatively, it might be suitable for situations where players can commit to their behaviour throughout the game.

Example: The Prisoners' Dilemma

Two suspects are arrested for a crime, and interviewed separately. If they both keep quiet (they *cooperate* with each other), they go to prison for a year. If one suspect supplies incriminating evidence (she *defects*), she is freed, and her partner is imprisoned for nine years. If *both* defect, they are imprisoned for six years. Their preferences are solely contingent on the jail term they serve.

- Turning this into a strategic form game:

  Players: The two suspects $i \in \{1, 2\}$.

  Actions: The action set for Player 1 is $x \in \{C, D\}$, and for Player 2 is $y \in \{C, D\}$.

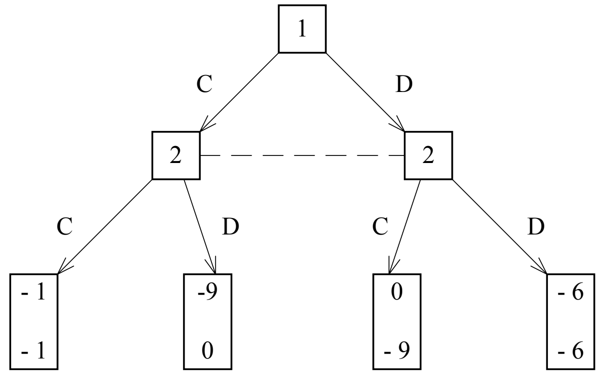
  Payoffs: The payoffs in the strategic form matrix are:

	Cooperate	Defect
Cooperate	-1 -1	0 -9
Defect	-9 0	-6 -6

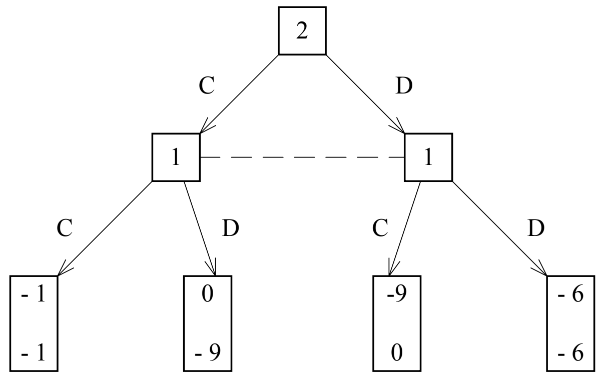
or

	C	D
C	-1 -1	-9 0
D	0 -9	-6 -6

- Alternatively, use an **Extensive Form** representation:



or



(The broken line indicates an *information set* — a player does not know which node in the set she is playing from.)

- These are two *different* extensive form games but they lead to *identical* strategic form games. The strategic form may be viewed as a *coarser* representation.

Strictly Dominant Strategies

- Let $a_{-i} = (a_1, a_2, ..., a_{i-1}, a_{i+1}, ..., a_n)$, so that an outcome of the game is $a = (a_i, a_{-i})$.

- **Definition**: A strategy $\widehat{a}_i \in A_i$ is a *strictly dominant strategy* for player i if

$$\pi_i(\widehat{a}_i, a_{-i}) > \pi_i(a_i, a_{-i})$$

  for all $a_i \in A_i$, $a_i \neq \widehat{a}_i$ and for all $a_{-i} \in A_{-i}$.

- A strictly dominant strategy gives a player a strictly higher payoff than any other, no matter what strategies are chosen by other players.
- Clearly, a rational player can do no better than play one.

- **Definition**: An outcome $(\widehat{a}_1, \widehat{a}_2, ..., \widehat{a}_n)$ is said to be an *equilibrium in dominant strategies* if $\widehat{a}_i$ is a dominant strategy for each player i.

