Electrodynamics before the Theory of Relativity, 1890 – 1905.

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Contents

1. Introduction

In the previous paper[1)], it was concluded that Einstein's relativity theory of 1905 did not succeed to the investigations on the influences of the annual motion of the earth on electric and optical phenomena, but was derived from a theoretical consideration of the electrodynamics of moving bodies, that is, of the theory of electromagnetic phenomena occuring in terrestrial bodies in motion. Recently G. Holton advanced a similar view and considered the question who was the immediate predecessor who exerted great influence on Einstein in his study of the electrodynamics.[2)] By investigating archival materials which are kept at the Institute for Advanced Study, Princeton, Holton concluded that A. Föppl's textbook[3)] on Maxwell's theory published in 1894 was most likely Einstein's teacher of the electrodynamics.

Holton's investigation has thrown a needed light on the origins of the relativity theory. But no investigation has yet been made on the question what

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were the general situations of the electromagnetic theory in the years about the turn of the nineteenth century. Many authors state that Lorentz has developed the most distinguished theory concerning the relative motion of the earth to the ether. But it is quite unknown what actually was the position of Lorentz's theory among other theories that existed in those days. We also hardly know about how it became accepted widely among physicists. A historical analysis of the process of formation of a scientific theory demands studies of historical background to that theory and of the motives and guiding ideas that brought about the birth of the theory. As regard to the origins of the relativity theory, too, such studies will be required in order to gain a full comprehension of the significance of the electrodynamics of moving bodies for the formation of the theory of relativity.

Present paper deals with the development of electrodynamics during one and a half decades from 1890 through 1905. At the beginning of this period, although a quarter of century had already passed since the formation of Maxwell's theory, it was not yet fully recognized by the majority of physicists. The deep impression raised by Hertz's experiment of 1887 - 88 caused a "sudden change of general opinion with extraordinary rapidity."[4] But the view which was formed as the result of that sudden change was that "even in the dielectrics (and, to be essential, in the air and the vacuum too) took place the electromagnetic processes"[5], and that there existed a medium of electromagnetic action, the ether.[6] To fill the gap between such an immature conception and the modern notion of electromagnetic phenomena, efforts had to be made to clarify the true meaning of Maxwell's theory and to re-formulate it in a way that could represent most relevantly the meaning of the theory. The main purpose of the present paper is to consider the development of those efforts.

The number of theoretical works dealing with the formulation of electrodynamics were relatively small in the period under consideration. Those cited in the following nearly exhaust the papers appeared in that period. But the lines of thought which guided those works were varied according to the difference of the author's conceptions of electromagnetic field. Roughly speaking, there were three streams of thought. The first originated from Hertz's theory. Its characteristic feature was not to bother about the atomic structure of matter or the constitution of the ether. The second was J. Larmor's theory which pursued a "dynamics of the ether", and the last was Lorentz's and Wiechert's theory of electrons.

2. Hertz's Electrodynamics - 1. Bodies at Rest

One of the remarkable features of Maxwell's theory was to introduce a quantity D called "electric polarization" for all kinds of medium including the ether and, by adding the "displacement current" $\partial D/\partial t$, to make all the current closed. The quantity D is today called electric displacement but in those days it was generally called polarization. The magnetic induction B too was called (magnetic) polarization. In reading literatures of those days, therefore, one must take care not to confuse them with the polarization of today. For the sake of definiteness, the word "polarization" used in the old meaning hereafter shall be put between quotation marks. Maxwell regarded the quantity D as representing a kind of strained state of the medium given rise to by a displacement of electricity under the action of electric force E. The magnitude of D was defined to be measured by the quantity of elctricity which had passed through a cross section of unit area. The relation between the electric force and the strain (displacement) was supposed to be same as in the case of elastic deformation: $D = (K/4\pi) \cdot E$. The constant K was called specific inductive capacity.[7)]

Hertz's electrodynamics[8)] was motivated by a criticism of ambiguities contained in Maxwell's formulation described above and was intended to give a consistent formulation of the theory of electromagnetic field.

Hertz's criticism to Maxwell's conception was as follows: Maxwell assumed that when an electric force E was exerted on a medium, a displacement D was produced there. But such a notion could, Hertz said, have any meaning only in the case when the force E stood by itself independently of the medium (ether). The case, however, was not thinkable without adopting the theory of action at a distance and consequently contradicted Maxwell's fundamental assumption of action at contiguity.

Hertz thought that this confusion in Maxwell's theory was caused by the fact that Maxwell preserved the old picture of electric fluid and based his concept of dielectric displacement on that conception of the electric charge.[9)] He claimed that the displacement itself should be taken as the foundation of formulation. Connected with this opinion is his famous dictum "Maxwell's theory is Maxwell's system of equations."[10)] What he meant by this dictum may be guessed from the following:

> "After these equations are once found, it no longer appears expedient to deduce them (in accordance with the historical course) from conjectures as to the electric and magnetic constitution of the ether and the nature of the acting forces, - all these things being entirely unknown. Rather is it expedient to start from these equations in search of such further conjectures respecting the constitution of the ether."[11)]

In accordance with this view, Hertz's approach to the electromagnetic theory takes, so to speak, an axiomatic way. He first postulates that the interior of all bodies, including the free ether, can experience two kinds of disturbances denoted as electrical and magnetic, the magnitudes of which are represented by electric force E and magnetic force H defined as the mechanical forces exerted upon a small body loaded with unit charge. Then the density of energy which is stored within an isotropic medium as a consequence of the electric or magnetic disturbance is proportional to E^2 or H^2 respectively. The coefficients of these proportionality are called the dielectric constant and the magnetic permeability:

$$\frac{1}{8\pi}E^2, \quad \frac{1}{8\pi}H^2 \qquad \text{for the ether, and}$$

$$\frac{\epsilon}{8\pi}E^2, \quad \frac{\mu}{8\pi}H^2 \qquad \text{for a ponderable body.}$$

Hertz does not explain why the energy density is proportional to the square of electric or magnetic force. These expressions are, so to speak, postulated axiomatically, though he does not here use the term "to postulate". The relation between the electric and magnetic forces in the free ether is (postulated to be) given by equations

$$A\frac{\partial H}{\partial t} = -\mathrm{rot}\,E, \qquad A\frac{\partial E}{\partial t} = \mathrm{rot}\,H,$$

$$\mathrm{div}\,H = 0, \qquad \mathrm{div}\,E = 0,$$

where A has the reciprocal dimension of velocity and is a intrinsic constant of the ether. By intrinsic constant is meant that its magnitude is independent of the presence of any other body, or of any arbitrary stipulations on our part. For homogeneous isotropic nonconductors too, the equations does not change qualitatively. Only the intrinsic constant alters. Thus one must replace $A\frac{\partial H}{\partial t}$ and $A\frac{\partial E}{\partial t}$ by $A\mu\frac{\partial H}{\partial t}$ and $A\epsilon\frac{\partial E}{\partial t}$ respectively.

A few remarks may be made here concerning the expression of mathematical formula. In the first half of 1890's the vector notations were not yet in general use. They had just been propounded by Heaviside and Föppl but most of the physicists including Hertz used to write the equations for each component. But throughout present paper, for the sake of brevity and comprehensiveness, the mathematical formulae are written in the way that is common today. In addition, since Hertz employed the left hand system of coordinates, terms containing rotation or vector product in his equations have reversed sign. In the following all equations are rewritten in the right hand system.

Thus far the equations are obtained for the ether and nonconductors. By them it is shown that when $H = 0$, E is kept constant as far as there is no external action. But there are many bodies in which under same circumstance E disappears more or less rapidly without any external disturbance. They are called conductor. For them Hertz assumes that, when left to itself, the electric force varies in accordance with

$$A\epsilon \frac{\partial E}{\partial t} = -4\pi\lambda AE.$$

Where magnetic forces are present, the variation takes place in accordance with

$$A\epsilon \frac{\partial E}{\partial t} = \mathrm{rot}\, H - 4\pi\lambda AE.$$

Here Hertz introduces new notations $D = \epsilon E$ and $B = \mu H$ and called them "polarization". In the case of non-isotopic bodies, ϵ and μ are no longer scalar constants but tensors and consequently the equations take complicated forms. But if one uses D and B, the equations can always be written in the same forms irrespective of the isotropy of the medium.

Finally come the definitions of the quantities of electricity and magnetism. With the aid of the fundamental equations, the volume integral of div D over a domain which comprises the whole system is shown to be constant. This integral is put equal to $4\pi e$. The indestructibility of e prompts the idea that e represents an amount of some substance contained in the system. The total amount of the electricity of the system has thus been defined. But the distribution of the electricity over each volume element cannot necessarily be put equal to $(1/4\pi)$ div D. By Gauss's theorem the integral which defines e can be transformed into a surface integral. But since all portions of the surface of integral lies in vacuum outside the system, we have $D = E$ on the surface of integral. The total quantity e, therefore, does not depend on whether we take as the density of electricity $(1/4\pi)$ div D or $(1/4\pi)$ div E. Hertz called the electricity whose density is defined by div D "true", and that whose density is given by div E "free". Herein originated the distinction of two kinds of electricity in old-fashioned electromagnetic theory.[12] By similar arguments "true magnetism" $(1/4\pi)$ div B and "free magnetism" $(1/4\pi)$ div H are defined. It is the true charge that serves as the source or sink of lines of force. Hence the lines of force in Hertz's theory are connected with the "polarizations" D and B.

Having seen Hertz's manner of deriving the fundamental relations, his conceptions of electromagnetic phenomena which underlie his formulation of electrodynamics may now be examined.

We must first admit that it is an indisputable merit of Hertz's electrodynamics that it has reduced the fundamental quantities of electromagnetic theory to electric and magnetic charges, E, H, ϵ, λ, and μ, and has brought into light the logical structure of the theory to a degree that was never reached before him. This result no doubt owes much to his axiomatic treatment of the problem. But at the same time attention should be paid to the fact that the same axiomatic treatment did impose a limitation on Hertz's theory. It certainly played a significant role of prohibiting speculations about mechanical processes in the ether. But it also decrees that, once E and H have been postulated, one ought not to seek further for the meaning of these quantities. This limitation reveals itself most clearly when one considers the electromagnetic field in ponderable bodies. Since the disturbances represented by E and H were postulated to exist equally in the ponderable matter as well as in the ether, one naturally has to regard E and H within ponderable bodies as being borne, at least partially, by the ponderable matter. In fact, discussing the electromagnetic equations for moving bodies, Hertz stresses that "the lines of force simply represent a symbol for special conditions of matter"[14] and that they cannot be regarded as realities independent of the matter. It was therefore impossible in Hertz's conception to separate the field from the matter by reducing the bearer of E and H to the ether alone.

3. Hertz's Electrodynamics - 2. Bodies in Motion

An attempt to extend the formulation described in the last section so as to include the electromagnetic phenomena in moving bodies was made in Hertz's second paper of 1890 entitled "Über die Grundgleichungen der Elektrodynamik für bewegte Körper".[15] Before entering into consideration of its contents, it may be well to see what problems the "electrodynamics of moving bodies" was to deal with.

The electrodynamics of moving bodies had two sdies, technological and theoretical. The technological side concerns the problem how to treat theoretically the electric machines having movable parts such as dynamos or motors. In connection with those problems, the phenomenon of unipolar induction drew special attentions of physicists. A cylindrical bar magnet rotates about its axis. If one connects the axis and the lateral surface of magnet with a wire through sliding contacts on the lateral surface and the axis of magnet, then an electromotive force is induced in the circuit consisting of the wire, the axis, and the body of magnet. Where in the circuit does the electromotive force reside? And do the magnetic lines of force rotate together with the body of magnet or are they left remained in fixed positions of the space? These questions had long been discussed since Faraday's time.[16] In particular, much discussion arose around 1900 about the unipolar induction. Einstein also referred to this problem in his first paper of the theory of relativity.

