APPLICAZIONI LINEARI

Stabilire se le seguenti funzioni sono applicazioni lineari e, in caso affermativo, determinarne nucleo e immagine.
Calcolare immagine e controimmagine dei vettori indicati.

- $f(x,y) = (3x-2y,\ x+5y)$

  $f(1,3) = ?$ $\qquad f^{-1}(6,2) = ?$

- $f(x,y) = (x+9y,\ 2x^3 - xy)$

- $f(x,y) = (x+3y,\ x+5y,\ 2x+2y)$

  $f(2,-1) = ?$ $\qquad f^{-1}(4,6,4) = ?$ $\qquad f^{-1}(0,1,0) = ?$

- $f(x,y) = (2x+6y,\ 2x+y,\ x-7)$

- $f(x,y,z) = (x,\ y+2,\ z-2x+8)$

- $f(x,y,z) = (2x+y,\ x+z,\ 2x+y-2z)$

  $f(1,-1,3) = ?$ $\qquad f^{-1}(1,0,4) = ?$

- $f(x,y,z) = (x+y-2z,\ y+z,\ x+3y+5z)$

  $f(1,0,0) = ?$, $f(0,1,0) = ?$, $f(0,0,1) = ?$ $\qquad f^{-1}(4,2,0) = ?$

- $f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+4y \\ -6x-24y \end{pmatrix}$, $\qquad f^{-1}\begin{pmatrix} 3 \\ h+1 \end{pmatrix} = ?$ al variare di h.

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- $f(x,y) = (-x+3y,\ x-3y,\ -2x+6y)$

  $f^{-1}(1,0,h) = ?$ al variare di h.

- $f(x,y,z) = (x-z,\ -x+y+5z)$

  $f^{-1}\left(-\frac{1}{2}, 1\right) = ?$ $\quad f(2,1,0) = ?$

- $f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}x - y + z \\ -x + 2y - 2z \end{pmatrix}$, $\quad f^{-1}\begin{pmatrix} 3 \\ -6 \end{pmatrix} = ?$ $\quad f^{-1}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = ?$

  $f\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} = ?$

- $f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & -1 & 0 \\ 2 & 0 & 2 \\ 1 & 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $\quad f^{-1}\begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix} = ?$ $\quad f\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = ?$

- $f(x,y,z) = (x-y+2z,\ -3x+3y-6z,\ -x+y-2z)$, $\quad f^{-1}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = ?$ $\quad f^{-1}\begin{pmatrix} 4 \\ -12 \\ -4 \end{pmatrix} = ?$

- $f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 3 \\ -1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & -1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $\quad f^{-1}\begin{pmatrix} 4 \\ 0 \\ 2 \\ 2 \end{pmatrix} = ?$ $\quad f^{-1}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = ?$

- $f\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 2y + z + t \\ 3x - y + z + 2t \\ x - y + t \end{pmatrix}$ $\quad f\begin{pmatrix} -2 \\ 6 \\ \frac{1}{3} \\ 1 \end{pmatrix} = ?$ $\quad f^{-1}\begin{pmatrix} \frac{1}{4} \\ 0 \\ -1 \end{pmatrix} = ?$

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- $f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $f^{-1}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = ?$ $f^{-1}\begin{pmatrix} 2 \\ 0 \\ 3 \\ 7 \end{pmatrix} = ?$

  $f(0,1,0) = ?$ $f(1, \sqrt{2}, \sqrt{5}) = ?$

- $f(x,y,z,t) = (2y+z+t,\ 3x-y+z+2t,\ x-y+t)$

  $f(5,0,0,0) = ?$ $f^{-1}(1,0,3) = ?$

- $f\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 1 & 1 & 2 & 6 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$, $f^{-1}\begin{pmatrix} 2 \\ 1 \\ -6 \end{pmatrix} = ?$ $f\begin{pmatrix} 2 \\ -1 \\ 7 \\ 3 \end{pmatrix} = ?$

- $f(x,y,z,t) = (2x-4z,\ -x+2z,\ 4x-16z,\ x-4z)$

  $f(0,1,0,1) = ?$ $f^{-1}(1, -\frac{1}{2}, 2, \frac{1}{2}) = ?$ $f^{-1}(1,-1,1,-1) = ?$

- $f\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} x+z+t \\ -y-z+t \\ x-y+2t \\ 2x+y+3z+t \end{pmatrix}$, $f^{-1}\begin{pmatrix} 3 \\ -1 \\ 2 \\ 7 \end{pmatrix} = ?$, $f\begin{pmatrix} 4 \\ 0 \\ -\sqrt{2} \\ \sqrt{2} \end{pmatrix} = ?$

- $f(x,y,z,t) = (x-z+t,\ y+z,\ -y,\ 2x+2z)$

  $f(1,0,0,1) = ?$ $f^{-1}(0,1,k,3) = ?$

- $f\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 5 & 4 & 0 \\ 0 & 0 & 2 & 2 \\ -1 & 0 & 3 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$; $f^{-1}\begin{pmatrix} 1 \\ 0 \\ 3 \\ -3 \end{pmatrix} = ?$ $f\begin{pmatrix} 1 \\ 0 \\ 4 \\ 8 \end{pmatrix} = ?$