- The Prisoners' Dilemma provides an easy example:

	Cooperate		Defect
Cooperate	-1 (top), -1 (bottom)	$\Rightarrow$	0 (top), -9 (bottom)
	$\Downarrow$		$\Downarrow$
Defect	-9 (top), 0 (bottom)	$\Rightarrow$	-6 (top), -6 (bottom)

- By defecting a player maximizes his payoff, regardless of opponent's behaviour.

$\Rightarrow$ Mutual defection is a *dominant strategy equilibrium*.

- There are no real strategic issues in the Prisoners' Dilemma.

- But the equilibrium $\{$D, D$\}$ is Pareto dominated by $\{$C, C$\}$, which unfortunately is not an equilibrium.
- Do you agree with the prediction? If not, then is the analysis flawed? Or is the game being played a different one?

Application: Price Competition

Two profit maximising firms produce the same good and choose whether to price High or Low. A firm with a lower price gains the entire market. If firms charge the same price, they split the market. The demand structure is:

	Demand
High Price ($3)	4 units
Low Price ($1)	10 units

Players: Two firms labelled $i \in \{1, 2\}$.

Actions: Player 1 chooses $x \in \{H, L\}$ and Player 2 chooses $y \in \{H, L\}$.

Payoffs: The payoffs in the strategic form matrix are:

	High		Low
High	6 6	⇒	10 0
	⇓		⇓
Low	0 10	⇒	5 5

- This is a Prisoners' Dilemma: It is a dominant strategy for each player to price low.

$\Rightarrow x = y = L$ is the dominant strategy equilibrium.

Iterated Deletion of Dominated Strategies

- A rational player will choose a strictly dominant strategy if one is available. The same logic leads one to assume that no player would want to play a strictly dominated strategy.

- **Definition**: Strategy a_i'' is strictly dominated by strategy a_i' if

$$\pi_i \left(a_i', a_{-i}\right) > \pi_i \left(a_i'', a_{-i}\right), \quad \forall a_{-i} \in A_{-i}.$$

- If it is common knowledge that players will never play dominated strategies, then we can eliminate them from the game.
- And we can then proceed to delete further strategies that are strictly dominated.
- At the end of this process we are left with a smaller game, including only undominated strategies.

- The *iterated deletion of strictly dominated strategies* proceeds as follows:
  - For each player, remove any strictly dominated strategies;
  - Focus on the smaller game, as though it were a game in its own right;
  - Remove any strictly dominated strategies;
  - Continue until it is not possible to remove any other strategies.
- If a unique strategy profile remains, then a game is said to be *dominance solvable*.

- But to apply this process we need to assume *common knowledge* of rationality: everyone is rational, everyone knows that everyone is rational, everyone knows that everyone knows that everyone is rational ...

- As an example consider the following game:

	Left		Middle		Right	
Top	4	3	2	7	0	4
Bottom	5	5	5	−1	−4	−2

- For the column player, M is strictly better than R. So we can eliminate R and we are left with the following game:

	Left		Middle	
Top	4	3	2	7
Bottom	5	5	5	−1

- For the row player, B is strictly better than T. So we are left with:

	Left		Middle	
Bottom	5	5	5	−1

$\longrightarrow$

	Left	
Bottom	5	5

- For the column player, L beats M, leaving us with the profile $\{B, L\}$.

- To eliminate M for column, we need to assume that column knows that row knows that column is rational.

Coordination

Two friends, Chris and Patrick, need to meet up to discuss their love for economics. They can meet in either the pub or the cafe. Patrick likes coffee, and would prefer to go to the cafe. Chris is a big fan of beer, and would prefer to meet in the pub. They would both rather meet (wherever it may be) than miss each other, but need to choose simultaneously where to go.

Players: Patrick (row player) and Chris (column player).

Actions: Patrick chooses $x \in \{\text{Cafe, Pub}\}$, and Chris chooses $y \in \{\text{Cafe, Pub}\}$.