The theoretical side of the electrodynamics of moving bodies concerns the question of the nature of electric current and of the constitution of dielectrics. That a moving electrified body would produce the same effects as a conduction current was first predicted by Faraday in 1838. This supposition was confirmed qualitatively in 1876 by Rowland's experiment. After that the experiments were repeated with quantitative measurements and under varied conditions. Among them the experiments performed by V. Crémieu during 1900 - 1902 provoked serious discussions. Crémieu reported that he had obtained results which disproved the Rowland effect. If Crémieu was right, the concept of convection current would have to be rejected and the foundation of electromagnetic theory would be undermined. But the repeated experiment of Crémieu in collaboration with H. Pender disproved Crémieu's earlier results and confirmed Rowland effect. The Röntgen current also may be regarded as a variation of Rowland effect. It is the effect discovered by Röntgen in 1888 that a dielectrics moving in an electrostatic field produce a magnetic field in the surrounding space. After Röntgen many experiments were repeated, and in 1903 A. A. Eichenwald obtained a qualitative result which confirmed the theoretical calculation.[17]

To formulate the electrodynamics of moving bodies, it was necessary to make an assumption about the motion of the ether. Hertz assumed that when a body moved, the ether contained within it shared the whole motion of the body. The reason for making this assumption was that if the ether had been regarded as independent of the motion of body, the field quantities at each point of space would have to split into two components, one for the ether and another for the ponderable matter.[18] In connection with this assumption Hertz remarked that his theory was to show how to formulate the theory under certain restrictions which were imposed arbitrarily and that it was scarcely probable that these restrictions corresponded to the actual facts of the case[19]. He moreover stated that the correct theory should rather distinguish between the conditions of the ether at every point, and those of the embedded matter. But at the same time he indicated that to propound a theory in accordance with the distinction of two kinds of electromagnetic conditions would require more numerous and arbitrary hypothesis than those of his theory. He thus attached value to his own theory from the point of view of systematic arrangement.

It must be noted here that Hertz has restricted himself to electromagnetic processes in narrower sense. In assuming the dragging of ether, he argues that among the phenomena so embraced there is no one which indicates that the ether can move independently of ponderable matter within this latter.[20] But at the same place he admits that if we consider the wider class of phenomena, there are some indications that even in the interior of tangible matter the ether moves independently of it. This makes a remarkable contrast to the attitudes of Lorentz and Larmor who constructed their theories on the basis of optical experiences.

Hertz imagined as the moving ponderable matter a most general compressible fluid and assumed that the influence of the motion was such a kind that, if it alone were at work, it would carry the lines of force with the matter. He then enumerated changes of lines of force induced by various causes. We fix our attention upon a surface-element in the interior of the moving medium, which at a certain instant lies perpendicular to x axis and during the motion is displaced and distorted with the matter. Then the variation during a time interval dt in the number of magnetic lines of force which traverses this surface element consists of:

1. the variation due to a displacement of the element:

$$(v\cdot\nabla)B_x dt.$$

2. the variation due to a rotation of the element:

$$-(B_y\frac{\partial}{\partial y}+B_z\frac{\partial}{\partial z})v_x dt.$$

3. the variation due to an increase of the area of the element:

$$b_x(\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z})dt.$$

4. the variation due to causes other than influences of the motion:

$$\frac{\partial B_x}{\partial t}\ dt.$$

The total variation is the sum of 1 to 4. On the other hand, Hertz asserts, we may also analyze the total variation into the amount which the presence of the electric forces alone would contribute and the amount which the motion alone would do. The first amount is given by $-\ c$ rot $\boldsymbol{E}\cdot dt$ and the second is, by Hertz's fundamental assumption, equal to zero. Hence the total variation is also represented by $-\ c$ rot $\boldsymbol{E}\cdot dt$. Thus equating two expressions we finally obtain

$$\frac{1}{c}\{\frac{\partial \boldsymbol{B}}{\partial t}-\mathrm{rot}[\boldsymbol{v}\boldsymbol{B}]+(\mathrm{div}\,\boldsymbol{B})\boldsymbol{v}\}=-\mathrm{rot}\,\boldsymbol{E}.$$

Similarly, for the electric lines of force we obtain

$$\frac{1}{c}\left\{\frac{\partial D}{\partial t} - \mathrm{rot}\,[v\,D] + (\mathrm{div}\,D)\,v\right\} = \mathrm{rot}\,H - \frac{4\pi}{c}\,i.$$

Here $\partial/\partial t$ denotes the time derivative at a point fixed to our system of coordinates.

Using these equations Hertz discussed various problems such as the movement of true charges accompanying the motion of a magnet or an electrified body, the induced electromotive force along a closed curve. Passing through the same material points of a moving medium, the treatment of surfaces of slip, and the ponderomotive force of electromagnetic origin. As for the electromagnetic induction, the time rate of the variation of magnetic flux through a closed curve s was shown to be equal to the line integral of electric force E along s. But Hertz denied to express this result in such a way that the induced electromotive force was proportional to the number of lines of force cut by closed circuit s per unit time, because he thought that an expression which admitted independent motions of the lines of force ought not to have been allowed. Such an argument of Hertz clearly shows that in his view the field have not been separated from the matter.

Hertz obtained the expression for the ponderomotive force by considering the energy balance. In all but one cases the expression gave results which agreed with experiences. The exceptional case was the resultant force acting on a portion of the ether surrounded by a closed surface. This force was such as to give rise to a translation of that portion of the ether. This result would thenceforth provide much trouble for physicists, but Hertz asserted that there was no reason to abandon his theory on this account, because in any case a flow of the ether could not practically be observed.

An interesting point in Hertz's theory is the covariance of theory. Hertz remarked that his derivation of the fundamental equations did not require the system of coordinates being absolutely fixed in space, and that on this account the equation could be transformed, without change of form, to any moving system provided that v, ϵ, μ, and λ were referred to this system. From this he concluded that the absolute motion of rigid system of bodies had no effect upon any internal electromagnetic processes in it, provided all the bodies under consideration, including ether as well, shared the same motion.[21] In view of this, Hertz may be said to be aware of a kind of covariance, though use is not made of this term.

Confining his argument to the electromagnetic phenomena in narrower sense, Hertz wanted to emphasize the value of systematic construction of the theory. This was in a sense a wise way. For, if Hertz had wanted to extend his theory so as to include optical phenomena, he would be troubled by how to treat the aberration and Fresnel's dragging coefficient. But, as we shall see later, it was on account of this restriction, especially of its inability of providing a satisfactory explanation of Fizeau's experiment, i.e. of the dragging of the ether, that many people criticized Hertz's theory. Even in the domain of electromagnetic phenomena in narrower sense, Hertz's electrodynamics was found to contradict experimental facts. But it was long before this that Lorentz advanced his theory of electrons. It was, therefore, not because of this kind of disagreement of Hertz's theory with experiments that Lorentz's theory was propounded.

4. The Dynamics of the Ether

Among the British followers of Maxwell who endeavoured to construct mechanical models of the ether, J. Larmor made an exceedingly comprehensive study on the subject. Though comprehensive discussions are also given in Lord Kelvin's *Baltimore Lectures*[22], its discussions are almost entirely directed to optical phenomena. As to the electrodynamics, Larmor should in the first place be referred to. During from 1893 to 1897, Larmor published three extended papers bearing the title "A Dynamical Theory of the Electric and Luminiferous Medium".[23] Each of these are summarized in three separate papers.[24] Larmor's intention throughout this series of papers is stated plainly at the opening of the first paper: "The object of this paper is to attempt to develop a method of evolving the dynamical properties of the ether from a single analytical basis."[25] In its summarizing paper, Larmor explains his point of view as follows:[26] Regarding all the interactions of the different classes of physical agencies as manifestations of some fundamental medium, efforts have been made to discover the properties of this medium. The most powerful method for this attempt is that of Lagrange, the precise force of which consists in its allowing us to leave out of account altogether the details of the mechanism that underlies the phenomena under discussion. The whole problem can be solved at once if the Lagrangian is suitably determined. What properties should be most conveniently and simply assigned to the medium in order that it has the Lagrangian obtained may be discussed. But this is rather of the nature of illustration and explanation,….

It may therefore be said that Larmor's object was to build up a dynamics of the ether but not necessarily to construct a so-called mechanical model. The dynamical theory as understood by Larmor was to give a Lagrangian for the ether and to derive analytically various conclusions from it. It appears that Larmor's position was not to look for a mechanical explanation of Maxwell's system of equations assuming its validity as the fundamental presumption. For him the fundamental entity was the ether as a dynamical substance. Maxwell's theory was, in his eyes perhaps, merely a clue to the dynamical properties of the ether. Larmor's purpose might be, therefore, to build up a general dynamics of the ether, parts of which would correspond to various results obtained by Maxwell.

However, he occasionally makes digressions into annoying discussions about mechanical analogies to conclusions from the theory of electromagnetism. Due to those discussions, Larmor's papers are very difficult to read through. Generally speaking they are not written in such a lucid way that the argument is evolved logically and systematically from a fundamental proposition, or inversely, the fundamental proposition is induced finally by bringing into association step by step various topics. They contain many digressions into details and repeated arguments, and lack the lucidity which characterizes Hertz's or Lorentz's paper. Though Larmor's papers might be said comprehensive in the sense that everything is alluded to in them, they are by no means systematic.

Now the skeleton of Larmor's inquiry shall be resumed.

It must first be noticed that in contrast to the case of Hertz, Larmor's inquiry has the optics for its ground. He wanted to build up a dynamics of the ether by re-interpreting appropriately J. MacCullagh's theory of luminiferous ether with rotational elasticity. Assuming an elastic body whose potential energy did depend solely on the rotation of volume element, MacCullagh succeeded in 1839 in deriving the laws of optics from the principle of mechanics. If we denote by e the linear displacement of each point of the medium, the potential energy W of a rotationally elastic body is expressed by a homogeneous quadratic function of the components (f, g, h) of rot e:

$$W = \tfrac{1}{2}\int\{a^2f^2 + b^2g^2 + c^2h^2\}d\tau.$$

The total energy of the medium also comprises the kinetic energy of each mass point

$$T = \frac{\rho}{2}\int(\partial e/\partial t)^2 d\tau.$$

The equation of motion for this medium will be derived from the variational principle

$$\delta\int(T - W)dt = 0.$$

Since the equation obtained guarantees the identity div $e = 0$, the difficulty of longitudinal wave will be eliminated.

Larmor first compares MacCullagh's theory with Fresnel's theory of elastic waves and with Kelvin's theory of an elastic medium which gives no resistance to a laminar compression, i.e. compression without lateral deformation. It is shown that both latter theories can be brought into association with MacCullagh's theory by appropriate transformations. Having thus confirmed that MacCullagh's theory may safely be taken as the basis of the dynamics of ether, Larmor proceeds to adapt the theory to electromagnetic phenomena. For this purpose he puts

$$\mathrm{rot}\,\boldsymbol{e} = \text{electric displacement } \boldsymbol{D},$$

and

$$\partial e / \partial t = \text{magnetic force } \boldsymbol{H}.$$

The unit employed is the rationalized electromagnetic unit. Then, in the first place, the condition for electrostatic equilibrium is obtained from $\delta W = 0$. The result is that using a function of space coordinates V, $\boldsymbol{D}$ is expressible as

$$D = -\left(\frac{1}{a^2}\frac{\partial}{\partial x}, \frac{1}{b^2}\frac{\partial}{\partial y}, \frac{1}{c^2}\frac{\partial}{\partial z}\right)V.$$

After this follow discussions concerning the case of medium with ordinary compressive elasticity, treatments of the electric current and oscillation, the relation of a dissipative term added to the Lagrangian with the absorption of light and the electric resistance, and so on. Of special interest is the discussion of ponderomotive force between conductors carrying electric currents. Since purely electromagnetic formulae were insufficient in explaining the ponderomotive forces, Maxwell, Neumann, and Helmholtz were all compelled to introduce a substance, the element of current, but, Larmor asserts, this substance must be rejected. For, if the element of current is admitted as a substance, an internal stress will be produced in the conductor which, when the conductor is a liquid such as mercury, will involve a change of fluid pressure. But such a conclusion is denied by FitzGerald's and Lodge's experiments. Larmor asserted that this difficulty would be avoided if one considered all the electric currents as convection currents. In this way he proposed to introduce into theory the particle which constituted the electric current. He called this particle "electron".[27)]

Larmor attached great importance to the molecular theory from more general point of view. He argues that though the electrodynamic problems concerning material bodies regarded as continuums may be solved by macroscopic equations, treatments by molecular theory inevitably become needed when one wants to discuss the porblems such as the force exerted upon matter, especially when it is in motion, the distinction between true current and Maxwell's total current including displacement currents, inhomogeneous dielectrics, and so forth. Thus Larmor supposes that the matter is composed of positive and negative electrons which are regarded as centres of rotational strains. According to him, the ether is an absolutely continuous medium and has an elasticity and an inertia as its sole ultimate and fundamental properties. The matter, on the other hand, is an assemblage of strain centres of the ether and its elasticity, inertia, and other properties are to be derived from the interactions between the strain centres through the ether. Larmor supposed that the strain centres could move through the ether independently of the substance of the ether. Larmor's ether is, therefore, a rest ether which does not accompany the motion of material body. The reason why Larmor has adopted the rest ether may be looked for in his being motivated by optics. As was also the case with Lorentz, the incontestable superiority of Fresnel's theory would naturally have induced him to adopt the rest ether. And in fact, when he introduced the electron, he also took into account the advantage of the theory of electrons in guaranteeing the rest state of the ether. In favour of the theory of electrons, Larmor argued that, if the current was considered to be composed of electrons, only the forces acting upon electrons would contribute to the ponderomotive force on a conductor carrying electric current, whereas in Maxwell's theory, since the current included the displacement current within the ether, the free ether too would be caused to move.

