PROCESSO STOCASTICO AR(1) MARKOVIANO

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + W_t$$

$$\forall \phi_0 = 0 \quad 1 - \phi_1 \mathrm{B} = 0 \rightarrow \mathrm{B} = \frac{1}{\phi_1}$$

È STAZIONARIO $\left(\left|B_1\right| > 1\right)$ SE: $\left|\phi_1\right| < 1$

PARAMETRI

VALORE ATTESO

$$\mathrm{E}(Y_t) = \mathrm{E}(\phi_1 Y_{t-1} + W_t)$$

$$= \phi_1 \mathrm{E}\left(Y_{t-1}\right) + \mathrm{E}\left(W_t\right)$$

$$= \phi_1 \mathrm{E}\left(Y_{t-1}\right)$$

SE È STAZIONARIO

$$\mathrm{E}(Y_t) = \phi_1 \mathrm{E}\left(Y_t\right) \rightarrow \mathrm{E}(Y_t) = 0$$

FUNZIONE DI AUTOCOVARIANZA

$$\gamma_k = \mathrm{E}(Y_{t-k}, Y_t)$$

$$= \mathrm{E}(Y_{t-k}\left[\phi_1 Y_{t-1} + W_t\right]$$

$$= \phi_1 \mathrm{E}(Y_{t-k}, Y_{t-1}) + \mathrm{E}(Y_{t-k}, W_t)$$

$$= \phi_1 \gamma_{k-1} + \mathrm{E}\left[\phi_1\left(Y_{t-k-1} + W_{t-k}\right) W_t\right]$$

$$\forall k = 0$$

$$= \phi_1 \gamma_{k-1} + \phi_1 E(Y_{t-1}, W_t) + E\left(W_t^2\right)$$

$$= \phi_1 \gamma_{k-1} + \sigma_w^2$$

$$\forall k \neq 0$$

$$\gamma_k = \phi_1 \gamma_{k-1}$$

quindi:

$$\gamma_1 = \phi_1 \gamma_0$$
$$\gamma_2 = \phi_1 \gamma_1 = \phi_1^2 \gamma_0$$
$$\ldots$$
$$\gamma_k = \phi_1^k \gamma_0$$

$$se \;\; \phi_1 > 0 \rightarrow \gamma_k > 0$$

$$se \; \phi_1 < 0 \rightarrow \gamma_1 < 0$$

$$\gamma_2 > 0$$

$$\gamma_3 < 0$$

$$\ldots$$

FUNZIONE DI AUTOCORRELAZIONE

$$\rho_k = \frac{\gamma_k}{\gamma_0}$$

$$se \; k=0 \;\; \rightarrow \rho_0 = 1$$
$$se \; k \neq 0 \rightarrow \rho_k = \phi_1^k$$

quindi:

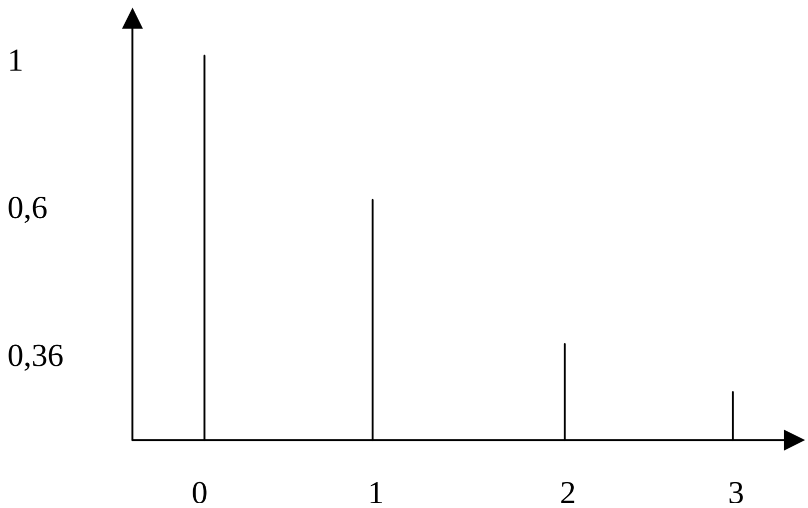
$$\rho_1 = \phi_1 \quad \textit{dipende dal segno di } \phi_1$$

$$\rho_2 = \phi_1^2 > 0 \textit{ non dipende dal segno di } \phi_1$$

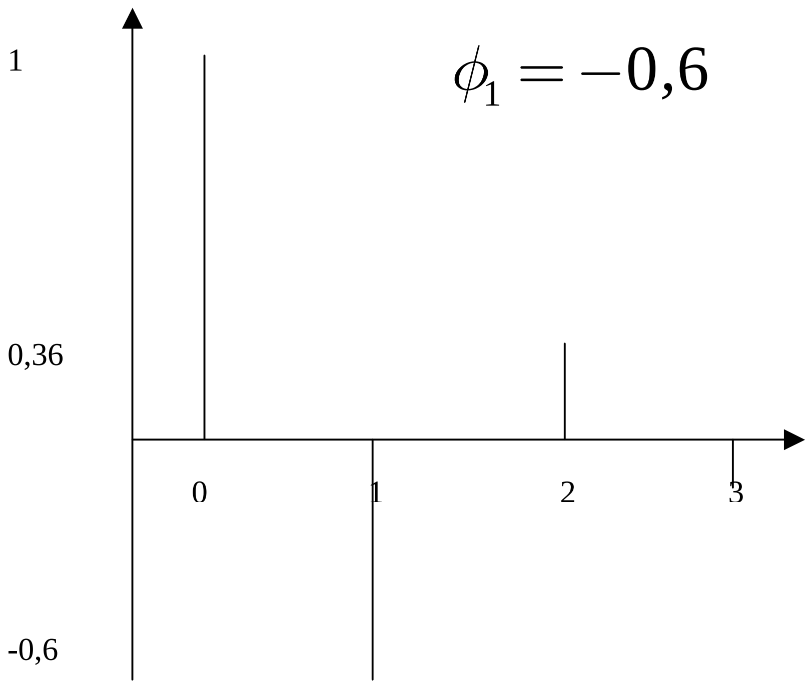
.....

CORRELOGRAMMA

$$\phi_1 = 0,6$$



$$\phi_1 = -0,6$$



SE: $\phi_1 = 1$

AR(1) NON STAZIONARIO → **Random Walk**

$$Y_t = Y_{t-1} + W_t$$

$se \ \ \phi_0 = 0$

$se \ \ \phi_0 \neq 0$ **Random Walk con drift**

$E(Y_t) = \mu$

$Var(Y_t) = t\sigma_w^2$ varia al variare del tempo

$\rho_k \rightarrow 1 \ \ \forall t \rightarrow \infty$ processo a memoria infinita

$$Y_t - Y_{t-1} = W_t \rightarrow \Delta Y_t \ \textit{è stazionario}$$

PROCESSO STOCASTICO AR(2)
DI YULE

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + W_t$$

IL PROCESSO E' SEMPRE INVERTIBILE MA E' STAZIONARIO SE:

$$1 - \phi_1 B - \phi_2 B^2 > 0$$

$$|B| > 1 \rightarrow \phi_1 + \phi_2 < 1 \text{ e } \phi_2 - \phi_1 < +1 \text{ e } -k\phi_2 < 1$$