Payoffs: The payoffs in the strategic form matrix are:

	Cafe	Pub
Cafe	3 4	1 1
Pub	0 0	4 3

⇒ Both Cafe and Pub can be *best responses*, depending on the expected move of the other player.

- Deletion of dominated strategies does not give a prediction. What should we expect?

(This game is known as "Battle of the Sexes".)

Nash Equilibrium

- **Definition**: An outcome $(a_1^*, a_2^*, ..., a_n^*)$ is a *Nash equilibrium* (in pure strategies) if, for every player i,

$$\pi_i\left(a_i^*, a_{-i}^*\right) \geq \pi_i\left(a_i, a_{-i}^*\right), \quad \forall a_i \in A_i.$$

- That is:

$$a_i^* \in \arg\max_{a_i \in A_i} \pi_i\left(a_i, a_{-i}^*\right).$$

⇒ Given that his opponents play a_{-i}^*, a player can do no better than play a_i^* (there is no incentive to deviate).

- **Proposition**: An equilibrium in dominant strategies is a Nash equilibrium (but the reverse is not necessarily true).
- If a strategy is part of a NE, then it survives iterated deletion of dominated strategies. But there exist strategies that survive iterated deletion of dominated strategies, but are not part of a NE.

Best Responses

- A tool, called "best-response" or "reaction" function helps the search for a Nash equilibrium.
- **Definition**: The *best-response function* of player i is a function $R_i(a_{-i})$ that, given the actions a_{-i} of other players, assigns an action $a_i \in A_i$ that maximises player i's payoff. That is:

$$R_i(a_{-i}) = \arg\max_{a_i \in A_i} \pi_i(a_i, a_{-i}).$$

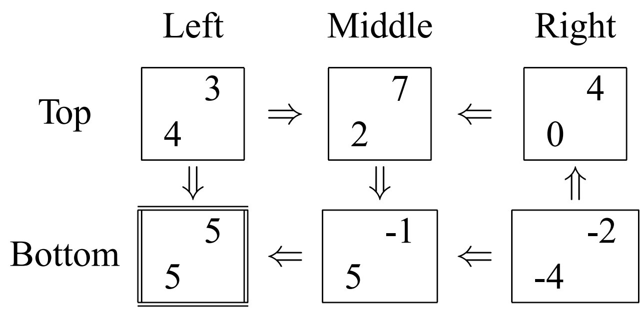
- **Proposition**: If $(a_1^*, a_2^*, ..., a_n^*)$ is a Nash equilibrium, then $a_i^* = R_i(a_{-i}^*)$ for every player i.

⇒ A Nash equilibrium includes strategies that are mutual best responses.

- Hence, to find a Nash equilibrium:
  1. Calculate the best-response function of each player.
  2. Check which outcomes (if any) belong to the best-response functions of all players.
  3. Those outcomes constitute the Nash equilibrium outcomes.

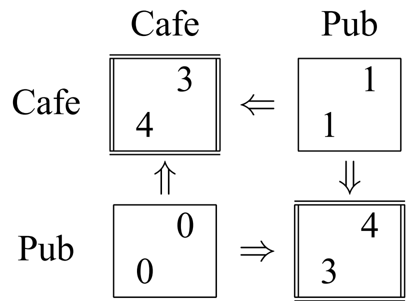
Examples

- Consider the best response structure of the earlier 2×3 example:



Notice that $\{B, L\}$ are mutual best response — this is a *Nash equilibrium*.

- Consider the best response structure of Battle of the Sexes:



Both {Cafe,Cafe} and {Pub,Pub} are mutual best responses — they are both (pure strategy) *Nash equilibria*.

Interpretation

(i) Assume players get together and communicate about how to play the game before they actually play it.

- Players may agree on a strategy combination which should be played but cannot make binding agreements. Thus, any agreement must be self-enforcing.
- Therefore, any solution reached in the pregame negotiation process must be a Nash equilibrium because otherwise it will not be self-enforcing.