Because of his emphasis on the molecular point of view and on the rest ether, Larmor's theory largely resembles Lorentz's theory of electrons. The very title *Theory of Electrons* was for the first time used in 1895 by Larmor as the subtitle of his second paper. But in spite of these resemblances there remains fundamental difference between two theories. In Lorentz's theory the ether and the electron are considered as two ultimate and fundamental entities on the same footing. Contrary to this, for Larmor, the fundamental entity is the ether alone. Larmor therefore made some comments on Lorentz's method of treating the phenomena in a moving medium which consisted in, after formulating the dynamics of a single electron, applying an averaging procedure to the equations obtained. In Lorentz's theory, he says, Maxwell's formal equations must take place of the dynamical hypothesis of a rotational ether, though one can state the theory independently of that hypothesis. But in his opinion, the abstract procedure of Lorentz's method "will neither be so simple nor so graphic, nor lend itself so easily to the intuitive grasp of relations, as a more concrete one" based upon a dynamical hypothesis of the ether.[29)]

Larmor's point of view reminds us the unitary theory of field of later date. But historically, this point of view cannot be said to have played a positive role at that stage of development. For it was on account of this point of view that Larmor was unable to separate the field from the matter and to make both of them stand on equal footing. For instance, he states that the displacement current in a body consists of that of the ponderable matter which constitutes the body and that of the ether within this latter.[30] In the same place Larmor also remarks reproachingly that Maxwell has disregarded the physical difference between displacement current of the ponderable matter and that of the ether occupying the same place. But these words rather prove the essential limitation of his conception. It may be concluded that it was because Larmor did not carry the point of view of molecular theory to its extreme that he had to remain at the position that the field was borne by the ponderable matter as well as by the ether.

5. The Theory of Electrons

The origins of Lorentz's theory of electrons has been already considered elsewhere.[31] We therefore need only lay an emphasis on the fact that Lorentz's motive has been deeply rooted in the optics. As has been pointed out in the earlier paper, of numerous papers written during seventeen years of his early scientific career before the theory of electrons of 1892, only two dealt with theoretical problems of electrodynamics. In this period he devoted himself chiefly to the problems of the optics and of the kinetic theory of gases. It was through considerations of the influences of the annual motion of the earth on optical phenomena that he adopted the hypothesis of rest ether which was later to be one of the keystones of his theory of electrons. What is sometimes called his theory of electrodynamics for moving bodies is a theory which aims at discussing influences of the motion of the earth on electrical and optical phenomena. Whereas Hertz took interests in the electromagnetic phenomena within a deformable and compressible moving medium, in Lorentz's theory the motion of the medium was considered always to be uniform translation. Lorentz did never show any interest in dealing with electromagnetic phenomena in a medium doing a non-uniform hydrodynamical motion. Here again we see a sharp contrast with Hertz who confined himself to electromagnetic phenomena in narrower sense.

Lorentz's theory of electrons was laid its foundation in his article "La théorie électromagnétique de Maxwell et son application aux corps mouvants" which appeared in 1892.[32] In this paper, Fresnel's dragging coefficient was derived but the influences of the annual motion of the earth were not dealt with. Latter problems were successively given solutions immediately after this paper was written,[33] and were discussed systematically in his booklet of 1895 *Versuch* *einer Theorie der elektrischen und optischen Erscheinungen in bewegter Körpern.* [34] Lorentz's exposition of the theory in this booklet shall presently be outlined.

Lorentz first postulates the equations for the electromagnetic field in vacuum:

$$\mathrm{div}\, \boldsymbol{d} = \rho \qquad \text{(I)}$$
$$\mathrm{div}\, \boldsymbol{H} = 0 \qquad \text{(II)}$$
$$\mathrm{rot}\, \boldsymbol{H} = 4\pi\rho \boldsymbol{v} + 4\pi \dot{\boldsymbol{d}} \qquad \text{(III)}$$
$$-4\pi c^2 \mathrm{rot}\, \boldsymbol{d} = \dot{\boldsymbol{H}} \qquad \text{(IV)}$$

and the expression for the force exerted upon a unit charge, "the electric force" as Lorentz calls it,

$$\boldsymbol{E} = 4\pi c^2 \boldsymbol{d} + [\boldsymbol{v}\boldsymbol{H}]. \qquad \text{(V)}$$

Here the quantities $\boldsymbol{d}$, called dielectric displacement, and $\boldsymbol{H}$, called magnetic force, represent the deviations of the ether from its state of equilibrium and have been introduced in accordance with Maxwell. Lorentz also follows Maxwell in distinguishing electric force $\boldsymbol{E}$ and displacement $\boldsymbol{d}$ even in the vacuum and putting them in proportion to each other. But he neither proceeds to discuss how to explain this relation by the constitution of the ether nor makes attempt to derive the equations (I) - (V) from certain hypotheses. Though in the article of 1892 Lorentz devoted considerable pages to derivation of equations by dynamical consideration, now in 1895 he no longer pays attention to such a discussion, and rather, like Hertz, takes fundamental equations as a postulate.

Lorentz's purpose was to examine the influences of the annual motion of the earth. He therefore takes a system of coordinates fixed to the body which performs a uniform translation. The velocity of the translation is denoted by $\boldsymbol{p}$ and is assumed to be in the direction of x-axis. He further introduces a quantity defined as

$$\boldsymbol{H}' = \boldsymbol{H} - 4\pi[\boldsymbol{p}\, \boldsymbol{d}], \qquad \text{(VIb)}$$

and writes the electric force exerted upon a unit charge which is at rest relative to the body as

$$\boldsymbol{F} = 4\pi c^2 \boldsymbol{d} + [\boldsymbol{p}\, \boldsymbol{H}],$$

and the velocity of an electric charge relative to the body as $\boldsymbol{v}$. Then (I) - (V) are transformed into

$$\operatorname{div} \boldsymbol{d} = \rho, \qquad \text{(Ib)}$$
$$\operatorname{div} \boldsymbol{H} = 0, \qquad \text{(IIb)}$$
$$\operatorname{rot} \boldsymbol{H}' = 4\pi\rho\, \boldsymbol{v} + 4\pi \dot{\boldsymbol{d}}, \qquad \text{(IIIb)}$$
$$\operatorname{rot} \boldsymbol{F} = -\dot{\boldsymbol{H}}, \qquad \text{(IVb)}$$
$$\boldsymbol{E} = \boldsymbol{F} + [\boldsymbol{v}\, \boldsymbol{H}], \qquad \text{(IVb)}$$

Electromagnetic phenomena in narrower sense are discussed with the aid of these equations. If one supposes a system at rest S' which has the same constitution as the system under consideration S, but is elongated in the direction of x axis by the ratio of $\sqrt{1-p^2/c^2}$: 1, then it can be shown that the electric force E' which operates in the system S' is related with E at the corresponding point in the original system S by equations

$$E_x = E'_x\ ,\ (E_y, E_z) = \sqrt{1-p^2/c^2}\ (E'_y\ , E'_z\ )\ .$$

This result accounts for why the motion of the earth has no influence on electrostatic phenomena. With regard to the mechnical force between a current and an electrified body, the force between two currents, and the electromagnetic induction, it is shown that various effects compensate each other so as to conceal the influences of the motion of the earth insofar as the first order effects are concerned.

Before proceeding to discuss the optical phenomena, Lorentz deduces the macroscopic equations for moving bodies by applying an averaging procedure to equations (Ib) - (VIIb). This is for the purpose of dealing not only with the aberration or Michelson-Morley experiment performed in a vacuum but also with traditional problems concerning experiments that make use of lens-systems. Representing the electric moment per unit volume by

$$\boldsymbol{P} = \frac{1}{V} \sum_{v} e\boldsymbol{r},$$

where e stands for the change of particle and $\boldsymbol{r}$ for the displacement of particle from its position of equilibrium Lorentz defines the quantity

$$\boldsymbol{D} = \bar{\boldsymbol{d}} + \boldsymbol{P}.$$

A bar on the letter signifies the averaged value. The equations finally reached are

$$\operatorname{div} \boldsymbol{D} = 0, \qquad \text{(Ic)}$$
$$\operatorname{div} \bar{\boldsymbol{H}} = 0, \qquad \text{(IIc)}$$
$$\operatorname{rot} \bar{\boldsymbol{H}}' = 4\pi \dot{\boldsymbol{D}}, \qquad \text{(IIIc)}$$
$$\operatorname{rot} \bar{\boldsymbol{E}} = -\dot{\bar{\boldsymbol{H}}}, \qquad \text{(IVc)}$$
$$\epsilon \bar{\boldsymbol{E}} = 4\pi c^2 \boldsymbol{D} + [\boldsymbol{p} \bar{\boldsymbol{H}}], \qquad \text{(Vc)}$$
$$\bar{\boldsymbol{H}}' = \bar{\boldsymbol{H}} - \frac{1}{c^2} [\boldsymbol{p} \bar{\boldsymbol{E}}]. \qquad \text{(VIc)}$$

In deriving these equations, P is assumed to be proportional to the electric force that operates at the point considered: $P = \chi \bar{F}$, a constant quantity $\epsilon = 1 + 4\pi c^2 \chi$ is introduced, and the terms multiplied by $\frac{p^2}{c^2}, \frac{v^2}{c^2}, \frac{pv}{c^2}$ are neglected. Thus the equations (Ic) - (VIc) are of the first order approximation. In this approximation we may write $\bar{E} = \bar{F}$. If one put $p = 0$, the system of equations thus obtained will agree with Maxwell's system of macroscopic equations for the body which is at rest with respect to the ether. Comparing Maxwell's equations with eqs. (Ic) - (VIc) with $p = 0$, we easily see the correspondences between Lorentz's notations and those in common use today:

$D \rightarrow D$:	the electric didplacement,
$\bar{H} \rightarrow B$:	the density of magnetic flux,
$\bar{E} \rightarrow E$:	the intensity of electric field,
$1/4\pi c^2 \rightarrow \epsilon_0$:	the dielectric constant of vacuum,
$d \rightarrow \epsilon_0 E$:	dielectric displacement of vacuum.

The system of units used by Lorentz is a kind of rationalized electromagnetic unit system where $\epsilon_0 = 1/4\pi c^2$, $\mu_0 = 1$. Since he confines himself to non-magnetic substance, $B = H$ by $\mu_0 = 1$.