(ii) The outcome of a game is considered a rational expectations equilibrium.

- Players are assumed to behave optimally in regard to their beliefs about their opponents' behaviour, and in equilibrium these beliefs have to be correct.

More on Nash

- Dominant strategy equilibria are based on payoff maximising behaviour. No other assumption about opponents was necessary. But these equilibria often do not exist.
- In contrast, a Nash equilibrium generally exists (at least in mixed strategies). The problem is the opposite: usually there are too many Nash equilibria.

- Nash equilibrium combines *optimising behaviour* with *rational expectations*:
  - all players maximise given their beliefs,
  - their beliefs are mutually consistent — they all expect the same strategy profile.

- Moreover:
  - If deletion of dominated strategies yields a unique profile, it is a Nash equilibrium.
  - There may be multiple Nash equilibria, e.g. Battle of the Sexes
  - There may be no pure-strategy Nash equilibria, e.g. Economics Fashion Guru.

Example: Cournot Quantity Competition

Two profit-maximising firms must simultaneously choose production quantities of a homogeneous good. Market price is decreasing in total quantity Q, with linear demand $P = a - bQ$. There are unit production costs of c.

Players: Two firms labelled $i \in \{1, 2\}$.

Actions: Player 1 chooses quantity $q_1 \in [0, \infty)$ and Player 2 chooses quantity $q_2 \in [0, \infty)$.

Payoffs: Payoffs are profits:

$$\pi_i = q_i \left[P\left(Q\right) - c\right] \qquad i = 1, 2.$$

To solve this game:

1. Fix the behaviour of firm 2.
2. Calculate the best response of firm 1, yielding a reaction function.
3. Fix the behaviour of firm 1.
4. Calculate the best response of firm 2, yielding a second reaction function.
5. Combine the two reaction functions (yielding two equations in two unknowns) and solve to find an equilibrium.

- Fixing q_2, profits for Player 1 are:

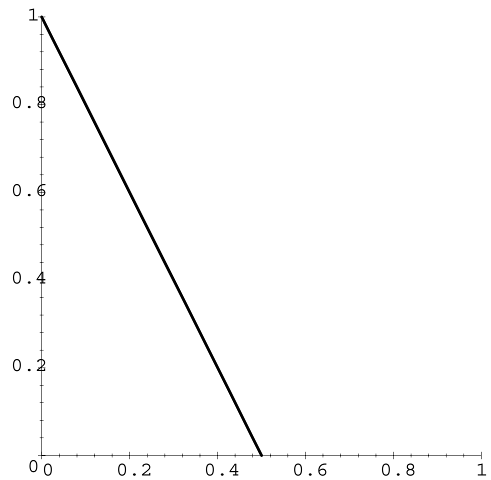
$$\pi_1\left(q_1\right) = q_1 \left[a - b(q_1 + q_2) - c\right]$$

- This is strictly concave in q_1. So compute FOC and rearrange to obtain the reaction function of 1:

$$\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - c = 0 \quad \Leftrightarrow \quad 2bq_1 = a - bq_2 - c$$

$$\Leftrightarrow \quad q_1 = \mathrm{BR}_1(q_2) = \frac{a - bq_2 - c}{2b}$$

- For $a = b = 1$ and $c = 0$, the reaction function is:



- By symmetry, the reaction function of Player 2 is:

$$\mathrm{BR}_2(q_1) = \frac{a - bq_1 - c}{2b}.$$

- Notice that the reaction functions are downward sloping:

$$\frac{\partial \mathrm{BR}_2(q_1)}{\partial q_1} = \frac{\partial \mathrm{BR}_1(q_2)}{\partial q_2} = -\frac{1}{2} < 0.$$

- This means that quantities are *strategic substitutes* : a *submodular* game.

- For a Nash equilibrium, we need:

$$\left.\begin{array}{l} q_1 = \mathrm{BR}_1(q_2) \\ q_2 = \mathrm{BR}_2(q_1) \end{array}\right\} \quad \Leftrightarrow \quad \left\{\begin{array}{l} q_1 = \dfrac{a - bq_2 - c}{2b} y \\ q_2 = \dfrac{a - bq_1 - c}{2b} \end{array}\right.$$

  so that both players mutually optimise and correctly anticipate the opponent's action.

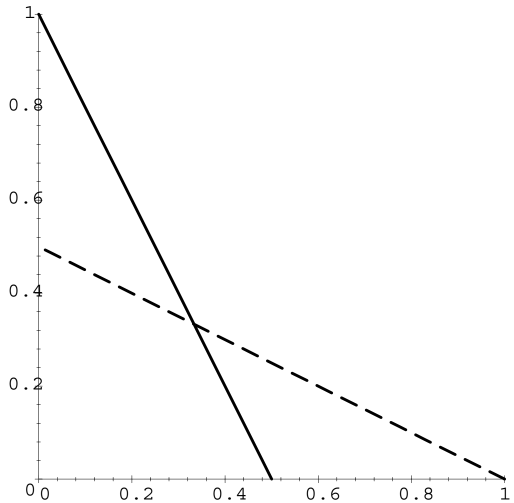
- Any solution of this symmetric game must be symmetric. Hence, in equilibrium, both firms produce the same quantity $q_1 = q_2$.

- Thus solve:

$$q_1 = \frac{a - bq_1 - c}{2b}$$

$$\Leftrightarrow \quad 2bq_1 = a - bq_1 - c$$

$$\Leftrightarrow \quad q_1^* = q_2^* = \frac{a - c}{3b}.$$



Cournot equilibrium: $\mathrm{BR}_1(q_2) = \mathrm{BR}_2(q_1)$

Example: Election Advertising

The Labour and Conservative parties choose their advertising budgets for an election, denoted by x and y respectively. They dislike campaign spending, but wish to obtain a higher share of votes. Advertising is the sole determinant of the election outcome, yielding vote shares of $x/(x+y)$ and $y/(x+y)$ respectively.

Players: The Labour and Conservative Parties.

Actions: Labour chooses $x \in [0, \infty)$ and the Conservatives choose $y \in [0, \infty)$.

Payoffs: Suitable payoff functions might be:

$$u_L = \frac{x}{x+y} - x \quad \text{and} \quad u_C = \frac{y}{x+y} - y.$$

- If Labour believes that the Tories will spend y, it solves:

$$\frac{\partial u_L}{\partial x} = \frac{(x+y)-x}{(x+y)^2} - 1 = 0 \quad \Leftrightarrow \quad y = (x+y)^2$$

$$\Leftrightarrow \quad x = \sqrt{y} - y.$$

⇒ The reaction functions are:

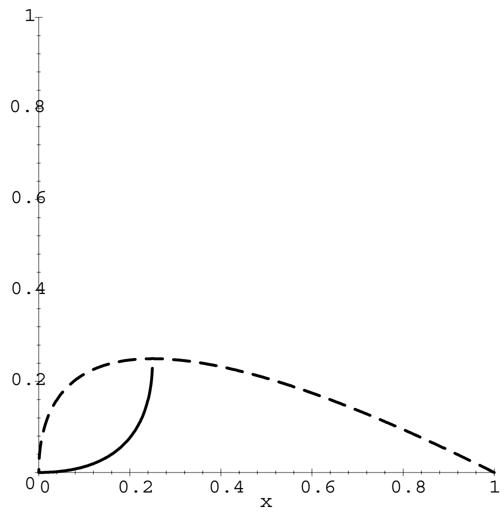
$$\text{BR}_L(y) = \sqrt{y} - y \quad \text{and} \quad \text{BR}_C(x) = \sqrt{x} - x$$