Now the equations (Ic) - (VIc) are tansformed by introducting the local time

$$t' = t - \frac{1}{c^2}(xp_x + yp_y + zp_z)$$

One denotes the derivatives with respect to t' and to relative coordinates by attaching accent. If one puts

$$4\pi c^2 D' = 4\pi c^2 D + [\, p \bar{H} \,],$$

and neglects the terms of higher order than p/c, one obtains

$$\mathrm{div}' D' = 0 \qquad \text{(Id)}$$

$$\mathrm{div}' \bar{H}' = 0 \qquad \text{(IId)}$$

$$\mathrm{rot}' \bar{H}' = 4\pi \frac{\partial D'}{\partial t'} \qquad \text{(IIId)}$$

$$\mathrm{rot}' \bar{E} = -\frac{\partial H'}{\partial t'} \qquad \text{(IVd)}$$

$$\epsilon E' = 4\pi c^2 D' \qquad \text{(Vd)}$$

The conditions at the boundary between two media are giben by

$$D'_{n(1)} = D'_{n(2)},\ \bar{E}_{t(1)} = \bar{E}_{t(2)},\ \bar{H}'_{(1)} = \bar{H}'_{(2)}.$$

Thus the macroscopic equations for moving bodies expressed in terms of new variables have the same forms as Maxwell's macroscopic equations for bodies at rest. This result immediatedly leads to following theorem of corresponding states:

> If for a system of bodies at rest a state in which D, $\bar{E}$, $\bar{H}$ are certain functions of x, y, z, t is known, then a state in which D', $\bar{E}$, $\bar{H}'$ are the same functions of x', y', z' and t' as above can take place in the same system when it is in motion with velocity p.[35]

A remark must be made here that the two systems, one at rest and the other in motion, have the same dimension, that is, there is no contraction in one of them.

Using this theorem of corresponding states, Lorentz accounts for the fact that no influence of the motion of the earth has been found in phenomena such as the reflection, the refraction, and the interference. Though the absence of second order effect in Michelson-Morley experiment cannot be explained by this theorem, the introduction of the contraction hypothesis removes the difficulty. Moreover, if the intermolecular forces which ultimately determine the configuration of the system of molecules are assumed to be electromagnetic origin or of the same character as electromagnetic action, Lorentz's theory makes it plausible that body in motion must contract in the direction of motion. It was immediately after this result had been obtained that Lorentz proposed the contraction hypothesis on November 26, 1892. Lorentz's contraction hypothesis was, therefore, not merely a kinematical one but a dynamical one.

The theory of electrons is connected today solely with the name of Lorentz. But historically, in addition to his name, at least one more name should be mentioned. It is E. Wiechert, who in 1890's propounded the fundamental ideas of the theory of electrons independently of Lorentz. The German term *Elektronentheorie* was proposed by Wiechert in 1900.

Wiechert's formulation was first given in his article "Über die Grundlagen der Elektrodynamik" published in 1896.[36] But the conceptions underlying this formulation was stated already in 1894 at the meeting of Physico-economical Society of Königsberg.[37] The aim of his article of 1896 was twofold. One is to give a systematization of vector calculus and another is to formulate a theory of electromagnetic phenomena from the molecular point of view. In the former part, the attention was chiefly directed to a classification of vector quantities, and in the latter, the macroscopic electromagnetic equations were derived. What draws our interest is, however, neither the classification of vectors nor the derivation of equations but Wiechert's underlying idea.

Wiechert considers it as the greatest advantage of basing the theory on the molecular structure of matter that thereby "the theory of electrodynamics gains a transparencey and simplicity."[38] He also adduces as an experimental ground the existence of electrified atoms or groups of atoms in matter revealed by the studies of electrolysis. He further emphasizes that all the actions of electromagnetic field on matter can be formulated on the basis of hypotheses that the matter consists of charged particles and the electric current is no other than a flow of charged particles.[39)] In Wiechert's view, though this hypotheses has sometimes been mentioned by that time, its important significance that one could build whole electromagnetic theory on this hypothesis alone has not yet fully recognized.

On the other hand, as to the electromagnetic field he advanced the following view. Adducing the aberration and Fizeau's experiment that confirmed Fresnel's dragging coefficient, he states that the electromagnetic field is independent of any motion of matter and can be supposed to be at rest everywhere, even within a moving body. In Wiechert's conception, therefore, the electromagnetic field is distinctly separated from matter. Now that the field and the matter are independent from each other, the modification of the propagation of light by the presence of matter should be an indirect, secondary effect. The intermediary of this effect is the charged particle that constitutes the matter. Then arises the question what is the conception which conforms with the independence of the field and the matter from each other. Wiechert considered the perceptible matter as not being the bearer of the field and assumed as the bearer the ether which pervaded the matter but at the same time did not share any motion of it.

As clearly seen from above, the fundamental ideas of Wiechert's theory largely agrees with that of Lorentz's theory. Though Wiechert's theory lacks the treatment of the problems concerning the relative motion of the earth to the ehter, the name of Wiechert as a founder of the theory of electrons should not be neglected. In fact, Wiechert developed his ideas independently of Lorentz's theory. In Wiechert's paper of 1896, no mention is made about Lorentz. He seems to have been unacquainted with Lorentz's *Versuch* by that time, not to have ignored it. In the paper written in 1898 "Hypothesen für eine Theorie der elektrischen und magnetichen Erscheinungen",[40)] Wiechert remarks that his theory agrees in many respects with Lorentz's theory. In the paper of 1900 "Elektrodynamische Elementargesetze",[41)] where the name *Elektronentheorie* was proposed for the first time, he said that Lorentz was the first to advance the theory of electrons.

He had two purposes in this paper of 1900. The first was to present the fundamental ideas of the theory of electrons. The second was, as the title tells, to derive the *Elementargesetze* and to discuss its applications. Here the Elementargesetze means the so called Liénard-Wiechert potential which gives the field produced by a moving charged particle. In passing, it was earlier than Wiechert, in 1898, that Liénard obtained this potential.[42] And Liénard too was an advocate of the theory of electrons who tried to promote Lorentz's theory in France.

There still remains one point to be mentioned concerning Wiechert's theory of electrons. It is Wiechert's experimental study on the cathode rays, which he was induced to perform by this theory. He writes in his paper of 1898 mentioned above as follows: When he wrote the earlier paper of 1896, he did not enter into details of the subject because it was pure hypotheses that the atom was divisible, but after that, being guided by his own theory, he has turned to experiments of the cathode-rays and on January 1, 1897, could report that the cathode rays consisted of particles having the mass 1/2000 - 1/4000 of that of the hydrogen atom. This is the result which Schuster, in his *Progress of Physics*[43], has appreciated as having concluded, prior to J. J. Thomson, that the cathode-ray particle is considerably smaller than atom. Wiechert says in his paper of 1898 that though at first many objections were raised to his and Thomson's result, subsequent efforts of experimenters have removed fundamentally all the objections and moreover support from the Zeeman effect has been added.

6. Criticisms to the Three Theories

In 1895 H. Poincaré, being prompted by Larmor's first paper mentioned in §4, published a series of considerations under the title "A propos de la théorie de M. Larmor."[44] This article was afterwards incorporated into his treatise *Electricité et optique* (1901, Paris) as its last chapter. Though the title bears the name of Larmor, the text is not confined to considerations of Larmor's theory. Various electromagnetic theories are examined and compared with each other.

Poincaré first considers, following Larmor, the problem of extending the theory of optics to the domain of electromagnetic phenomena. As a goal to be reached by extended theory, Poincaré adopts Hertz's formulation of electromagnetic theory on the ground that Hertz's theory reveals most clearly the analogy between optics and electrodynamics.[45] Now, there were two systems of optical theory, those of Fresnel and of F. Neumann. In Fresnel's theory, the direction of oscillation of the ether is taken as perpendicular to the plane of polarization and the difference of refractive indices of various media are accounted for by the differences in the densities of the ether within media. In Neumann's theory, the oscillation of the ether is assumed to take place in the plane of polarization and the refractive index is determined by the elastic modulus of the ether within the medium. Comparison of these theories with Hertz's theory reveals that the vector representing the velocity of each point of the ether in Fresnel's theory corresponds to electric force, and similar vector in Neumann's theory corresponds to the magnetic force. It follows from this that Larmor's theory is no other than an electromagnetic adaptation of Neumann's theory.

The most general and fundamental difficulty of Larmor's theory, as Poincaré points out, is the following. Since in Larmor's theory, as mentioned above, the intensity of magnetic field represents the velocity of the ether at each point, if there is a constant magnetic field, the ether will continue to flow stationarily and the displacement of each point of the ether will increase without limit in proportion to time. But this contradicts the fundamental assumption of the theory of elasticity that the displacement of each point is considered to be small.

Though, strictly speaking, there is no theory of elastic ether that explains satisfactorily all the phenomena, Fresnel's theory can be considered as most satisfactory if one ignores the dispersion. But the electromagnetic adaptation of Fresnel's theory also involves similar difficulties as above. In this case, the velocity of the ether is represented by the vector of electric force. If one imagines a spherically symmetric electrostatic field around a point charge, there must always be a flow of the ether to (from) the central charge and the density of the ether at the position of the charge will increase (decrease) indefinitely. The ether then must have an infinite compressibility. This might be favourable for the elimination of longitudinal waves, but is quite unreal in other respects. After all, whichever theory, Fresnel's or Neumann's, one would adopt, the ether must be endowed with quite strange properties. But Poincaré, though emphasizing this strangeness, does not demand to abandon the mechanical conception of the ether on this account. He avoids to draw decisive conclusion merely saying "It is advisable in every case to insist on these strangeness, whether one would like to familiarize his mind with them or to regard them as insurmountable obstacles which would not allow us to adopt these explanations."

Poincaré next considers the electrodynamics for moving bodies with special attention to the conservation of electric charge, the principle of action and reaction, and Fizeau's experiment.

As for Hertz's theory, Poincaré first remarks that since Hertz's equations are settled on some audacious reasonings, it is quite doubtful that it might be accepted. Concerning the conservation of electric charge it has no difficulty because it confirms that the true electric and magnetic charges are accompanied by the ponderable matter in its motion. The theory also shows that the ponderomotive force operating in an electromagnetic field satisfies the principle of action and reaction. Furthermore it is proved that, under certain general restrictions, no other theory than Hertz's one could be compatible with both these two conditions. But when dealing with the propagation of light in moving media, it leads to full dragging and consequently contradicts Fizeau's experiment. On this account Poincaré draws conclusion that Hertz's theory should be modified.

On the other hand, Lorentz's theory gives the correct Fresnel's coefficient and assures the conservation of electric charge, because in this theory the electric charge is considered to be carried by the minute particles which constitute the matter. But it violates the principle of action and reaction. Besides Hertz's and Lorentz's theories, the theories of the propagation of light in dielectrics by Helmholtz and J.J. Thomson are also examined and shown to be unsatisfactory. Helmholtz's theory violates the conservation of charge and Thomson's one violates the principle of action and reaction. Poincaré asserts that any theory which accords with Fizeau's experiment will necessarily violate the principle of action and reaction.[50] Consider, for example, an electrified sphere placed in an electric field. It suffers an electric force and at the same time must exert a reaction to this force. But, what is the substance which would receive this reaction? According to Hertz's theory, there is no necessity to distinguish the ether and the ponderable matter within a material body since the ether always accompanies the body. Hence the above question can be answered in such a way that the reaction acts on the air surrounding the sphere. But in order that a theory should accord with Fizeau's experiment, the complete dragging ought to be denied. An explanation like above, therefore, becomes impossible. Poincaré rejected the answer that the reaction was exerted upon the ether. The reason for this was that it was only upon ponderable body that we can experimentally make a force to operate. This argument is very remarkable since it shows that Poincaré opposes to admit the ether being a mechanical substance which stands side by side with the ponderable matter.

Since all existing theories have thus been shown to be imperfect in some respect, there is no other way than to choose one that is least defective. Lorentz's theory was that which Poincaré chose.[51] He, on the other hand, considered it a "grave difficulty" that Lorentz's theory did not satisfy the principle of action and reaction. His criticism to this defect of Lorentz's theory was so firm that thenceforth it was repeatedly expressed by him. Then, what was the reason that he dared to choose Lorentz's theory as most favourable? Poincaré gives no explicit mention. But in view of the importance Poincaré attached to Fizeau's experiment, we may infer that the reason was that it gave most plain explanation of this experiment. As we shall see later, those who investigated the electrodynamics of moving bodies about 1900 all took a serious consideration of Fizeau's experiment and demanded modification or rejection of Hertz's theory for the reason that it contradicted this experiment.

Poincaré at this stage, however, is not conclusively recommending Lorentz's theory. He says that Lorentz's theory, though it is able to render certain services to us, "can neither meet completely our demands nor be regarded as definite one." [52] This is, first of all, because it violates the principle of action and reaction. Furthermore, Poincaré says, it is not perfect even in the proper domain of optics. For, the Michelson-Morley experiment has proved that the second order effect too could not be detected, whereas Lorentz's theory only explains the absence of the first order effects.

Succeeding to Poincaré, A. Liénard published in 1898 two articles dealing with Lorentz's theory. [53] A criticism Liénard made in one of them motivated Lorentz to improve his theory of 1895.

The aim of Liénard's articles was primarily to present in a simplified manner the gist of Lorentz's theory. He considered the gist of Lorentz's theory as consisting in the hypotheses of rest ether and the interpretation of electric displacement in the interior of matter. In the first paper, he presented the reconstruction of the theory on the basis of these two and derived the macroscopic equations for electromagnetic phenomena in moving bodies. He then showed that Fresnel's coefficient could be obtained from them and examined the principles of conservation of energy and of action and reaction. The energy principle is satisfied but the principle of action and reaction is, as Poincaré pointed out, violated. But Liénard does not attach so much importance to this result as Poincaré did. Even though, he says, a result is obtained that the total sum of the forces which all parts of the body exert upon each other does not vanish, one need not be surprised by this, because once action at a distance has been abandoned, the action and the reaction are no longer simultaneous. Rather, he asserts, since the time average of the resultant force is proved to vanish, the principle of action and reaction is satisfied in the sense of average. Poincaré stressed that in Hertz's theory this principle was satisfied. But Liénard does not regard this as showing the superiority of Hertz's theory to Lorentz's one. For, since in Hertz's theory there is no essential difference between the ether and the common matter except the numerical values of the dielectric constant, one cannot make any definite assertion about the principle of action and reaction without taking account of the forces acting on the ether all over the whole space. This argument shows that Liénard has recognized that in Lorentz's theory the field is separated from the matter owing to the notion of rest ether.

In this second paper, Liénard considered Lorentz's *Versuch* of 1895 and made a criticism against the explanation of Michelson-Morley experiment given therein. He also compared some results from Larmor's theory with those from Lorentz's theory. Liénard obtained equations for moving body retaining terms to the second order, and using them showed that a moving body having dielectric constant which differed from 1 became doubly refractional having the direction of motion for the optical axis. From this result he drew a criticism to Lorentz's contraction hypothesis. The experiment performed in the air may be accounted for by this hypothesis because the dielectric constant of the air is approximately 1 and practically no double refraction arises. But if part of the light path is laid in a material body such as water or glass, the difference of the speeds of ordinary and extraordinary rays must produce some effects which the contraction hypothesis is unable to eliminate. Lorentz's reply to this criticism was given in the next year (See next section).

As for Larmor's theory, Liénard points out that it involves not a few difficulties and that different parts of Larmor's paper are not without some mutual contradictions. But he also remarks that what is of great interest in Larmor's papers are the original views which, if they don't give any entirely satisfactory explantion of phenomena, at least show possible directions in which one might except a way to success. However, the views indicated by Liénard cannot be said truly hopeful, because they are such as the use of Carnot's principle in deriving Fresnel's coefficient, the explanation of the independence of the speed of electric current from the mass of electrons, and the like.

7. The Evolution of Lorentz's Theory

In 1899 Lorentz wrote a paper entitled "A simplified theory of electrical and optical phenomena in bodies in motion"[54], where a simplified formulation that led to the same result as in *Versuch* of 1895 was given, and moreover, the insight that the second order effects too would never be detected was obtained in connection with the reply to Liénard's criticism. In passing, the term *electron* was used for the first time by Lorentz in this paper. In *Versuch* of 1895 he called the charged particle contained in molecule "ion", and in his papers concerning Zeeman effect written in 1898, no use was made of the term electron.

The reason for calling new theory "simplified" seems to be that it reaches directly the wanted result without making detour through macroscopic equations as was done in *Versuch.* The starting point is again the equations for the field in vacuum. Assume the velocity of the body to lie in the direction of x-axis, and let x, y, x the relative coordinates in the system of reference that moves with the body. Putting

$$\frac{1}{\sqrt{1-p^2/c^2}} = k,$$

following new variables are introduced:

$$x' = kx,\ y' = y,\ z' = z,\ t' = t - k^2px/c^2,$$
$$F' = \{ 4\pi c^2 d_x,\ k^2(4\pi c^2 d_y - 4\pi[\boldsymbol{pH}]_y),\ k^2(4\pi c^2 d_z - 4\pi[\boldsymbol{pH}]_z)\},$$
$$H' = \{ kH_x,\ k^2(H_y + 4\pi[\boldsymbol{pH}]_y),\ k^2(H_z + 4\pi[\boldsymbol{pd}]_z)\}.$$

Then the equations are transformed by means of these new variables. From the equations obtained it is shown that with respect to electrostatic phenomena there is a correspondence of states between the moving body and a body at rest which is elongated by the ratio $1 : k$ in the direction of x-axis, and that the electromagnetic effects of first order in p/c compensate each other and consequently are not detectable.

The optical phenomena are treated as follows. Since both the range and the velocity of the oscillation of the electron in molecule, which is the sole motion of electron to be considered, are small, the H produced by the motion of electron too is small. Accordingly, not only the terms multiplied by p^2 but also those multiplied by pv, vH as well are neglected. Here v denotes the velocity of electron with respect to the relative coordinate system. In this approximation k may be put = 1, and the F' introduced above agrees with the force E acting on the electron. Now regarding the electron as a point, only the translational motion is considered. Let F_0' be the solution of the equations in the case that the electron is at rest at the position of equilibrium in molecule. Since F_0' does not, by definition, contribute to the motion of electron, what is essentially effective is the difference of general solution F' and this F_0' : $F_1' = F' - F_0'$. The equations to be satisfied by the effective F_1' is obtained as follows:

$$\mathrm{div}'\, \boldsymbol{F}_1' = -4\pi c^2(\boldsymbol{a}\ \mathrm{grad}')\rho_0, \qquad \text{(Id)}$$
$$\mathrm{div}'\, \boldsymbol{H}' = 0 \qquad \text{(IId)}$$
$$\mathrm{rot}'\, \boldsymbol{H}' = 4\pi\rho_0 \frac{\partial \boldsymbol{a}}{\partial t'} + \frac{1}{c^2}\frac{\partial \boldsymbol{F}'}{\partial t'}, \qquad \text{(IIIc)}$$
$$\mathrm{rot}'\, \boldsymbol{F}_1' = -\frac{\partial \boldsymbol{H}'}{\partial t'}, \qquad \text{(IVd)}$$

where a denotes the displacement of electron and ρ_0 the density of electric charge at the position of equilibrium. These equations are of similar form to those for the body at rest. From this similarity follows the same theorem of corresponding states as that in *Versuch*. As has been remarked in § 5, both the rest system and the moving system that are in correspondence by this theorem have same dimension.

Lorentz then refers to Liénard's criticism and, in order to refute it, considers the probability that the second order effect too would not be detected. For this purpose he introduces following doubly accented quantities:

$$x = (\epsilon/k)x'', \; y = \epsilon y'', \; z = \epsilon z'', \; t' = k\epsilon t'',$$
$$F' = F''/\epsilon^2, H' = (k/\epsilon^2)H'',$$
$$\rho_0 = (k/\epsilon^3)\rho_0'',$$
$$a_x = (\epsilon/k)a_x'', \; a_y = \epsilon a_y'', \; a_z = \epsilon a_z''.$$

As is immediately seen, if ϵ is put = 1, these equations give the correct Lorentz transformation for x, y, z, t, and H. But Lorentz does not proceed to show that these new variables transforms the equations into the exactly same form as those for rest bodies even when higher terms than p/c are included. If he had done so, he might have immediately concluded the non-detectability of the effects higher than first order. In deriving previous equations (Id) - (IVd) he neglected several 2nd order terms. This approximation is here still kept unchanged. Then what for the doubly accented quantities? They were introduced in order to show that Lorentz contraction was not a conventional fiction but a necessary requisite for the correspondence of states. In the theorem of corresponding states obtained in *Versuch*, there was neither elongation nor contraction of corresponding moving or rest body. But with the above transformation, one obtains equations having completely same form as those for rest body except for E being replaced by F''. E and F'' differ by following factors:

$$E_x = \frac{1}{\epsilon^2}F_x'', \; E_y = \frac{1}{k\epsilon^2}F_y'', \; E_z = \frac{1}{k\epsilon^2}F_z''.$$

From this result follows the correspondence of states between a rest system S_0 in which x'', y'', z'' and t'' represent the true position and true time and a moving system S in which the true position and true time are represented by x, y, z, t. But the dimensional relation between S_0 and S just agrees with Lorentz contraction. Finally since this correspondence of states has been concluded independently of the medium through which the light passes, the criticism raised by Liénard does not apply itself to this conclusion.

But it must again be noted that Lorentz explains the absence of influences of the motion of the earth not by showing the covariancy of Maxwell's equations to higher orders. He always remained faithful to approximative approach, and satisfied himself with confirming the necessity of the contraction. This probably stemmed from his fundamental conviction that the experimental result concerning the relative motion of the earth to the ether should be explained as a consequence of the compensation of opposing effects. He did not grasp the matter as one concerning a universal principle of physics.

8. Acceptance of Lorentz's Theory

The impression that Lorentz theory was possibly the most promising one began to spread about the turn of the century. The first example to be adduced is G. T. Walker's monograph *Aberrations and Some Other Problems Connected with the Electromagnetic Field* (Cambridge, 1900) to which the Adams Prize was awarded in 1899.

Walker considers the "polarization" D and B, following Maxwell and Larmor, as being borne both by the ether and ponderable matters and classifies electromagnetic theories according to the interpretations of the "polarizations" in the matter and the ether respectively. He calls *tubular* the point of view of representing the "polarization" by abstract lines of force and *molecular* that of picturing the "polarization" by means of displacements of particles. According as one considers the "polarization" as tubular or molecular, the time rate of variation in lines of force should be reckoned differently and accordingly the equations for moving media would assume different shape.

Hertz's theory is a kind of those which consider the "polarization" as tubular both in the ether and in the matter. This theory fails in explaining the aberration. A theory also can be conceived that considers both "polarizations" as tubular and at the same time assumes the ether to be rest not accompanying body's motion. But such a theory is not able to explain Fizeau's experiment. If one assumes partial dragging as Fresnel did, then the theory becomes identical with Lorentz's theory. The view that both "polarizations" are tubular must therefore be rejected. The second point of view that the "polarization" in the ether is tubular while that in the matter is molecular leads to the result that agrees with Lorentz's theory. The theory based on this view explains the influences on optical phenomena of the motion of the earth as well as the experimental results concerning Röntgen current. The theory derived from the third point of view that both "polarizations" are molecular turns out to contradict the experimental fact concerning the electric force acting on a conducting wire moving through a magnetic field.

Walker's discussions extend over wide range of theoretical and experimental problems, of which only the cardinal part has been mentioned in the above. From those discussions Walker drew conclusion that the most advantageous was the theory which considers the "polarization" in the ether as tubular and that in the matter as molecular, or definitely speaking, Lorentz's theory. He closes his essay with following words:

> "The evidence afforded by chemical and magneto-optical phenomena in favour of an interpretation in terms of ions is extremely strong; and a comparison of the result of Part IV with those of Parts II, III will give that evidence additional weight."[55]

But Walker's conception of the electromagnetic field, throughout this monograph, cannot be said as clear and definite as Lorentz's. For him the ether is a dielectric medium as well as ponderable matters are. They are distinguished by him only with respect to the interpretation of the "polarized" state arising in them. The field is, therefore, not made completely independent from the matter. Walker might have recommended Lorentz's theory only in the sense that the electric and magnetic phenomena in a material body should be explained by the presence of charged particles in the ponderable matter.