- For a Nash equilibrium we need:

$$\left\{ \begin{array}{l} y = \text{BR}_C(x) \\ x = \text{BR}_L(y) \end{array} \right\} \quad \Leftrightarrow \quad \left\{ \begin{array}{l} y = \sqrt{x} - x \\ x = \sqrt{y} - y \end{array} \right\}$$

$$\Leftrightarrow \quad x = \sqrt{x} - x \quad \Leftrightarrow \quad x\,(4x-1) = 0$$

$$\Rightarrow \quad x^* = y^* = \frac{1}{4}.$$



- The reaction functions slope upward then downward: variables are both strategic substitutes and complements, according to the region.

Example: Bertrand Competition

Two firms selling identical products must simultaneously choose what price to charge. A firm with a lower price gains the entire market, but firms would rather charge high prices. If firms charge the same price, they split the market. A unit mass of consumers will only buy if the price is less than $\overline{p}$. W.l.o.g., the marginal cost of production is zero.

Players: Two firms denoted by $i \in \{1, 2\}$.

Actions: Player 1 chooses $p_1 \in [0, \infty)$ and Player 2 chooses $p_2 \in [0, \infty)$.

Payoffs: Payoffs are profits. For example, for Player 1:

$$\pi_1 = \begin{cases} p_1 & \text{if} \quad p_1 < \min\{\overline{p}, p_2\} \\ \dfrac{p_1}{2} & \text{if} \quad p_1 = p_2 < \overline{p} \\ 0 & \text{if} \quad p_1 \geq \overline{p} \text{ or } p_1 > p_2 \end{cases}$$

⇒ There is a unique pure strategy NE with $p_1 = p_2 = 0$.

- In fact:
  - If the lowest price were negative, then the firm would makes a loss.
  - If the lowest price were strictly positive, then opponent should undercut.
  - If, e.g., $0 = p_1 < p_2$ then firm 1 should raise its price.
  - Hence only possibility is $p_1 = p_2 = 0$, and there is no better response.

- Notice that, strictly speaking, reaction functions do not exists:
  - Suppose that $0 < p_2 < \overline{p}$.
  - Then it's always better for Player 1 to undercut Player 2.
  - But if Player 1 undercuts by ε, then ε should be as small as possible but > 0.
  - Mathematically, the set of best response (feasible payoffs) is open above.
- The Bertrand specification is degenerate due to the discontinuity in payoffs. Be careful when using continuous action sets!

Submodular and Supermodular Games

Players: Two players labelled $i \in \{1, 2\}$.

Moves: 1 chooses $x \in X \subseteq \mathbb{R}$, 2 chooses $y \in Y \subseteq \mathbb{R}$.

Payoffs: Payoffs are $\pi_1(x, y)$ and $\pi_2(y, x)$. Consider the symmetric case $\pi_i(\cdot, \cdot) = \pi(\cdot, \cdot)$.

Information: Payoffs are commonly known, actions are taken simultaneously.

- Consider the reaction of Player 1 to Player 2. Fix y and maximize 1's payoff w.r.t. x:

$$\max_{x \in X} \pi_1(x, y) \quad \Rightarrow \quad \left.\frac{\partial \pi(x, y)}{\partial x}\right|_{x=\mathrm{BR}_1(y)} = 0.$$

- To calculate the slope of the reaction function, totally differentiate w.r.t. y:

$$\frac{\partial^2 \pi(x, y)}{\partial x \partial y} + \frac{\partial^2 \pi(x, y)}{\partial^2 x}\frac{dx}{dy} = 0$$

$$\Leftrightarrow \quad \frac{dx}{dy} = -\frac{\partial^2 \pi(x, y)/\partial x \partial y}{\partial^2 \pi(x, y)/\partial^2 x}.$$

- SOC for a maximization ensure that $\partial^2 \pi(x, y)/\partial^2 x < 0$.