In the same year 1899 as Walker wrote above mentioned monograph, Poincaré in France was lecturing at Sorbonne on the theory of optical phenomena in moving bodies.[56] As early as in 1895, in the article "A propos de la théorie de Larmor" mentioned in §6, he wrote: "the experience has revealed ------ it is impossible to make the absolute motion of matter, or rather the relative motion of ponderable matter to the ether, manifest itself. All that can be evidenced is the motion of ponderable matter relative to another ponderable matter".[57] In the lecture of 1899 at Sorbonne, he stated that it was highly probable that the optical phenomena depended only on the relative motion of ponderable bodies to each other, and asserted that a well worked out theory should prove this not by approximation but with complete strictness. He recommended Lorentz's theory as the most nearest to this goal:

> "Lorentz's theory has not yet attained this. Of all the theories which have been proposed, it is the one which most likely will attain this. One can therefore expect that it may be rendered perfectly satisfactory without profound modifications."[58]

In the following year he delivered an address[59] at the International Congress on Physics held at Paris, in which, while pointing out the grave difficulty of violation of the principle of action and reaction, he appreciated the great capability of

Lorentz's theory in treating the optical phenomena in moving bodies, theory of refraction, Zeeman effect, Faraday effect, and so forth. He says:

> "The most satisfactory theory is that of Lorentz; it is unquestionably the theory that best explains the known facts, the one that throws into relief the great number of known relations, the one in which we find most traces of definitive construction. -------- Lorentz's theory is not a mere artificial combination which must eventually find its solvent. It will probably have to be modified, but not destroyed".[60]

The appreciation of Lorentz's theory drawn from theoreitcal considerations was largely enhanced by the experimental establishment of the electron.

The electron is commonly said to be discovered experimentally in 1897. This is mainly due to the fact that in this year, J. J. Thomson determined the specific charge of the cathode ray particle, and at the same time Lorentz's theory of Zeeman effect which had been discovered in previous year was perfectly confirmed, and moreover the specific charge of luminiferous particles was shown to be of the same value as that for the cathode ray particle. But historically, the existence of particles which are much smaller than an atom was not immediately recognized publicly in this year 1897. Doubts were expressed as to the smallness of the particle. The existence of electron would not be recognized commonly until the individual values of the charge and the mass of electron, not the specific charge, were measured separately and the mass of electron actually was proved to be extremely smaller than that of hydrogen atom. It was in 1899 that J. J. Thomson's attempt to measure the charge of electron achieved a reliable result and physicists accepted his assertion. In the following year, 1900, the β rays emitted from radioactive substances were proved to consist of same particles as cathode rays. And in the next year, 1901, W. Kaufmann proved experimentally that the mass of the β ray particle depended on its velocity. Thus it may be said that about 1900 the electron became a confirmed real thing for physicists.

As we have seen in §5, Wiechert wrote in 1898 that the fundamental conceptions of the theory of electrons had not yet been generally recognized.[61] But in W. Wien's paper "Über die Möglichkeit einer elektromagnetischen Begründung der Mechanik" which was contributed to the celebrating volume, published in 1900, for the silver julilee of Lorentz's doctorate[62], we find a passage: "it is hardly a special assumption but is now approved by almost every physicist that the matter consists of such charged particles".[63] The change of climate of opinion during these two or three years must have been caused obviously by the experimental establishment of electron.

Wien's paper cited above was to propose the so called electromagnetic view of nature. His proposal was that since all the attempts to construct the electromagnetic theory on the basis of mechanics had finally proved the inadequacy of mechanics for expressing the electromagnetic processes, one should rather make attempt to derive mechanics from the electromagnetic equations considered as the fundamental of the world picture. In order to carry out this programme he made hypothesis that the matter consisted solely of positively and negatively charged particles. M. Abraham's paper "Dynamik des Elektrons" published in 1902[64] was an attempt to substantiate Wien's proposal, in which he discussed the velocity dependence of the electromagnetic mass of electron. Referring to Lorentz and Wiechert he wrote at the opening of this paper:

> "the so called theory of electrons is superior to all other theories of electrodynamics insofar as it embraces not only the propagation of light through moving bodies but the cathode rays as well."

We also see the Largrangian formulation given for the first time by K. Schwarzschild in 1903[65] be based on the Lorentz's formulation of Maxwell theory.

Superiority of Lorentz's theory with regard to the electrodynamics for moving bodies was emphasized by W. Wien. He wrote at the beginning of his paper "Über die Differentialgleichungen der Elektrodynamik für bewegte Körper" published in 1904:[66]

> Of hitherto formulated differential equations that have enlarged Maxwell's theory so as to include the case of moving bodies, that of Lorentz has been proved to be the best. Hertz's theory, according to which the bearer of electromagnetic actions moves with same velocity as the matter, is refuted by the Fizeau's experiment repeated by Michelson and Morley."

As for Cohn's theory, which will be discussed in the following section, he remarked that it had grave difficulties such that the speed of propagation of light depended on the velocity of light source, although it had superiority to Lorentz's theory of being able to explain Michelson's interference experiment without framing new hypothesis. He also appreciated Lorentz's theory from more general point of view. He observed that most of the difficulties in the electrodynamics of moving bodies arose from attempting to separate the ether from the matter while attributing, at the same time, mobility to the ether. But, he argued, if one adopted Lorentz's hypothesis according to which interactions between the ether and the matter were caused solely by the elementary charges of atoms, all the difficulties would disappear, and moreover, the equations for moving bodies would be derivable from Maxwell's equation without any additional hypothesis.

Thus it may be concluded that Lorentz's theory became to find broad acceptance shortly after 1900. Its remaining defect concerning higher order effects, which Poincaré stressed, was removed by the final formulation of his theory that Lorentz arrived at in 1904.[67] Lorentz could prove the correspondence of states that was strict in every order of the ratio p/c by making full use of doubly accented variables which were introduced at the last part of the paper of 1899, and by abandoning all extraneous approximations which had been retained there. This result utterly satisfied Poincaré's awaited requirement and solved completely the problems of relative motion of the earth to the ether as formulated at that time. Lorentz's theory was thus thought to have achieved a position of authenticity.

9. Cohn's Electrodynamics

We have seen in the previous section that Lorentz's theory became to be broadly accepted about 1900. But even in that period theories that inherited Hertz's standpoint did by no means disappear completely. The reason that Lorentz's theory was considered to be most promising was not only its capacity of explaining varied phenomena but rather its transparency and consequentiality as a theoretical scheme. This may be said to be the very reason that Lorentz's theory were appreciated by those such as Poincaré or Wien and Abraham, who examined thoroughly the fundamental problems of theoretical physics at that time, or intended to frame a new unified picture of nature. But on the other hand, forthose who attached importance to the treatments of separate problems, Lorentz's theory was not yet perfectly convincing. Comparing explanations by various theories of separate phenomena, they hardly could form final conclusion as to which one was the best. It was by those people that Cohn's electrodynamics inheriting Hertz's view was rated highly.

Cohn's theory was propounded during from 1900 to 1904. Here it is said to inherit Hertz in the following sense. Firstly, after the standpoint of Hertz's electrodynamics, it regards the "polaization" and the intensity of field as being borne by dielectrics including the ether, and consequently the field is not separated from the matter. Secondly, it rejects the idea to base the theory on the molecular structure of matter. Mechanical interpretation of electromagnetic phenomena is rejected as well. But Cohn could not disregard the fact that Lorentz's theory had got success at problems where Hertz's theory had failed. Cohn's aim was therefore to modify Hertz's formulation so as to embody into it the successful part of Lorentz's theory.

Cohn's equations were given for the first time in his paper "Über die Gleichungen der Elektrodynamik für bewegte Körper"[68] which was contributed to celebration volume for Lorentz referred to in § 8. Cohn first enumerates fundamental experimental facts concerning electromagnetic phenomena in moving bodies. Of those phenomena, electromagnetic phenomena in narrower sense (those not involving the propagation of electromagnetic wave) depend solely on the relative motion of ponderable bodies to each other. As for the radiation four facts are mentioned: a) Absolute propagation through media that are electromagnetically indistinguishable from void space, such as the vacuum or the atmosphere, is not affected by motion --- the aberration. b) In ordinary transparent body, Fizeau's experiment indicates a dragging coefficient $(1 - 1/n^2)$. c) The laws of reflection and refraction are not affected by the earth's motion. d) Michelson-Morley's experiment. Now Hertz's theory accords with c) and d), but contradicts a) and b). On the other hand, Lorentz's theory accords with all the fact in the first order approximation, but, if one takes into account second order effects, it contradicts d).

Cohn therefore attempted to find out such equations that would agree in the first approximation with Lorentz's ones and would give the optical length of light rays travelling along closed path an expression that would not be altered by the motion. For this purpose he sought for a system of equations which, when the local time $t' = t - \epsilon_0\mu_0(u_x x + u_y y + u_z z)$ was introduced, would be transformed into strictly same form as Maxwell's system of equations for rest bodies:

$$\mathrm{rot}'\, H = \frac{\partial(\epsilon E)}{\partial t'},$$

$$-\mathrm{rot}'\, E = \frac{\partial(\mu H)}{\partial t'},$$

$$\mathrm{div}'\,(\epsilon E) = 0, \quad \mathrm{div}'\,(\mu H) = 0.$$

Here ϵ_0 and μ_0 denote the electric and magnetic inductive capacities of the vacuum and $\epsilon_0\mu_0 = c^{-2}$, and u_x, u_y, u_z represent the components of velocity of the body. By simple calculations, the system of equations satisfying above requirement is found to be

$$\left.\begin{aligned} &\mathrm{rot}\, H = \frac{\partial D}{\partial t}, \quad D = \epsilon E - \epsilon_0\mu_0[\,u H\,], \\ &-\mathrm{rot}\, E = \frac{\partial B}{\partial t}, \quad B = \mu H + \epsilon_0\mu_0[\,u E\,], \\ &\mathrm{div}\, D = 0, \quad \mathrm{div}\, B = 0 \end{aligned}\right\} \quad (A)$$

in the system of coordinates fixed to the body. The units used here is indetermined apart from the condition $\epsilon_0\mu_0 c^2 = 1$. In order to get expressions in Gaussian units, one should put $\epsilon_0\mu_0 c^2 = 1/c^2$ and replace B by B/c, and H by cH. Cohn's peculiar notations for operators and vectors are replaced by those in common use today. Once the equations (A) are postulated as the equations for moving bodies, then the theorem of correspondence of states will hold good not approximately but exactly. This is to be rightly expected in view of the way of derivation of (A). It should be remarked that here too is no elongation nor contraction of systems in correspondence.

The equations (A) were for a body moving with uniform and constant velocity u, and were written in the frame of reference that was fixed to the moving body. In the paper "Ueber die Gleichungen des elektomagnetischen Feldes für bewegte Körper" published in the next year, they were generalized to a body moving with arbitrary velocity which varied point to point and was not constant in time. The generalized equations, written in the frame of reference which is rest with regard to fixed stars, are:

$$\left.\begin{aligned} -\mathrm{rot}(E-[uB]) &= \partial B/\partial t + u\,\mathrm{div}B, \\ \mathrm{rot}(H+[uD]) &= \partial D/\partial t + u\,\mathrm{div}\,D + i, \\ D=\epsilon E-\epsilon_0\mu_0[uH],\quad & B=\mu H+\epsilon_0\mu_0[uE]. \end{aligned}\right\} \qquad (B')$$

That this system of equations is the generalization of (A) will be easily seen if one considers u in these equations as uniform and constant and transforms (B') to a system of coordinates that moves with the body. In that case (B') reduces to (A).

Let the velocity of the earth be p and write $u = p + v$. Imagine the case $v = 0$, that is, the case when all the bodies on the earth are in a state of relative rest to the earth and to each other. Then the conclusion will be drawn from the theorem of correspondence of states that if, in a system which is rest with regard to fixed stars, the field at a point P and at an instant t is represented by E and H, then within a moving body a process is possible to take place by which a field represented by the same E and H is produced at each point at an instant $t' = t - p\cdot r/c^2$. t' differs from point to point. Now, if the time required by the light to propagate from one point P_1 to another point P_2 is written as $t_2' - t_1'$ in the moving system and as $t_2 - t_1$ in the rest system, then from the conclusion just obtained we have

$$t'_2 - t'_1 = t_2 - t_1 + p(r_1 - r_2)/c^2.$$

Hence, though the time required by the propagation of light indeed increases, the increase of time interval; $p(r_1 - r_2)/c^2$ depends only on the positions of P_1 and P_2 and not on the path of light. It is thus concluded that the motion of the earth does not bring about any change in the interference pattern. This will explain the Michelson-Morley experiment.

Cohn's explanation was criticized in 1903 by Lorentz in latter's article

"Weiterbildung der Maxwellschen Theorie: Elektronentheorie", which was written for *Encyklopädie der mathematischen Wissenschaften*, Bd. V, 2. Teil.[70) Lorentz argued as follows: Cohn's theory was intended to treat electromagnetic processes in moving ponderable bodies and its basis of explaining Michelson-Morley experiment was the conclusion that the interference pattern in a moving body did not differ from that in a rest body. The explanation of Michelson-Morley experiment is, therefore, possible only when the light passes through some ponderable media. In the case of experiment performed on the surface of the earth, the atmosphere might be considered as the moving ponderable body required. But if the experiment were performed in a vacuum, there would be no ponderable body through which the light would have to pass, and consequently Cohn's above argument could not be applied. Contrary to this, Lorentz's own theory leaves no room for such a criticism, because it admits nothing of macroscopic ponderable body, which has been decomposed into the vacuum and electrons, and consequently the light always passes through the vacuum (the ether).

Lorentz's criticism should be said to have hit upon the very shortcoming of Cohn's theory that the field was not made independent of the matter and remained being borne by the matter. Cohn's reply to this was given in his paper of 1904 "Zur Elektrodynamik bewegter System."[71) For the purpose of defending his explanation, he adduced Maxwell's experiment, carried out in 1866, on the viscosity of gases. The experiment proved that the coefficient of viscosity of gases did not depend on the density of the gases. But this result no doubt would be invalid at very low density. Similarly, argues Cohn, his whole argument will become invalid when the air becomes extremely rare, since the concept of bodily velocity of the air then loses its significance. In such a case, therefore, certain effect would be detected. And he concluded that on the ground of physical experiences we could not decide ourselves in favour of either standpoint, to assume contraction as Lorentz had done, or to explain by influences of the air as Cohn had done.

Cohn's paper of 1904 was, in the first place, intended to show that Lorentz's theory of 1904 agreed in its results with Cohn's theory. Since Lorentz's equations are for the microscopic charged particles, in order to compare it with Cohn's theory it is required to derive macroscopic equations by applying averaging procedure. Taking

$$x' = kx, \quad y' = y, \quad z' = z, \quad t' = t/k,$$

with

$$k = 1/(1 - u^2/c^2)$$

as the independent variables, where x, y and z are the coordinates in a system fixed to the earth, and putting

$$E' = (1, k, k)\{E + [uH]\},$$
$$H' = (1, k, k)\{H - [uE]\},$$

we obtain the transformed equations

$$\mathrm{rot}'E' = i + \partial D/\partial t' + u\,\mathrm{div}'D - \mathrm{rot}[uD],$$
$$\mathrm{rot}'H' = -\partial B/\partial t' - u\,\mathrm{div}'B + \mathrm{rot}'[uB],$$
$$\mathrm{div}'D = \rho',\ \mathrm{div}'B' = 0,$$

where

$$D = \epsilon E' - [uH'],\ B = H' + [uE'].$$

These equations have the same form, apart from their bearing accents, as Cohn's (B') specialized to the present case.

Having obtained above result, Cohn set forth remarkable discussions. They are remarkable because in some respects they remind us the point of view taken in the theory of relativity. Cohn first remarks that though two theories lead to equations of the same form, the interpretations thereof differ from each other. According to Lorentz's theory, the intensities of field are to be represented by E and H, and the problem is to find E and H as functions of x, y, z, and t. Quantities E', H', x', y', z', and t' introduced into above equations are considered merely to be subsidial quantities for the sake of calculation. On the other hand, in Cohn's theory all that appear in the equations are the true intensities of field which are functions of the true time and the true space coordinates. In Lorentz's theory, Cohn points out, it has been required to distinguish subsidial quantities x', y', z', and t' from the true variables x, y, z, and t but the theory fails to indicate any method to distinguish them experimentally. Furthermore he even asserts that the so-called *true* time and *true* length have no more meaning than that certain form of equations of electrodynamics would ensue when they are taken as independent variables.[72)] From this to reach the repudiation of the apriority of space and time, only a step forward would be sufficient.

Cohn derived his equations from the requirement that the equations when transformed to a system of coordinates that was fixed to the moving body, should assume the same form as for the rest body. But this requirement of covariancy was not clearly stated as the basic principle, and moreover has disappeared from Cohn's second and subsequent papers. The equations are put forward from the outset as, so to speak, a hypothesis. Owing to this, Cohn's equations gave impression that they had only flimsy ground. Thus F. Hasenöhrl, in his paper "Über die Grundgleichungen der elektromagnetischen Lichttheorie für bewegte Körper" published in 1902[73], criticized Cohn's theory saying that though it explained all experiences hitherto obtained, "it was not derived from whatever hypothesis." He was also dissatisfied with Hertz's theory because of its leading to crucial contradiction with the experience. He appreciated Lorentz's theory as agreeing with all experiences except for Michelson-Morley interference experiment, and attempted to derive equations for moving bodies by assuming the rest ether but not adopting ion hypothesis. He obtained results which were similar to those of Lorentz, but as to the conception of electromagnetic field he appears to have been still on the line of thoughts of Maxwell and Hertz. For he had not yet definitely abandoned the view that the field was borne by both the ether and the matter.

That Cohn's theory was at that time regarded by some physicists as being effective may be evidenced by R. Gans's paper "Zur Elektrodynamik in bewegten Medien" which was published in 1905.[74] According to Gans, in the discussion to Abraham's paper presented to Naturforscherversammlung held at Karlsbad in 1904, Planck raised a question what a result would be obtained if one took as the basis of Abraham's "dynamics of electrons" Cohn's theory instead of Lorentz's one. Gans's paper was intended to extend Cohn's theory so as to embrace the problems concerning dispersion, and to give answer to Planck's question as well. As to the latter, he obtained the same velocity dependence of longitudinal mass as Abraham did, but could not calculate transversal mass because Cohn's theory lacked the concept of electromagnetic momentum. In order to extend Cohn's theory, Gans tried to derive Cohn's equation on the basis of electronic constitution of matter. The result Gans obtained was to some extent satisfactory. In some respects it was even more satisfactory than Lorentz's theory. But Michelson-Morley and Trouton-Noble experiments had to be explained by the influences of the presence of air and consequently were concluded to give positive results when carried out in an absolute vacuum. As to the ponderomotive force of electromagnetic origin that acts on a body in the vacuum, it involves much more difficulties. Thus Gans ended his paper without reaching any definite conclusion as to the superiority of Lorentz's or Cohn's theory to the other.

Before closing this section, Cohn's equation may be compared with those derived from the theory of relativity. According to the theory of relativity, [75] Maxwell's macroscopic equations will hold good in every inertial system of coordinates if one defines the field quantities by means of two tensors F_{ik} and H_{ik} as

$$i\boldsymbol{E} = (F_{41}, F_{42}, F_{43}), \qquad \boldsymbol{B} = (F_{23}, F_{31}, F_{12}),$$

$$i\boldsymbol{D} = (H_{41}, H_{42}, H_{43}), \qquad \boldsymbol{H} = (H_{23}, H_{31}, H_{12}).$$

But it is only in a system of coordinates which is fixed to the body that $\boldsymbol{E}$ and $\boldsymbol{H}$

appearing in the equations represent the forces acting on unit charges of electricity and magnetism. In a system of coordinates to which the body moves with velocity $\boldsymbol{u}$, the forces of that kind are represented by vectors

$$E^* = E + [\boldsymbol{u}B]$$

and

$$H^* = H - [\boldsymbol{u}D].$$

Using E^* and H^*, Maxwell equations are transformed into

$$\text{rot}(E^* - [\boldsymbol{u}B]) = -\partial B/\partial t - \boldsymbol{u}\,\text{div}\,B,$$
$$\text{rot}(H^* + [\boldsymbol{u}D]) = \partial D/\partial t + \boldsymbol{u}\,\text{div}\,D + \boldsymbol{i}.$$

D and B are expressed as

$$D = (1, k, k)\{\epsilon E^* - \frac{1}{c^2}[\boldsymbol{u}H^*]\},$$
$$B = (1, k, k)\{\mu H^* + \frac{1}{c^2}[\boldsymbol{u}E^*]\}.$$

Since the E and H in Cohn's equations should be regarded as the same physical quantities as E^* and H^* here, Cohn's equations agree with the relativistic equations insofar as $k = 1/(1 - u^2/c^2)$ may be approximated by 1.

10. De-mechanization of the Ether

Thus far we have seen that about the turn of the century physicists' conceptions about the electrodynamics still were not uniform. But despite the variety of conceptions, the development of electrodynamics in 1890's produced one common result. It is the de-mechanization of the concept of the ether.

The substance *ether* had got rooted in physics when the wave theory of light was established: "Since where exist vibrations there must be something that vibrates, the propagation of light proves the existence of a medium in the seemingly void space between perceptible bodies."[76] The ether thus supposed actually to exist was naturally to be mechanical substance on account of the function it was charged to perform. When Hertz's experiment had convincingly demonstrated that this ether also functioned as the medium of electromagnetic actions, the properties of the luminiferous ether became to have to embrace the electromagnetic phenomena. But despite of its enlarged functions, the ether still remained a mechanical substance. And in Maxwell's original conception, the electromagnetic field was considered as a state of medium in which a kind of elastic strain was produced.

From this view comes out his fundamental idea that the electric force E produces in a medium a displacement D, the magnitude of which is, as is the case with elastic body, proportional to the force E.

It was therefore very natural that those who at the beginning of 1890's undertook the theoretical elaboration of the electrodynamics payed good deal of attention to the mechanical model of the medium of electromagnetic actions. It is very well known that the construction of mechanical model was vigorously pursued among British scientists.[78] In his treatise *Vorlesungen über Maxwell's Theorie der Elektricität und Lichtes* (3 Bde, Leipzing, 1891 - 93), Boltzmann considered extensively the construction of mechanical systems that were dynamically equivalent to the electromagnetic field. But on the other hand, not a single physicist, while considering the ether as a mechanical substance, did not attach so much importance to the construction of mechanical model. For example, in his treatise cited in §1, Föppl on one hand regarded it as an essential feature of Maxwell's theory that the electric and magnetic forces were considered to be produced by elastic strains in the medium, while on the other hand he remarked that it was doubtful that the hypothetical mechanism imagined by Faraday and Maxwell of producing strains actually corresponded to the reality and that what was important was the elastic energy stored up in dielectrics.

Even Larmor, who made a considerable effort to develop a dynamical theory of the ether, cannot be definitely said to have had the mechanical model for his ultimate goal. He writes in his first abstract paper that the discussion of what properties may be assigned to the medium in order to realize the Lagrangian required by the theory is rather of the nature of illustration and explanation.[80] He also says in the third abstract paper of 1897 that the significance of mechanical model lies in "that it gives an insight into the character of the formal relations that are possible or probable between the actual physical quantities involved in it."[81] Larmor's reserved attitude towards model construction seems to have followed from his point of view that the ether is the fundamental and ultimate reality. He considered material particles as centres of strain in the ether. And the ether was assumed to be a pure continuum and to have certain inertia and elasticity for its sole ultimate and fundamental properties. Properties of common macroscopic matter were, in his view, merely secondary ones which originated from interactions between material particles transmitted by the ether.[82] From this point of view, it naturally loses meaning to conceive a mechanical model which explains the properties of the ether in terms of those mechanical properties which are observed in usual matter. But at the same time, Larmor ardently argued here and there about particular mechanical models.

His ultimate and real conception is, therefore, difficult to be guessed precisely. But in any case, the certain thing is that for him the ether was to the final analysis a dynamical substance. That the electromagnetic theory could be presented in a Largrangian form seemed to him to provide a firm ground thereof. In this respect Larmor differed definitely from Wiechert who, as we will soon see, also held the view that the ether was more fundamental than the matter.

The development of electromagnetic theories up to about 1900, produced a considerable change in physicists' view of the ether. Mechanical character of the ether had almost entirely disappeared. To symbolize this vicissitude is the emergence of the elctromagnetic view of nature mentioned in § 8. This view of nature was to re-construct the mechanics on the ground of electrodynamics rejecting positively the mechanical interpretation of properties of the ether. The electromagnetic view of nature was maintained not only by Wien and Abraham in Germany but by J. J. Thomson in England as well. In his lectures delivered at Yale University in 1903, he developed a view that all the reality was an aggregate of electric lines of force.[83)]

The electromagnetic view of nature was foreshadowed as early as 1894 in Wiechert's address.[84)] Emphasizing that most of forces in the nature such as electromagnetic actions and the light had been yielded from the matter to the ether, he raised a question what then was the matter which was perceptible to our senses. He argued that what still remained in our hands in order to endow the matter with an independent *Dasein* was its mass alone. But even the mass, at least a part thereof, is of electromagnetic nature and consequently has its origin in the ether. Therefore one rather ought to attempt a unification of electricity and matter at higher level by considering the electric charge of atom or atom groups as the ultimate attribute of matter. Whatever the matter may be, "the only thing that we can assert with certainty about the matter which is perceptible to our senses is that it would be an aggregate of centres of excitement, modified positions; on the other hand the ether is the proper bearer of the material world." Wiechert thus concluded: "where our naive sensory observation beholds nothing, our spiritual eyes see in the "ether" a "being" of fundamental significance for the world."