- Hence the slope of the reaction function is determined by:

$$sign\left\{\frac{dx}{dy}\right\} = sign\left\{\frac{\partial^2 \pi(x, y)}{\partial x \partial y}\right\}.$$

- We define:

(i) π is **supermodular** (strategies are *strategic complements*) if:

$$\frac{\partial^2 \pi(x, y)}{\partial x \partial y} > 0 \quad \Leftrightarrow \quad \mathrm{BR}_1(y) \text{ is upward sloping.}$$

- In a supermodular game a player's marginal utility of increasing his strategy raises with increases in his rival's strategy.

  (True definition of supermodularity is more general: it does not require differentiability).

(ii) π is **submodular** (strategies are *strategic substitutes*) if:

$$\frac{\partial^2 \pi(x, y)}{\partial x \partial y} < 0 \quad \Leftrightarrow \quad \mathrm{BR}_1(y) \text{ is downward sloping.}$$

Bulow, Geanakoplos and Klemperer (1985): "Multimarket Oligopoly: Strategic Substitutes and Complements" *Journal of Political Economy*.

Examples of Super/Submodular Games

(i) Cournot competition is typically a submodular game:

If x and y are quantities, then a typical payoff function is:

$$\pi(x,y) = xP(x+y) - c(x)$$

$$\Rightarrow \quad \frac{\partial \pi}{\partial x} = \underbrace{P\left(x+y\right) + xP'(x+y)}_{\text{marginal revenue}} - \underbrace{c'\left(x\right)}_{\text{marginal cost}}$$

$$\Rightarrow \quad \frac{\partial^2 \pi}{\partial x \partial y} = P'(x+y) + xP''(x+y)$$

- Hence, the mixed partial derivative is negative if marginal revenue is decreasing (as it is usually the case).
- Quantities are strategic substitutes. An *increase* in my opponent's quantity *reduces* my incentive to raise my quantity.

(ii) Bertrand competition is typically a *supermodular* game.

- Prices are strategic complements. An *increase* in my opponent's price *increases* my incentive to raise my own price.

(iii) Reaction functions can be non-monotonic, e.g. Election Advertising game.

The Economics Fashion Guru

A MEF student is a fashion guru, and always wishes to stand out from the crowd by wearing the latest designer fashions. Her remaining classmates wish to copy her, but that would compromise her individuality.

Players: The student (row player) and the others (column player).

Actions: The student chooses $x \in \{$Gucci, Chanel$\}$, and the others choose $y \in \{$Gucci, Chanel$\}$.

Payoffs: The payoffs in the strategic form matrix are:

	Gucci		Chanel
Gucci	0, 1	$\Leftarrow$	1, 0
	$\Downarrow$		$\Uparrow$
Chanel	1, 0	$\Rightarrow$	0, 1

- Any sequence of best responses leads to a *cycle:* there is no (pure strategy) Nash equilibrium to this game — we must consider mixed strategies.
- How would you play?

To review the material, suitable readings, increasing in difficulty, are:

- Chapter 1 of Gibbons (1992).
- Chapters 7 and 8 of Mas-Colell, Whinston and Green (1995).
- Chapters 1– 4 of Osborne and Rubinstein (1994).
- Chapters 1 and 2 of Fudenberg and Tirole (1991).

A great source of examples and a fun read is Dixit and Nalebuff (1991). An excellent undergraduate textbook is Dixit and Skeath (1999).

References

- Dixit and Nalebuff (1991): *Thinking Strategically*, Norton.
- Dixit and Skeath (1999): *Games of Strategy*, Norton.
- Fudenberg and Tirole (1991): *Game Theory*, MIT Press.
- Gibbons (1992): *A Primer in Game Theory*, Harvester Wheatsheaf – Il Mulino.
- Mas-Colell, Whinston and Green (1995): *Microeconomic Theory*, Oxford University Press.
- Osborne and Rubinstein (1995): *A Course in Game Theory*, MIT Press.