Wiechert made also considerations of the concepts of matter and of the ether in both papers of 1898 and 1900. At the end of the paper of 1898 is inserted a section entitled "Einige Bemerkungen über die Begriff 'Materie' 'Aether' und 'Elektricität', where he asserts that the matter is not an independent substance but it means an aggregate of special *Dinge*, where as the ether means a possibly really existing *Stoff* which corresponds to the electromagnetic field without motion and is its bearer.[85] But Wiechert reserved his attitude towards the view that sensibly perceptible matter was merely a particular form of the ether (the view, for example, of Lord Kelvin). The paper of 1900 begins with a section entitled "Grundlagen der Theorie" where he alludes to the concepts "ether" and "matter".[86] Because the aberration and Fizeau's experiment had revealed that a moving body did not carry together the light waves, it had become necessary to assume within the matter a bearer of electromagnetic action which was independent of matter. This was the way in which the rest ether had been introduced. But both the matter and the ether, argued Wiechert, were for us merely pictures (*Bilder*) which we saw in the nature from our human standpoint. He concluded that the question what they corresponded to in reality was opened for future. Thus it may be said that the ether as conceived by Wiechert is rather of the nature of conceptual construct than a mechanical substance.

A similar but more redical view was advanced by Poincaré. His view is most comprehensively presented in the chapters 10 and 12 of his *La science et l'hypothèse* (1902). The chapter 10 of this book is the latter part of his address "Relations entre la physique expérimentale et la physique mathématique" which was delivered at the International Congress of Physics held at Paris in 1900. The former part of this address constitutes the chapter 9 of the same book. The chapter 12 named above consists of excerpts from the introductions to his treatises *Théorie mathématique de la lumière* (Paris, 1889) and *Electricité et optique* (Paris, 1901).

Poincaré first criticizes Kelvin and Larmor who consider the matter as an aggregate of vortices or strain centres of the ether. They have considered the ordinary matter only as an apparent false matter but, he asks, by what right do they apply to the ether the mechanical properties observed in ordinary matter, which is but false matter?[87] He also says: "does our ether actually exist?" People have believed, argues Poincaré, that the light coming from distant stars revealed the ether filling the interstellar space and that Fizeau's experiment showed us that the ether and the ordinary matter penetrated each other. And in Lorentz's theory, the ether has been called for in order to save the principle of action and reaction. But, in view of Poincaré, "it is very likely that things do not happen in this way" (i.e. the compensation by the ether of the reaction).[88] May it be so, if any influence of the motion of the earth to optical and electromagnetic phenomena might be found, it would become necessary to introduce the ether as the reference frame against which the earth moves. But this prospect too seemed to Poincaré doubtful. He rather believed that however accurate experiment might be done, any influence would by no means be found. This was the prediction which was later to be considered as enunciating the principle of relativity.[89] But the legitimacy of such an assessment is very dubious.[90] We cannot, however, enter here into detailed discussion of this point. In any way, considering these situations Poincaré declares "whether the ether exists or not matters little." It is only a convenient hypothesis for the explanation of phenomena and "some day, no doubt, the ether will be thrown aside as useless."[91) He also asserted that we had to satisfy ourselves with the theory being formulated on the basis of the principle of least action, because the true and only aim of physics is not the mechanical explanation but the unity.[92)

Though Poincaré in this way reduced the ether to a mere "convenient hypothesis", he never ceased to hold to the ether, at least as a hypothesis. But in Germany some physicists went further and dared to assert that the concept of ether was unnecessary. For example, Cohn's electrodynamics was formulated without any reference to the ether, the fundamental equations being postulated at the outset. To his paper of 1901 Cohn appended an account of his view as to the ether. He stated there as follows: In his formulation "there is no reason to introduce an 'ether' beside the ponderable body. It is sufficient to assume that the electromagnetic energy can propagate also through a space which is void of matter."[93) But in order to make our fundamental assumption more intuitive, we may introduce something which exists everywhere and permeates even within a material body. We will call it "ether" without retaining any notion which in the course of time has been attached to this word. The ether as conceived by Cohn, therefore, was not to correspond to anything which really exists. His opinion about the "ether" was that a metaphorical word like this would not add slightest to the theory but might be of heuristic value for the further development of the theory.

German physicists appear generally not to have clung to the ether as a mechanical substance. For example, in his textbook on optics published in 1900, P. Drude wrote: "the ether is conceived to be not a substance but merely space endowed with certain physical properties."[94) This was written in connection with his adoption of the hypothesis of rest ether in dealing with optical phenomena in moving bodies. His argument for this hypothesis was that if the ether was of such character as described above, the conception of an ether absolutely at rest was the most simple and most natural.

From above observations it may be concluded that at about the same time as Lorentz's theory had become widely accepted, the ether was deprived entirely of mechanical character and moreover doubts arose as to even its being independent substance. As Drude's words suggest, two processes of the acceptance of Lorentz's theory and of the de-mechanization fo the concept of ether were interrelated to each other.

11. Conclusions

In the early years of 1890's Maxwell's theory at last became to absorbe much interest of all physicists owing to, among others, Hertz's successful experiment. But the conception of electromagnetic field in those days was considerably different from that of today. But after the lapse of over ten years, at the time just before the advent of the theory of relativity, a conception of classical electromagnetic field which is very close to that we have today is seen to have been established. In the following we will consider some features of the development during this one and a half decades as a whole.

When, entering into 1890's, Maxwell's theory had begun to be recognized widely, what seemed to physicists most essential was that various electromagnetic phenomena such as displacement current could take place even in dielectrics which "for the old electricians ... had been entirely inert and whose role had been limited to the opposition to passage of electricity." [95] In view of the electromagnetic theory of light and of the fact that light passes through the vacuum, the dielectrics mentioned here ought to include the ether as well. Such being the general circumstances at that time, it was very natural that most of people saw in the dielectrics as material substance the bearer of whole electromagnetic phenomena. This explains the conception, on which are based various systems of electrodynamics, that the field is borne partly by common ponderable matter and partly by the ether. For, if the gist of Faraday-Maxwell's theory lay in the dielectrics bearing and transmitting electromagnetic processes, the electromagnetic field would necessarily be considered as being borne by a medium and as representing a particular state of the medium. In fact, such was Maxwell's conception of the field. As stated in §2 and 3, Hertz's electrodynamics too was derived from the same point of view. Hertz declared unequivocally: "the lines of force simply represent a symbol for special conditions of matter."

There were several different views as to what parts of the field are borne by the matter and the ether respectively. In the case of Hertz's electrodynamics, the field is borne entirely by the ether in the vacuum, where as in a ponderable body it is borne entirely by the matter. For, his assumption that the ether within a body shares the whole motion of the body implies in the last analysis not to distinguish the roles of the matter and of the ether within a body. The point of view that the electromagnetic processes within a body is wholly borne by the matter have been inherited by Cohn even after 1900. But the lapse of time over ten years from Hertz to Cohn brought about considerable change in the conception of the ether. Hertz took for granted the existence of a substance called ether as the medium of electromagnetic processes although he did not enter into the problem of the constitution of the ether. Contrary to this, Cohn declared definitely that there is no need to introduce an "ether" as really existing entity.

Larmor and Walker took the position that there co-exist the ether and the matter in the interior of a ponderable body and that one need to make a distinction between electromagnetic fields borne by either of them. They at the same time stressed that the theory should be based on the atomic structure of matter, but unlike Lorentz, they were not lead by this to separating completely the field from the matter. Why was it so? To answer this question, it is well to think of the difference in their conceptions of the ether. Lorentz's ether was no more than the seat of the electromagnetic field and did not maintain slightest of the mechanical character. He never went into the consideration of the nature and the constitution of the ether. Larmor was contrary in this very respect. Though he remarked that the construction of mechanical model was rather the nature of illustration or explanation, he did not cast slightest doubt about the reality of the ether as a mechanical substance. His statement that only the ether has the inertia and the elasticity as the ultimate and fundamental properties suggests the idea that the notion of matter that have been familiar to us should properly apply itself only to the ether. Larmor thus may be said to have taken the way to have the matter absorbed in the ether instead of separating the ether from the matter. The ambiguity of his position has arisen from this.

Compared with various views described above, the leading significance of the theory of electrons in the development of the electromagnetic theory in the period under consideration makes itself manifest. The conception of electromagnetic field as we have today was formed, in the framework of classical theory, by the theory of electrons. But in spite of the superiority of its point of view, the theory of electrons had to wait for a while until it obtained general acceptance. In 1898 and 1900, Wiechert repeated in his papers that the true significance of the theory of electrons had not yet fully recognized. But the situation turned at about 1900 and the theory of electrons began to be accepted rapidly. This was caused most likely by the experimental establishment of the electron during few years from 1897 to 1900. On the other hand, the theoretical consideration that Hertz's theory was decisively incompatible with Fizeau's experiment prompted the recognition of the theory of electrons. Poincaré, Walker, Wien, Hasenöhrl, and even Cohn indicated that Hertz's theory had to be modified in this respect.

Almost simultaneously with the theory of electrons being widely accepted as most reliable theory, the notion of the ether as a mechanical substance became rejected. Lorentz deprived the ether of its mechanical character and endowed it only with the function of being the seat of electromagnetic field. The de-mechanization of the ether was a necessary consequence of his adopting the hypothesis of rest ether. But at the same time the latter was made possible by the former. They were intrinsically related to each other. The de-mechanization of the ether called forth a further radical view. Poincaré degraded the ether to a mere "convenient hypothesis" and even predicted that some day the ether would be thrown aside as useless. Cohn declared it unnecessary to introduce an ether, and Drude stated that the ether was not a substance but merely space endowed with certain physical properties. As a conclusion we may say that the conception of electromagnetic field as we have today was finally established shortly after 1900.

The establishment of the modern conception of electromagnetic field provided the ground for Einstein's theory of relativity. Einstein, however, does not appear to have followed up the evolution of electrodynamics described thus far. But Lorentz's rest ether alone was sufficient for him. He could understand that according to this conception "the immobility was the only remaining mechanical property" [96] of the ether. What Einstein had to do was to complete the "electro-magnetization" of the ether by depriving the ether of its final mechanical property, the absolute rest. It might inversely be said that his attempting to carry the electromagnetic grasp of the ether to its extreme might enable him to deprive the ether of its mechanical property, the absolute rest. The principle of relativity for the electromagnetic theory could be postulated only on this ground.

Thus Einstein's theory may be said to succeed Lorentz's theory in the sense that it has completed the electromagnetization of the ether. But it does not so in respect of the problems concerning influences of the motion of the earth to electric and optical phenomena. Einstein's special theory of relativity has its root rather in a theoretical consideration about the formulation of electrodynamics of moving bodies. Here it should be noted that Lorentz's theory was by no means an "electrodynamics of moving bodies". The electrodynamics of moving bodies stood essentially on the ground of macroscopic grasp of the material body. Its aim was to consider the peculiar phenomena that took place in bodies in motion. Contrary to this, Lorentz's theory did not admit any macroscopic body at all, and only intended to consider from a moving system of reference the phenomena that were taking place in the ether at absolute rest. Once, as the theory of electrons did, one has decomposed moving bodies into the rest ether and flows of electrons and has regarded the electromagnetic processes as taking place in the rest ether, it would be mentally very difficult to conceive the principle of relativity for the electromagnetic theory. It is probably for this reason that by the word "principle of relativity" Poincaré could mean no more than that the future theory should *ex-*

plain the imperceptibility of motion relative to the ether.

But what has been stated in the last two paragraphs is no more than a surmise. More thorough consideration about the formation of the special theory of relativity shall be reserved to another occasion.

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